Unawareness and Strategic Announcements in Games with Uncertainty *

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February 19, 2008

Abstract

This paper studies games with uncertainty where players have different awareness regarding a chance player's moves (contingencies). An announcer, who is fully aware of the contingencies, can announce some of them to an unaware decision maker (DM) before the DM takes an action. An equilibrium concept and a refinement is introduced to study the way the DM generates her belief on her extended awareness.

JEL-*Codes*: C72, D82, D83

Keywords: Unawareness, games with uncertainty, communication

*I am indebted to Ariel Rubinstein for his guidance at every stage of this work. I am also grateful for suggestions and comments to Emel Filiz-Ozbay, David Pearce, Burkhard Schipper, Andrew Schotter, participants in TARK 2007 and in Microeconomic Theory Workshop at New York University.

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1 Introduction

In many economically relevant problems, agents interact without having a clear picture of the exact nature of the problem: they may not be aware of all the relevant aspects of uncertainty. For example, an investor in the stock market hardly foresees all the relevant contingencies that affect the future value of the stocks. Similarly, the financier of an innovation, an insurance buyer, a teenager deciding on her career path can be thought of as the agents dealing with unawareness. On the other hand, stock brokers, inventors of innovations, insurance companies and advising parents foresee more contingencies than their counterparts. Due to the strategic nature of these environments, the aware agents may inform the unaware ones regarding the contingencies that were not initially foreseen by them. The key question is how an unaware agent assigns probabilities to the newly announced contingencies after becoming aware of them. This paper introduces a game theoretical setup and an equilibrium notion that will enable to analyze these kinds of situations.

Specifically, this paper considers games with uncertainty where players have different awareness regarding the contingencies (the moves of a chance player - nature). There will be two players such that one of them (the announcer) is fully aware while the other player (the decision maker - DM) is unaware of some moves of nature. Furthermore, the announcer knows that the DM is unaware of these moves. However, the DM is not aware that the announcer is aware of some additional moves of nature. In their awareness, both players know the payoffs associated with the moves. Initially, each player has a probability distribution on the moves of nature that they are aware of.

Moreover, the announcer can make an announcement and inform the DM about some moves of nature that were not originally in the DM's state of mind. There is no language barrier between the players. In other words, when some contingencies are announced, the DM comprehends not only the announced moves of nature, but also the payoffs corresponding to those moves. After the announcement, the DM's action and the realization of nature determine the payoffs the players receive.

In the context of the examples above, a fully informed stock broker who is advising an investor, may strategically decide which contingencies to make the investor aware. The investor's portfolio choice and realization of the contingencies determine the return. After listening to the broker, the investor extends her awareness to consider the newly announced contingencies. In order to evaluate stocks within her extended awareness, the investor needs to assign probabilities to those newly mentioned contingencies. This paper proposes a belief formation that is a part of the equilibrium concept.

Standard game theory assumes that players are fully aware of the game (see e.g. Rubinstein, 1998). Indeed, Dekel, Lipman and Rustichini (1998) showed that standard state spaces preclude non-trivial unawareness. Recently, Heifetz, Meier and Schipper (2006) and Li (2006b) introduced the generalized state space constructions that captured non-trivial unawareness among multi-agents as a generalization of Aumann (1976). Motivated from the possibility of non-trivial unawareness, Copic and Galeotti (2006) and Halpern and Rego (2006) considered extensive form games where players might be unaware of the complete structure of the game (such as unawareness regarding actions of other players) (see also Heifetz, Meier and Schipper, 2007, and Li, 2006a). These papers described the game from each player's point of view and introduced a solution concept which is a generalization of Nash equilibrium. Alternatively, Feinberg (2004) epistemically defined unawareness and reasoning about unawareness in dynamic games. He showed that cooperation in a finitely repeated prisoner's dilemma game could be achieved when a player was not fully aware of all the aspects of the game. Feinberg (2005) studied normal form games with incomplete awareness based on the epistemic formulation in Feinberg (2004). He defined the awareness of players regarding the other players' awareness and extended Nash equilibrium as a solution concept.

In the unawareness literature, the players can extend their awareness only if they come to a node in the game that they were not aware of before. If this happens, then the players fully understand those new aspects of the game. However, in many real life examples people communicate with each other regarding the nature of the game they are playing. Before an investor experiences a surprise by the nature at an unforeseen realization of the relevant contingency, some financial advisor may mention those contingencies in his media cover. Once the investor becomes aware of new contingencies, still she needs to determine what to believe. This paper not only allows for extending awareness through communication but also proposes an equilibrium concept that includes a belief generating process where the unaware player takes into account the fact that the announcement is strategic. That is, the DM thinks that the announcer is expecting higher payoff from his announcement than from another, less informative announcement.

In Section 2, the formal model is introduced. In Section 3, an equilibrium concept for this type of games is defined, and the existence of the equilibrium is studied. A refinement in this setup is suggested. In Section 4, the difference between assigning zero probability and unawareness is discussed, and also the concept of contingency in this setup is related with the generalized state space notion. Some possible applications of the setup are then provided in the conclusion. In order not to alter the flow of the paper, the relaxations of the assumptions are explained in the footnotes.

2 The model

There are two players: an announcer and a DM, indexed by 1 and 2, respectively. The set of all moves of nature (contingencies) is a finite set Ω, and the contingencies are distributed by π where $\pi(\omega) \neq 0$ for any $\omega \in \Omega$.

Players have different awareness regarding the contingencies. The announcer is aware of the full set of contingencies [1], Ω, and believes that the distribution on Ω is π. On the other hand, the DM is only aware of Ω_o, which is a non-empty subset of Ω. The DM believes the conditional distribution of π on Ω_o, $\pi(.|\Omega_o)$ [2]. Moreover, the announcer is aware of the DM's limited awareness, while the DM is unaware of the fact that the announcer has superior awareness. Rather, the DM perceives that the announcer is aware of Ω_o and $\pi(.|\Omega_o)$. The strategy of the announcer is an announcement regarding the moves of nature that the DM is unaware. The announcer does not observe the realization of nature before the announcement. Therefore, $\mathcal{M} := 2^{\Omega \setminus \Omega_o}$ is the set of all strategies of the announcer.

[1] Announcer does not need to be aware of all the contingencies. Since there is no source of information for the announcer to extend his awareness, Ω can be interpreted as the set of contingencies announcer is aware of.

[2] It is possible to set the initial belief of the DM arbitrarily. In that case conditional on Ω_o the DM and the announcer will not hold common beliefs. Given these first order beliefs, if the higher order beliefs are defined accordingly, all the arguments of this paper will still hold. In order not to introduce additional notation due to higher order beliefs, it is assumed that the DM and the announcer agree on the conditional distribution on the initial awareness of the DM. In all the examples in the paper, Ω_o is a singleton to highlight that the initial belief on Ω_o is not deriving the relevant arguments.

After an announcement, in addition to what she is already aware of (Ω_o), the DM extends her awareness to include all newly announced contingencies [3]. Then, she takes an action from a finite action set, A, which is the same set independent of the announcement. The strategy of the DM is a decision function, $d : \mathcal{M} \rightarrow A$, which specifies the action of the DM after each announcement. It is important to note that since the announcer is aware of Ω but the DM is not, the decision function can be thought of as the announcer's belief regarding the behavior of the DM after each announcement; the announcer's belief is confirmed by the action of the DM.

The payoffs received by both players depend on the realization of nature and the action chosen by the DM. Formally, the payoff of player i is determined by utility function $u_i : \Omega \times A \rightarrow \mathbb{R}$ for $i = 1, 2$. Observe that utility of the DM is defined on Ω even though she is not aware of Ω initially. When the DM becomes aware of a contingency, she also becomes aware of the associated payoffs.

In order to illustrate the notions introduced so far, an example is constructed:

Example 1 *Let* $\Omega = \{\omega_1, \omega_2\}, \Omega_o = \{\omega_1\}$, *and* $\pi(\omega_1) = \pi(\omega_2) = 0.5$.

The DM has two actions: left and right. The payoffs are as follows:

		Actions	
		left	*right*
Contingencies	ω_1	*1, 1*	*0, 0*
	ω_2	*0, 0*	*2, 2*

[3] Alternatively, the announcement of a contingency may make the DM consider some additional unannounced contingencies as relevant. One can easily show that all the statements of the paper would still hold when a remind function which is commonly known by both the DM and the announcer is defined.

Figure 1 is the game the DM understands initially. Figure 2 is the game that will be understood by the DM when the announcer announces $\{\omega_2\}$.

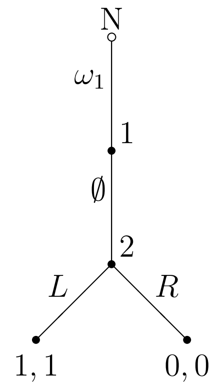

Figure 1: Game that is initially understood by the DM.

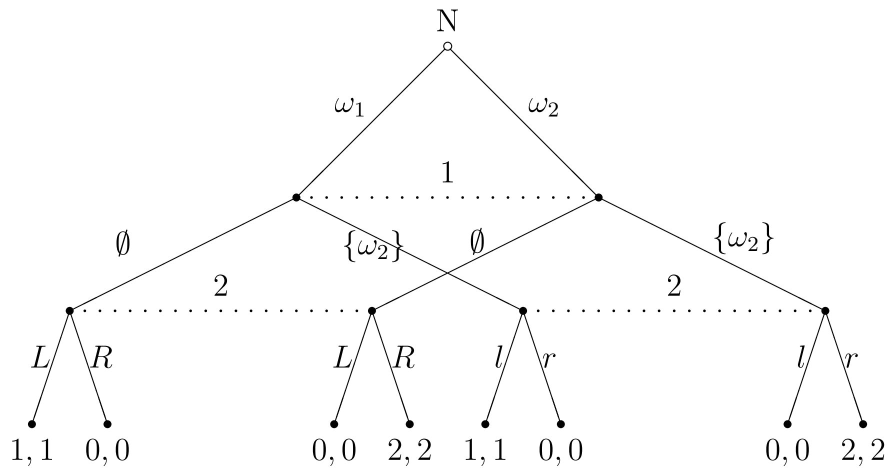

Figure 2: Game that will be understood by the DM when $\{\omega_2\}$ is announced.

3 Solution concept

Standard game theory assumes that players' awareness regarding the moves of nature cannot change throughout the game. Relaxing this assumption prevents us from using a solution concept from standard game theory. This issue will be discussed in detail in

Section 4.

A solution concept imposes some conditions on an assessment, (M, d, F), which is a triplet containing strategies of each player, M and d, as well as a belief function F. A belief function, F, assigns to each announcement, M, a probability distribution, F_M, on the union of the sets of contingencies that are announced and the contingencies that are in the initial awareness of the DM, i.e. $M \cup \Omega_o$. The announcer believes that the DM will play based on F, and the belief of the announcer will be confirmed by the DM in the equilibrium[4].

The deductive interpretation of the solution concepts in standard game theory assumes that each player deduces how the other player will act simply from the rationality principles (see e.g. Osborne and Rubinstein, 1994). Similarly, the solution concept in this paper can be interpreted as the deductive interpretation within the awareness of the players. Both the announcer and the DM evaluate any strategy by calculating their expected utilities. For any action, $a \in A$, and an announcement, $M \in \mathcal{M}$, the expected utility of the announcer is defined as:

$$EU_1(a) := \sum_{\omega \in \Omega} u_1(\omega, a)\pi(\omega)$$

and the expected utility of the DM, with respect to the probability distribution, F_M, is defined as

$$EU_2(M, a|F_M) := \sum_{\omega \in M \cup \Omega_o} u_2(\omega, a)F_M(\omega)$$

[4]Potentially, the DM and the announcer can have different belief functions. By the existence of equilibrium, we will show that it is possible to find one belief function such that both the DM and the announcer commonly hold.

The first requirement, rationality, states that given the action of the other player and the belief function, both the announcer and the DM respond in order to maximize their expected utilities.

Definition 1 *An assessment* (M^*, d^*, F) *is* ***rational*** *if*

$$\begin{aligned} M^* &\in \arg\max_{M \in \mathcal{M}} EU_1(d^*(M)); \\ d^*(M) &\in \arg\max_{a \in A} EU_2(M, a|F_M), \textit{ for any } M \in \mathcal{M}. \end{aligned}$$

The DM knows that the announcer does not observe the realization of nature before the announcement. Therefore, an announcement informs the DM about the existence of those contingencies. After hearing an announcement, the DM understands that the announced contingencies are relevant. It is assumed that the announcer cannot announce an impossible contingency, and therefore the DM should believe that any announced contingency is possible.

Definition 2 *A belief function,* F*, has* ***full support*** *if* $\forall M \in \mathcal{M}$*, and* $\forall \omega \in M \cup \Omega_o$*,* $F_M(\omega) \neq 0$.

This is a reasonable restriction in a setting where the realizations of nature are independent events and announcement is made before uncertainty is resolved. Here, after any announcement there is no reason for the DM to question his initial conditional belief. For example, a first time roulette player, who thinks that there are only reds and blacks, beliefs that red and black are equally likely. However, in reality there are two more contingencies, 0 and 00 which are neither red nor black. If someone informs her the possibility of 0 and 00, she assigns some positive probabilities to red, black, 0 and 00; moreover in her new belief, she keeps assigning equal probabilities to red and black. Next, it is required that an announcement does not alter the relative weights of the contingencies in the DM's initial awareness. In other words, the conditional of the belief held after each announcement should agree with the initial belief.

Definition 3 *A belief function, F, **respects the initial belief** if*

$\forall M \in \mathcal{M}$, *and* $\forall \omega \in \Omega_o$, $F_M(\omega|\Omega_o) = \pi(\omega|\Omega_o)$.

The DM knows that the announcer is rational. So, the belief of the DM should justify the behavior of the announcer. In other words, after the announcement, the DM forms her belief so that the expected utility of the announcer, according to this belief, is not lower than the expected utility of any other announcement that is a subset of the current one. Since the DM can only reason within her revised set of contingencies, she cannot evaluate any other announcement that is not contained in this set.

Definition 4 *An assessment, (M^*, d^*, F), is **justifiable** if* $\forall M \subseteq M^*$,

$$\sum_{\omega \in M^* \cup \Omega_o} u_1(\omega, d^*(M^*)) F_{M^*}(\omega) \geq \sum_{\omega \in M^* \cup \Omega_o} u_1(\omega, d^*(M)) F_{M^*}(\omega)$$

In the above definition, the left hand side of the inequality corresponds to the announcer's expected utility from the point of view of the DM when M^* is announced. The right hand side is the expected utility of the announcer from the DM's point of view when any other subset of M^* is announced. Note that both of the expected utilities are calculated with respect to F_{M^*}, because while evaluating any subset of M^*, the DM is already aware of M^*. After hearing the announcement, M^*, the DM thinks that the announcer could have announced a subset of M^* but he preferred to announce M^*. Therefore, the justifiability requires that announcing M^* should be better than announcing any subset of M^* from the point of view of the DM.

Definition 5 *An assessment, (M^*, d^*, F^*) is **awareness equilibrium** if it is rational, justifiable, and F^* has full support and respects the initial belief.*

A natural question arising at this point is the existence of assessment that satisfies the equilibrium conditions.

Theorem 1 *Awareness equilibrium always exists.*

Proof. Under no announcement, the DM holds her initial belief, $\pi(.|\Omega_o)$. Set $d^*(\emptyset)$ as a maximal action for the announcer, among the actions that maximize the DM's expected utility.

Define the belief function, F, such that for every announcement, $M \in \mathcal{M}$, F_M respects the initial belief and assigns a small but non-zero probability to all contingencies in the announcement, M, to guarantee that the best response of the DM, $d^*(M)$, is one of the actions that maximizes expected utility of the DM under no announcement. Since A and $\mathcal{M}$ are finite, those probability distributions exist. Since for any $M \in \mathcal{M}$, $d^*(\emptyset)$ is a maximal among all $d^*(M)$ for the announcer, $M^* = \emptyset$. The assessment, $(\emptyset, d^*, F)$, is justifiable since $M^* = \emptyset$. ■

In the proof of Theorem 1, under no announcement, the DM takes an action which is a maximal action for the announcer, among the actions that maximize the DM's expected utility. This is a key point for the existence of equilibrium.

Example 2 *Let* $\Omega = \{\omega_1, \omega_2, \omega_3\}$, $\Omega_o = \{\omega_1\}$, *and* $\pi(\omega_1) = \pi(\omega_2) = \pi(\omega_3) = \frac{1}{3}$.

The DM has two actions: left and right. The payoffs are as follows:

		Actions: *left*	Actions: *right*
	ω_1	$1, 1$	$1, 1$
Contingencies	ω_2	$1, 0$	$0, 1$
	ω_3	$0, 0$	$2, -4$

The announcer prefers the DM to play "right". The DM is indifferent between "left" and "right" under no announcement. Next, it is shown that in any equilibrium of this game, $d^*(\emptyset) = right$.

Assume on the contrary that $d^*(\emptyset) = left$.

Clearly, $d^*(\{\omega_2\}) = right$ *and* $d^*(\{\omega_3\}) = left$.

If $d^*(\{\omega_2, \omega_3\}) = left$, *then* $M^* = \{\omega_2\}$ *which is not justifiable.*

If $d^*(\{\omega_2, \omega_3\}) = right$, *then* $M^* = \{\omega_2, \omega_3\}$ *which is not justifiable.*

To see that the announcement $\{\omega_2, \omega_3\}$ *is not justifiable under* $d^*(\{\omega_2, \omega_3\}) = right$, *let* $F_{\{\omega_2,\omega_3\}}(\omega_1) = p$, $F_{\{\omega_2,\omega_3\}}(\omega_2) = q$, $F_{\{\omega_2,\omega_3\}}(\omega_3) = 1 - p - q$ *for* $p, q \in (0, 1)$. $EU_2(\{\omega_2, \omega_3\}, right|F_{\{\omega_2,\omega_3\}}) \geq EU_2(\{\omega_2, \omega_3\}, left|F_{\{\omega_2,\omega_3\}})$ *since* $d^*(\{\omega_2, \omega_3\}) = right$, *so* $p + q - 4(1 - p - q) \geq p$, $4p + 5q \geq 4$.

Justifiability of $\{\omega_2, \omega_3\}$ *requires* $p + q \leq p + 2(1 - p - q)$, $2p + 3q \leq 2$, *which is a contradiction.*

Hence, for the existence of the equilibrium we must have $d^*(\emptyset) = right$ *which is a maximal action for the announcer among the ones that maximize expected utility of the DM.*

The proof of Theorem 1 suggests that there is always an awareness equilibrium in which the announcer does not make any announcement, and the DM remains within her initial awareness. In that equilibria, the belief is such that the newly announced contingencies are not probable enough to convince the DM to change her action in a favorable way. This behavior of the DM with the belief construction is rational. Still, we can push the mind process of the DM one step further and ask the DM to take into account that the announcement is driven from a strategic decision. Next, a refinement that addresses this point is suggested.

In Example 1, at the initial awareness of the DM, she plays "left" which gives some small utility to both players. At the contingency that the DM cannot originally foresee, w_2, playing "right" brings higher utility to both players. The equilibrium suggested by the proof of Theorem 1 makes the DM assign very small probability to this second contingency if she hears about it. Therefore, she keeps playing "left". Since the announcer cannot change the action of the DM by announcing the second contingency, he does not announce it in the equilibrium. In reality, if such a behavior is observed, then one cannot conclude that the DM is irrational. However, if a new contingency is announced, then the DM may reason why this new contingency is announced. When she starts reasoning, she may see that there is an action which improves the payoff of both players if the new announced contingency is probable enough. The DM may think that the announcer wants her to change her action by announcing the new contingency. If there is an extension of awareness, the DM should switch to another action by assigning high enough probability to the newly announced contingency.

Let $\Delta(M \cup \Omega_o)$ be the set of all probability distributions on $M \cup \Omega_o$. Let $\varphi(M, a|d^*)$ be the set of probability distributions with full support on $M \cup \Omega_o$, which make the action $a \in A$ maximal for the DM, respect the initial belief and justify the announcement, $M \in \mathcal{M}$, for a given decision function d^*. Formally,

$$\varphi(M, a|d^*) :=$$
$$\Big\{P \in \Delta(M \cup \Omega_o) : \quad P(\omega) \neq 0 \text{ for any } \omega \in M \cup \Omega_o;\ P(\omega) = \pi(\omega|\Omega_o) \text{ for any } \omega \in \Omega_o;$$
$$\sum_{\omega \in M \cup \Omega_o} u_1(\omega, a)P(\omega) > \sum_{\omega \in M \cup \Omega_o} u_1(\omega, d^*(M'))P(\omega) \text{ for any } M' \subset M;$$
$$\text{and } a \in \arg\max_{a' \in A} EU_2(M, a'|P)\Big\}$$

A probability distribution $P \in \varphi(M, a|d^*)$ is called a **reasonable probability distribution** that supports the action $a \in A$, after the announcement $M \in \mathcal{M}$, and $d^* : \mathcal{M} \to A$.

Definition 6 *An awareness equilibrium, (M^*, d^*, F^*) satisfies the* ***reasoning refinement*** *if for any non-empty $M \in \mathcal{M}$ and $a \in A \setminus \bigcup_{M' \subset M} \{d^*(M')\}$ such that $\varphi(M, a|d^*) \neq \emptyset$, $F^*_M \in \varphi(M, d^*(M)|d^*)$.*

The reasoning refinement states that when new contingencies are announced, the DM thinks that the announcer made this announcement to make the DM change her action: After an announcement, $M \in \mathcal{M}$, the DM considers all of the subsets of M and all of the actions she would have taken if these subsets were announced. Then, the DM asks the following questions: Is there an action, $a \in A$, that she would not play if one of the subsets of M was announced? If so, then is there a reasonable probability distribution for

M and a? If the DM answers these questions affirmatively, then after the announcement M, she should hold a belief that is a reasonable probability distribution for M and $d^*(M)$. Hence, she plays an action that would not have been played after any subset of M.

In Example 1, under no announcement, the DM plays "left". When $\{\omega_2\}$ is announced, the DM thinks that the announcer wants her to take another action, "right". The DM can support playing "right" by a reasonable probability distribution which assigns ω_2 a probability that is greater than or equal to $\frac{1}{3}$. So, the reasoning refinement disallows assigning a very small probability to ω_2, as the equilibrium in the proof of Theorem 1 suggests. Under this belief, the announcer prefers announcing $\{\omega_2\}$ to making no announcement. Therefore, no announcement is not always part of an awareness equilibrium that satisfies the reasoning refinement.

Theorem 2 *Awareness equilibrium that satisfies reasoning refinement always exists.*

Proof. Let F be the belief function constructed in the proof of Theorem 1.

Under no announcement, the DM holds her initial belief, $\pi(.|\Omega_o)$, and set $d^*(\emptyset)$, as a maximal action for the announcer among the actions that maximize the DM's expected utility.

Construct F^* and d^* inductively:

For $n \in \mathbb{N}$, let for any $M' \in \mathcal{M}$ s.t. $|M'| < n$, $d^*(M')$ and $F^*_{M'}$ be constructed.

Construct F^*_M and $d^*(M)$ for $|M| = n$:

If there is an action, $a \in A$, that would not have been played after any announcement that is a proper subset of M, and if there is a reasonable probability distribution, P, on $M \cup \Omega_o$, supporting the action a after the announcement M, then set $F^*_M = P$ and

$d^*(M) = a$.

Otherwise, set $F^*_M = F_M$, and $d^*(M)$ as one of the actions that maximizes expected utility of the DM under no announcement (same as in the proof of Theorem 1).

Given the decision function, d^*, the announcer announces M^*, which maximizes his expected utility.

This construction of the assessment (M^*, d^*, F^*) is an awareness equilibrium satisfying the reasoning refinement. ■

The reasoning refinement emphasizes the strategic nature of the announcement. The announcer, who would like the DM to take the right action for the announcer, may prefer to hide some of the contingencies. Even if there is no conflict of interest, in order to correct the DM's action, it is not always optimal to reveal all of the contingencies. For example, consider a teenager who is not taking the correct action for herself due to her limited awareness. The parent, who has the same preference as his child, would like to advise her in order to direct her to the right action. In such a case, too much advice might confuse the child and lead her to a wrong action. In Example 3, there is a strategy-wise unique awareness equilibrium that satisfies reasoning refinement and in this equilibrium all of the contingencies are not announced.

Example 3 *Let* $\Omega = \{\omega_1, \omega_2, \omega_3\}$ *with* $\pi(\omega_1) = 0.1$, $\pi(\omega_2) = 0.8$, $\pi(\omega_3) = 0.1$. *and* $\Omega_o = \{\omega_1\}$.

		Actions		
		left	*middle*	*right*
	ω_1	$3,3$	$0,0$	$2,2$
Contingencies	ω_2	$0,0$	$7,7$	$2,2$
	ω_3	$2,2$	$0,0$	$2,2$

Here, one can easily check that the unique strategies in any awareness equilibrium satisfying reasoning refinement are as follows: The announcer announces only ω_2; the DM plays "left" under no announcement, "middle" when ω_2 is announced, "left" when ω_3 is announced, and "right" when both ω_2 and ω_3 are announced.

In the above example, the announcer would like the DM to take "middle" action. Within the initial awareness, the DM plays "left". If the announcer announces ω_2, then the DM reasons that the announcer wants her to change her action. Under reasoning refinement, the DM concludes that with respect to a certain belief switching to "middle" improves both the announcer's and the DM's payoffs. When ω_3 is announced, there is no such a belief. So, she keeps playing "left". Therefore, announcing only ω_2 is enough to correct the action of the DM. However, when all of the contingencies are announced, the DM reasons that the announcer wants her to take an action different than "left" and "middle". She thinks that if the announcer wanted her to play "left" or "middle", he could do it by announcing less. So, the DM seeks for another action and a belief such that playing that action is better for both the DM and the announcer from the DM's point of view, and "right" is that action. Hence, the announcer does not want to announce everything in order to prevent the DM to make the wrong reasoning.

4 Discussions

Zero Probability vs. Unawareness

The major question regarding the unawareness literature is its difference from assigning zero probabilities. In other words, may one reformulate our setup so that the DM is aware of all the contingencies, but assigns zero probability to some of them? The announcer does not know the realization of chance player, but he knows the distribution of contingencies. This distribution may be considered as the type of the announcer, and a type space can be defined as a set of distributions on contingencies. The DM does not know the true type of the announcer and holds a prior on the type space. Then, the announcement can be thought of as cheap talk (see Crawford and Sobel, 1982). Since the DM assigns zero probability to some of the contingencies, every type in the type space should assign zero probability to those contingencies. Then, whatever the initial belief of the DM is, the posterior assigns zero probability to these contingencies.

Even if in the type space, some of the types assign non-zero probability to those contingencies, still there are examples where perfect Bayesian equilibrium cannot capture some behavior that awareness equilibrium can explain:

Example 4 *Let* $\Omega = \{\omega_1, \omega_2\}, \Omega_o = \{\omega_1\},$ *and* $\pi(\omega_1) = \pi(\omega_2) = 0.5$.

The DM has two actions: left and right. The payoffs are as follows:

		Actions	
		left	*right*
Contingencies	ω_1	*1, 1*	*0, 1*
	ω_2	*1, 1*	*0, 3*

Observe that there is an awareness equilibrium where the announcer does not make any announcement, $M^* = \emptyset$*; and the DM plays "left" under no announcement, and "right" when* ω_2 *is announced.*

One can easily show that with any type space and any initial belief of the DM assigning non zero probability to a type that is distribution with positive probability to $\{\omega_2\}$*, in any perfect Bayesian equilibrium, the DM plays "right" independent of the announcement. Therefore, this cannot capture the behavior where the DM is taking different actions before and after the announcement.*

States vs. Contingencies

The main point of the paper does not need explicit construction of the generalized state space. Nonetheless, it can be constructed as in Heifetz, et al. (2006). It is important to note that a contingency in this paper is only a move of chance player, and it should not be confused with a state. A state defines the game that is generated by the vocabularies of the players (see Heifetz, et al. 2006).

For example, in order to construct the generalized state spaces in Example 1, we need two vocabularies p and q as specifications of games in Figure 1 and 3, respectively.

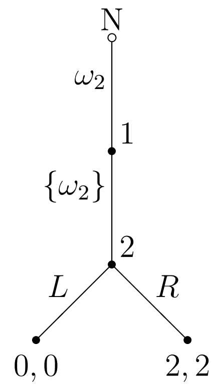

Figure 3: Game that is generated by vocabulary q.

The announcer is endowed with vocabulary p and q, while the DM is aware of only p. The state space of the announcer is $S_1 = \{pq\}$, the state space of the DM is $S_2 = \{p\}$. Observe that S_2 is a projection of S_1 by using the vocabulary p. Being at state pq means that player is considering the game in Figure 2. On the other hand, being at state p means that player is considering the game in Figure 1.

The interactive unawareness is defined as the conjunction of the state spaces of each player a l*à* Heifetz, et al. (2006), i.e. $S_1 \bigwedge S_2 = \{p\}$. In other words each player thinks that the other player is aware of the game in Figure 1.

5 Conclusion

This paper studies games with uncertainty where players have different awareness regarding the moves of nature. The fully aware player decides which contingencies to announce to the less aware player. The key element in this game is the belief formation procedure. In equilibrium, the belief formation is based on the fact that “the announcement is a strategic decision of a rational announcer”. Additionally, a refinement concept that further emphasizes the strategic nature of the announcement is introduced. In a simple example, it is shown that even if there is no conflict of interest, it is not always optimal to announce all of the contingencies in order to change one's action. Announcing too many contingencies can lead the DM to make wrong inferences.

Insurance contracts, financing innovations, pricing future options and bilateral trade agreements are some of the examples as natural applications of this setup. Indeed, Filiz Ozbay (2008) considered this model and solution concept in a contract setting since a contract can be thought of as a communication device that extends the awareness of the contracting parties. She showed that although it is feasible to sign a complete contract, in equilibrium an incomplete contract is signed. It may be fruitful to revisit the other economically relevant settings by the concepts introduced in this paper.

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