best interests of the community when full insurance is not possible, the informational requirements for the planner are huge, the impact on self-selection must be considered, and the objective function should be carefully defined. In any event, a uniform reward system may not yield a first-best solution where utility functions differ across the set of actual and potential managers.

REFERENCES

Arrow, Kenneth J., and Lind, Robert C., "Uncertainty and the Evaluation of Public Investments." *Amer. Econ. Rev.* **60,** 3:364–378, June 1970.

Baron, David, "Price Equilibrium, Utility and Industry Equilibrium in Pure Competition." *Internat. Econ. Rev.* **11,** 3:463–480, Oct. 1970.

Bergson, Abram, "Managerial Risks and Rewards in Public Enterprises." *J. Comp. Econ.* **2,** 3:211–225, Sept. 1978.

James, Estelle, "A Note on Uncertainty and the Evaluation of Public Investment Decisions." *Amer. Econ. Rev.* **65,** 1:200–206, March 1975.

Kihlstrom, Richard, and Laffont, Jean-Jacques, "A General Equilibrium Theory of Firm Formation Based on Risk Aversion." Mimeo, 1978; forthcoming in *J. Pol. Econ.*

Pratt, John W., "Risk Aversion in the Small and in the Large." *Econometrica* **32,** 1–2: 122–136, Jan.–April 1964.

Ross, Stephen A., "The Economic Theory of Agency: The Principal's Problem." *Amer. Econ. Rev.* **63,** 2:134–139, May 1973.

JOURNAL OF COMPARATIVE ECONOMICS 3, 407–415 (1979)

COMMENT

The Problem of Equity in Determining Managerial Rewards in Public Enterprises

A Comment on A. Bergson, "Managerial Risks and Rewards in Public Enterprises"

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Received July–August 1979

In a recent article in this Journal, Professor Abram Bergson[1] addresses the important issue of managerial rewards in a public enterprise. Of particular concern is the question whether it is possible to design a system of managerial rewards that will encourage the appropriate level of risk taking on the part of enterprise managers. Bergson concludes that while such a reward scheme can be designed, the rewards may have to be so large that the resulting income distribution may "conflict with accepted precepts of equity" (Bergson, 1978, p. 222).

In this note we generalize Bergson's analysis and show that an appropriate reward structure need not create bonuses that conflict with accepted equity precepts. We show that the size of the rewards depends on the way in which the reward function is constrained, and that under a reasonable set of constraints the rewards to managers can be quite small. Thus we argue that it is possible to establish managerial incentive schemes that encourage appropriate decision making and at the same time involve relatively small bonuses.

I

Drawing on Arrow's (1970) theorem, Bergson argues that public enterprise managers should be encouraged to make decisions so as to maximize expected returns to the community.[2] The formal problem

[1] Bergson (1978).

[2] Bergson used the term community to mean everyone except the manager.

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Bergson poses is to find a bonus function that will encourage risk-averse managers to act as if they were risk-neutral decision makers in maximizing expected returns for the community.

In Bergson's model managerial income is

$$I = w + b + c, \tag{1}$$

where w is the managerial wage, b is the bonus, and c is the career reward. The bonus depends on B, total realized benefits from enterprise activity; thus:

$$b = b(B). \tag{2}$$

In addition, the manager can anticipate that, based on this period's performance, his wages may be either higher or lower next period. The present value of these future rewards (penalties) for a level of performance better than (worse than) B_0 are then captured by the career function,

$$c = c(B); \quad c'(B) \geq 0; \quad c(B_0) = 0. \tag{3}$$

If the utility function of the manager is

$$U = U(I), \tag{4}$$

Bergson argues that an appropriate bonus scheme can be found by solving the following equation for b:

$$U(I) - U(w) = \alpha(B - b) + \beta, \tag{5}$$

where α and β are two arbitrary constants and $\alpha > 0$.

Equation (5) is an example of the similarity condition.[3] For any given bonus scheme, b, which satisfies Eq. (5) the risk-averse manager will make decisions consistent with risk-neutral maximization of the community's benefits.

Having identified the bonus function as being determined by Eq. (5), Bergson then analyzed 18 numerical examples. In these examples he shows that the bonus payments may be as high as 4.75 times the manager's wage. He then concludes that while it is possible to find a bonus scheme that leads to appropriate decision making, the rewards may be too high. Since the bonus scheme is a function of the parameters α and β, and these can be arbitrarily chosen, there are, however, an infinite number of bonus schemes satisfying Eq. (5). If outside constraints are placed on the problem, then the values of α and β will also be constrained. We will show that if these constraints are very restrictive then an appropriate bonus scheme will require rewards several times the managerial wage. On the other hand, under reasonable constraints rewards need only be a fraction of the wage.

[3] See Ross (1973, 1974).

II

In order to examine the relationship between the constraints and the size of the rewards, we first characterize the bonus function in a general way. Taking the first derivative of Eq. (5) with respect to B, we find

$$b' = \frac{\alpha - U'c'}{\alpha + U'}. \tag{6}$$

The second derivative of the bonus function is then

$$b'' = -\frac{\alpha U''(b' + c')(c' + 1) + U'c''(\alpha + U')}{(\alpha + U')^2}.$$

Simplifying and assuming, as Bergson does in his numerical examples, that $c'' \leq 0$, this reduces to

$$b'' \geq -\frac{U''(b' + c')^2}{(\alpha + U')}.$$

With the usual assumptions that $U' > 0$ and $U'' < 0$, b'' will be positive. The bonus function will therefore be U-shaped, and its minimum will occur when $\alpha = U'c'$.[4]

The shape of the bonus function reflects the fact that the community is less risk-averse than the managers. The first derivative of the bonus function will be negative over some range of B. This may appear to discourage managers from trying to increase B; however, as long as $b' + c' > 0$ there will be an incentive for managers to increase B. This will be the case as long as α and c' are positive.[5]

A reward structure with $b' < 0$ over some values of b may at first appear to be somewhat unusual, but there are clearly times when firms ask individuals who have been successful in their jobs to take a short-term reduction in income for the benefit of the company and their careers. For example, it is common for large firms to request that executives move from one area to another. Not infrequently this can mean a reduction in real current income for the executive, if the new location has a higher cost of living. Thus, an executive, who has recently been successful (had a high B), will suffer a short-term income loss with the understanding that long-term career prospects will be improved. Another example occurs in sales departments. Sometimes experienced salesmen are asked to give up a lucrative sales territory in order to take over new accounts that need their attention. Again the short-term income of a productive employee is

[4] Assuming that $\lim_{I \to \infty} U'(I) = \infty$.

[5] $b' + c' = (\alpha - U'c')/(\alpha + U') + c' = \alpha(1 + c')/(\alpha + U') > 0$. Bergson assumes that $b' > 0$. To restrict b' in this way even over the range of B that Bergson considers in his numerical examples can produce very high rewards.

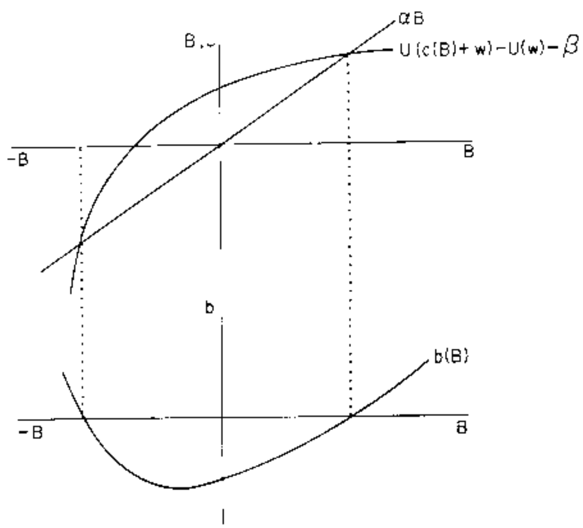

FIGURE 1

reduced with the understanding that the individual's career prospects will be improved.[6]

These examples, of course, do not have institutional arrangements exactly equivalent to those specified in Bergson's framework. However, they do show that, in the real world, it is possible that current income can be negatively correlated with performance if, at the same time, career prospects are enhanced. In the real world, reward structures, especially career functions, are usually implicit rather than explicit. Thus, one would not expect to observe real-world bonus schemes that are exactly analogous to those derived from the similarity condition. The fact that one can observe some situations in which current income is negatively correlated with output would seem to indicate that it can be institutionally acceptable to have $b' < 0$ when the career function is explicitly known by all participants.

To further characterize the bonus function, it is useful to find those points where $b = 0$. Setting $b = 0$ in Eq. (5) and rearranging terms, we get

$$U(c(B) + w) - U(w) - \beta = \alpha B. \tag{7}$$

The upper part of Fig. 1 is a graphical representation of Eq. (7). (We are assuming $c(B)$ is linear throughout.)[7] The curved line represents the left-hand side and the straight line represents the right-hand side. The two lines cross at values of B where $b = 0$. The bonus function will therefore look

[6] We would like to thank Edwin Wood for suggesting these examples.

[7] If as in some of Bergson's numerical examples the career function is zero over the positive range of B, the curved line will become horizontal to the right of the vertical axis.

like the line in the bottom half of Fig. 1 reaching its minimum where the slope of the curved line is equal to α. Another important feature of Fig. 1 is that in the Bergson case where the career function is zero at $B = 0$, the vertical intercept of the curved line is $-\beta$.[8] Changing β, therefore, moves the curved line upward or downward and changes those values of B where $b = 0$. In Bergson's numerical examples $\beta = 0$; in this case the curved line will intersect the straight line at the origin. Bergson then determines α by adjusting the straight line so that the two lines also intersect at $B = -10$.

From Fig. 1 it is possible to see that if we desire the bonus to be nonnegative everywhere the curved line representing the left-hand side of Eq. (7) must lie everywhere at or below the straight line representing the right-hand side. Negative bonuses can be avoided by raising β and shifting the curved line downward until it is tangent to the straight line. The range of possible α and β which satisfy this condition is quite large. If α is reduced and bonuses are to remain nonnegative, β must be increased, and the tangency will occur at a higher B. For these reasons, if $b \geq 0$ and $b' > 0$ over a range from $B = -10$ to $B = 30$ (as Bergson seems to suggest is true of his numerical examples) α will be relatively high. This in turn will require high bonus payments when $B = 30$.[9]

If the constraint on b' is relaxed while retaining the condition that $b' + c' > 0$, it is possible to construct bonus schemes that have "relatively modest" rewards even if we constrain the function to be nonnegative everywhere.[10] Table 1 gives examples of possible rewards under this set of restrictions for the three utility functions and the six career functions that Bergson describes.[11] Bonuses are given for the range with which Bergson was concerned: $B = -10$ to $B = 30$.

From the table it is possible to see that the size of the reward payments can be reduced to levels well below those calculated by Bergson (which appear in parentheses). While somewhat higher rewards must be paid in situations where the career function is steeply sloped, in general the degree of inequality created by the bonuses appears to be less than the

[8] This can, of course, be generalized. $-\beta$ is the height of the curved line $U(c(B) + w) - U(w) - \beta$ at B_0 when $c(B_0) = 0$.

[9] Under the stated conditions, the best we can do in the case where the career function is linear with $c' = 0.05$ and the logarithmic utility function is a reward 8.9 times the wage as compared with Bergson's 3.69. Actually the bonus function that Bergson uses in computing his result does have a negative slope in the range from $B = -10$ to $B = -4.75$. The point is, however, that these are an unnecessarily restrictive set of constraints.

[10] Since what might be considered a "relatively modest" bonus payment is debatable, we use the term in a way consistent with Bergson's usage. He refers to bonuses of 36% of the managerial wage as "relatively modest." As can be seen from Table 1, none of our bonuses exceed 40%.

[11] Bergson describes the bonus for $U(I) = \log I$, $U(I) = I^{1/2}$, and $U(I) = I$.

TABLE I

INDICATED BONUSES (b) FOR ALTERNATIVE BENEFITS (B) IN BERGSON'S 18 CASES[a]

B	Case I: $c(-10) = -0.1$		Case II: $c(-10) = -0.3$		Case III: $c(-10) = -0.5$	
	$c(B) = 0$ (for $B > 0$)	$c(B) = B/100$ (for $B \geq 0$)	$c(B) = 0$ (for $B > 0$)	$c(B) = 3B/100$ (for $B \geq 0$)	$c(B) = 0$ (for $B \geq 0$)	$c(B) = B/20$ (for $B \geq 0$)
	Relative risk aversion of unity					
−10	0.08 (0)	0.02 (0)	0.24 (0)	0.11 (0)	0.40 (0)	0.28 (0)
0	0 (0)	0[b] (0)	0 (0)	0.03 (0)	0 (0)	0.07 (0)
10	0.02 (0.11)	0 (0.01)	0.06 (0.41)	0 (0.12)	0.12 (0.88)	0 (0.44)
20	0.04 (0.23)	0[b] (0.03)	0.13 (0.97)	0.04 (0.41)	0.25 (2.39)	0.09 (1.58)
30	0.07 (0.36)	0.02 (0.07)	0.20 (1.74)	0.16 (0.92)	0.40 (4.75)	0.38 (3.69)
α	0.002 (0.011)	0.009 (0.011)	0.006 (0.036)	0.023 (0.035)	0.0115 (0.069)	0.033 (0.069)
β	0 (0)	0.0052 (0)	0 (0)	0.032 (0)	0 (0)	0.072 (0)
	Relative risk aversion of one half					
−10	0.08 (0)	0[b] (0)	0.24 (0)	0.07 (0)	0.38 (0)	0.21 (0)
0	0 (0)	0 (0)	0 (0)	0.01 (0)	0 (0)	0.08 (0)
10	0.02 (0.10)	0[b] (0)	0.06 (0.34)	0[b] (0.05)	0.12 (0.61)	0.02 (0.11)
20	0.04 (0.21)	0.01 (0.02)	0.12 (0.73)	0.02 (0.15)	0.25 (1.39)	0.03 (0.47)
30	0.06 (0.33)	0.02 (0.03)	0.19 (1.16)	0.08 (0.30)	0.39 (2.28)	0.11 (0.93)
α	0.001 (0.005)	0.005 (0.005)	0.003 (0.016)	0.0133 (0.016)	0.006 (0.029)	0.019 (0.029)
β	0 (0)	0 (0)	0 (0)	0.008 (0)	0 (0)	0.04 (0)
	Risk neutrality					
−10	0.08 (0)	0 (0)	0.22 (0)	0 (0)	0.37 (0)	0 (0)
0	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
10	0.02 (0.10)	0 (0)	0.08 (0.29)	0 (0)	0.12 (0.48)	0 (0)
20	0.04 (0.20)	0 (0)	0.16 (0.58)	0 (0)	0.24 (0.96)	0 (0)
30	0.06 (0.30)	0 (0)	0.24 (0.87)	0 (0)	0.36 (1.43)	0 (0)
α	0.002 (0.01)	0.01 (0.01)	0.008 (0.03)	0.03 (0.03)	0.012 (0.05)	0.05 (0.05)
β	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)

[a] Bergson's numbers appear in parentheses.
[b] Less than or equal to 0.005 before rounding.

implied inequality in the wage structure suggested by the career function. For example, when $c' = 0.05$, $B = -10$ will cause a reduction in the discounted value of next year's wage equivalent to 50% of this year's wage. Nowhere in our table does the bonus affect the manager's income as dramatically as this.[12] It can be seen, therefore, that managerial bonus schemes need not have rewards that are excessively high.

III

In Section II, we showed that the bonuses paid to managers need not be as large as those suggested by Bergson. However, even these results

[12] This argument suggests that if there were no career function, then the bonuses could be made infinitesimally small. This appears to be the case. If $c = 0$, then the curved line in Fig. 1 becomes a horizontal line at height $-\beta$. If $\beta = 0$, and α is very small then the bonus will be very small over any reasonable range of B.

do not necessarily represent the lower bound of the bonuses that must be paid to solve the problem of risk-averse managerial behavior. While the bonus function for the similarity condition is relatively easy to find, minimization of the bonus requires finding a solution to a more complex problem, which Ross (1973) calls the principal's problem. The principal's problem can be summarized in the following manner:

> For any public enterprise, then, the problem is how to share between managerial personnel and the community generally the benefits produced so that, in choices among alternatives with uncertain outcomes, managerial personnel may be induced to maximize expected benefits to the community. (Bergson, 1978, p. 215)

Thus, the problem is to choose a bonus function that maximizes expected benefits to the community.

In order to give an exact representation of the principal's problem, it is necessary to introduce an explicit representation of managerial choice. Following Bergson, managers choose between prospects, and the total benefit accruing to an enterprise is a function of the prospect chosen and the outcome of an environmental event. Of course, the environmental outcome is unknown when the prospect is chosen. Thus,

$$B = B(a,\theta),$$

where a is a variable denoting which prospect has been chosen and θ is a variable denoting which environmental event has occurred. Thus, a manager will choose the value of a that is a solution to the following problem:

$$\max_{a} \{ \underset{\theta}{E} \, [U(c(B(a,\theta)) + b(B(a,\theta)) + w) - U(w)]\}. \tag{8}$$

Therefore, in order to solve the problem stated in the above quotation from Bergson, central planners will solve

$$\max_{b} \{ \underset{\theta}{E} \, [B(a,\theta) - b(B(a,\theta))]\}, \tag{9}$$

given that managers are choosing a using criterion (8).

In deriving the similarity condition as a necessary condition for an optimal bonus function, Bergson reformulates the problem to be solved:

> The desideratum then is that managers, in choosing a prospect that maximizes [(8)] will be induced by the rewards offered to them to choose at the same time one that maximizes $[E_\theta(B(b(a,\theta)) - b(a,\theta))]$. (Bergson, 1978, p. 218)

This is not a statement of the principal's problem. The second quotation refers to the choice of bonus function that will ensure that the manager's prospect choice, given the bonus function, is the same as the prospect that the community would choose, given the bonus function.

Bergson has formulated two different approaches to the problem of choosing an appropriate bonus function. The similarity condition, although a necessary and sufficient condition for the solution of one problem, is neither necessary nor sufficient for the choice of a bonus function that maximizes (9). If the community chooses a bonus function that maximizes (9), the result will be that the community obtains a higher expected value of net benefits than if it chooses a bonus function satisfying the similarity condition.[13]

Given that the similarity condition is not necessarily a solution to the principal's problem, one may question the use of the similarity condition in assessing the size of managerial bonuses. We shall show that in fact our use of the similarity condition can be justified. Let us suppose that $b^*(B(a,\theta))$ is the solution to the principal's problem and a^* is the prospect chosen by the manager, given that b^* is the bonus function. Also, assume that $\bar{b}$ is any bonus which satisfies the similarity condition and that confronted with $\bar{b}$, the manager will choose the prospect $\bar{a}$.[14] Thus, from the definitions of the principal's problem and the similarity condition, the following inequalities can be derived:

$$\underset{\theta}{E}\,[B(a^*,\theta) - b^*(B(a^*,\theta))] \geq \underset{\theta}{E}\,[B(\bar{a},\theta) - \bar{b}(B(\bar{a},\theta))],$$

$$\underset{\theta}{E}\,[B(\bar{a},\theta) - \bar{b}(B(\bar{a},\theta))] \geq \underset{\theta}{E}\,[B(a^*,\theta) - \bar{b}(B(a^*,\theta))].$$

Combining these inequalities, one obtains:

$$\underset{\theta}{E}\,[\bar{b}(B(a^*,\theta))] \geq \underset{\theta}{E}\,[b^*(B(a^*,\theta))].$$

a^* is the prospect chosen by the manager when the optimal bonus function is chosen. Therefore, the above inequality says that when the optimal prospect is chosen, the expected value of bonus payments for a bonus solving the principal's problem will not be greater than the expected value of the bonus satisfying the similarity condition. Therefore in analyzing the size of bonus payments for bonuses satisfying the similarity condition, one may be overestimating the size of bonus payments that must be paid. Thus, one may view the figures in Table 1 as upper bounds for the bonuses that must be paid to encourage appropriate managerial behavior. Our use of the similarity condition to show that bonuses will not be excessive is therefore justified.[15]

[13] It can be shown that the similarity condition is *not* a necessary condition for solution of the principal's problem. Construction of a counterexample in which the bonus solving the principal's problem does not satisfy the similarity condition involves an extended discussion which is not central to the argument of this paper.

[14] It is assumed that all relevant constraints are satisfied by b^* and $\bar{b}$. For example, one may assume that we have placed the constraint that $b(B) \geq 0$ for all B in the problems. The following argument holds whatever constraints are assumed.

[15] It can be noted that the similarity condition allows us to draw much more general conclusions than use of the solution to the principal's problem, since deriving a solution to that problem would require us to make an assumption about the nature of $B(a,\theta)$ and the distribution of θ.

IV

In conclusion, we are in agreement with Bergson that it is theoretically possible to find a bonus scheme that overcomes the problem of managerial risk aversion. However, we disagree with his conclusion that such a bonus scheme will create a reward structure that conflicts "with accepted precepts of equity." Although the similarity condition does not necessarily give a characterization of the optimal bonus structure, we have shown that even with the similarity condition, one can derive bonuses described by Bergson as relatively modest.

REFERENCES

Arrow, Kenneth J., and Lind, Robert C., "Uncertainty and the Evaluation of Public Investments." *Amer. Econ. Rev.* **60,** 3:364–378, June 1970.

Bergson, Abram, "Managerial Risks and Rewards in Public Enterprises." *J. Comp. Econ.* **2,** 3:211–225, Sept. 1978.

Ross, Stephen A., "The Economic Theory of Agency: The Principal's Problem." *Amer. Econ. Rev.* **63,** 2:134–139, May 1973.

Ross, Stephen A., "On the Economic Theory of Agency and the Principal of Similarity." In M. S. Balch, D. L. McFadden, and S. Y. Wu, eds., *Essays on Economic Behavior under Uncertainty.* Amsterdam: North-Holland, 1974.