The Performance of Multiperiod Managerial Incentive Schemes

By PETER MURRELL*

A number of recently published articles have focused on the design of incentive schemes for economic agents in planned economies. (See John Bonin; Liang-Shing Fan; Jeff Miller and James Thornton; Martin Weitzman.) The incentive schemes have been designed with two objectives in mind.[1] First, before the beginning of the plan period, an enterprise manager must be induced to supply accurate information to plan authorities about production possibilities. The information supplied will be in the form of a target value of output level. Secondly, at the end of the plan period, the enterprise manager must be induced to report accurately the achieved value of production, whatever target value was reported initially.

On the surface, the Soviet incentive scheme (analyzed by Miller and Thornton, and Weitzman) is different from the scheme analyzed by Bonin (itself a generalization of Fan's scheme). The Soviet scheme contains not only a self-imposed production target but also a production target imposed by the planners. However, in the single period case, to which all authors restrict themselves, the planner's target is exogenous and therefore does not affect any enterprise decisions. Thus, effectively the Soviet scheme and Bonin's scheme are very similar and, not surprisingly, give exactly equivalent single period results.

For the single period case with uncertainty in production levels, the two objectives of incentive design are satisfied by the incentive schemes of the above papers.[2] First: "A plan resulting from self-imposed targets can reflect any risk of underfulfillment desired by the planner" (Bonin, p. 685). Thus, the planner can manipulate at will the tautness of the enterprise plan. In particular, in Fan's scheme the reported target will be that one with a probability of fulfillment of one-half. Secondly, the accurate reporting of accomplishments is also obtained: ". . . given any self-imposed target, the manager will always report the highest possible level of performance, i.e., the realized value of [the planned variable]" (Bonin, p. 685).

The aforementioned studies have all limited their analysis to a single period framework. The reason may be that the authors are keen to ensure that the "dynamic incentive problem" or "ratchet effect" does not appear in their model (see Bonin, p. 687; Weitzman, p. 252). The dynamic incentive problem arises when planners use present performance as a basis for future target setting. With such target setting by planners, producers will tend to bias downward present reports of output achieved in order to leave themselves with an easier target in the future. However, if the results reported above are a reasonable representation of the properties of the incentive systems, then the dynamic incentive problem will greatly diminish in importance. The incentive schemes, encouraging honest target setting, will obviate the need for planners to set targets. Planners will be able to rely safely on the enterprises' self-imposed targets. In a sense, one may say that the incentive schemes are much more powerful than previous authors have claimed. The incentive schemes solve not only the single period incentive

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[1]In the following description I use the terminology appropriate to an environment where central planners are solely concerned with the output targets of enterprises. As many authors have pointed out, the incentive scheme can be applied in situations where variables other than output are the chief object. Indeed, the appropriate environment can be thought of as planning within a large organization rather than in a whole economy I use the planning-production terminology solely for ease of discussion.

[2]Miller and Thornton extend the results to the case where managerial effort affects output.

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problem, but also the dynamic incentive problem.[3]

There is, however, a lacuna in the above argument. Previous results, having been restricted to single period analysis in order to justify ignoring the dynamic incentive problem, do not reflect on the behavior of the incentive schemes in a multiperiod framework. Thus one must extend the analysis of the incentive schemes to a multiperiod framework in which the planners do not impose targets.

There is one change in environment which is crucial to the move from a single period to a multiperiod analysis: the presence of inventories. In the single period case, there is no reason why producers should disguise their production levels because undeclared output has no value. In the multiperiod case, unreported output from one time period can be kept as producers' inventory and be reported in future time periods. Thus the producer will have to weigh the value of honest declaration of production against the value of keeping extra inventories. In turn the change in the nature of the production reporting decision will alter the way in which the production target is chosen. In the ensuing sections, I show that such considerations will affect the performance of the incentive schemes.

I. Incentives In a Multiperiod Framework

The model of production will be identical to that of Weitzman and of Bonin. Actual output at time t, Y_t, is subject to uncertainties which are represented by the probability density function $f_t(Y_t)$. For ease of notation it will be assumed that $f_t = f$ for all t. None of the results presented in this paper depend upon this assumption. The function f is known by the enterprise manager, but not by the planners. It will be assumed that f is continuously differentiable at all points, and that $f(Y_t) \equiv 0$ for $Y_t \in (-\infty, 0]$.

At the beginning of every plan period, the enterprise manager picks the target level Z_t, and communicates this target to the planners. The target level Z_t is used by the planners for plan construction purposes. In order to simplify the analysis, I assume that Z_t has no influence on Y_t. (Martin Loeb and Wesley Magat have examined the performance of the incentive scheme when such an influence is explicitly introduced.) At the end of the plan period, the enterprise manager knows the realized value of Y_t, and must report and deliver an amount of output X_t. The manager, in fixing the level of X_t, can reduce or increase the level of inventories K_t. Therefore, there are two constraints:

$$X_t \leqq Y_t + K_t, \text{ and } K_{t+1} = K_t + Y_t - X_t$$

Once K_{t+1} is known, Z_{t+1} can be chosen.

I will focus on one particular case of the incentive schemes: the scheme introduced by Fan. Fan's scheme is a special case in that the costs of overprediction and underprediction of target level are symmetric. Thus, in each time period the bonus is given by[4]

$$\alpha X_t - \epsilon |X_t - Z_t| \quad \alpha > \epsilon > 0$$

As the scheme is operating in a multiperiod framework, it must be assumed that the manager has a time horizon T. Inventory left over after time T, K_{T+1}, is assumed to be valued at $V_{T+1}(K_{T+1})$ at time T. I will make the assumption that $V_{T+1}(0) = 0$, and that V_{T+1} is continuously differentiable with $0 \leqq V'_{T+1}(K_{T+1}) \leqq \alpha + \epsilon$ and $V''_{T+1} \leqq 0$. The valuation of an increment of terminal capital is set at less than or equal to $\alpha + \epsilon$, because this term represents the maximum bonus payment which the enterprise manager can obtain from producing and delivering one unit of output.

Define E_t as the expectation operator over $f(Y_t), \ldots, f(Y_T)$ and E as the expectation

[3]Some authors also use the term ratchet effect to refer to a situation where performance in time t affects the parameters of the incentive system in time t + 1. Such an effect might arise if planners adjust the incentive system for time t + 1 on the basis of the manager's output level in time t. In this paper, it is assumed that the incentive system is not changed from one time period to the next.

[4]In the more general case the value of ϵ when $X_T > Z_T$ is different from that when $X_T < Z_T$. In Fan's scheme, X_T and Z_T are interpreted as profit levels.

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operator in any one time period. The manager's task in choosing Z_t is to maximize[5]

$$(1)\quad E_t\{\sum_{s=t}^{T}\delta^s[\alpha X_s - \epsilon|X_s - Z_s|] + \delta^T V_{T+1}(K_{T+1})\}$$

where $\delta \leqq 1$ is the manager's rate of time preference. Let us call the maximum value of (1) $V_t(K_t)$. Also define

$$W_t(K_t, Z_t) = \max_{X_t, Z_{t+1}, X_{t+1}} E_t\{\sum_{s=t}^{T}\delta^s[\alpha X_s - \epsilon|X_s - Z_s|] + \delta^T V_{T+1}(K_{T+1})|Z_t, K_t\}$$

$$\text{Now } V_t(K_t) = \max E\{(\alpha X_t - \epsilon|X_t - Z_t|)\delta^t + E_{t+1}[\sum_{s=t+1}^{T}\delta^s(\alpha X_s - \epsilon|X_s - Z_s|) + \delta^T V_{T+1}(K_{T+1})]\}$$

$$(2)\quad = \max_{Z_t, X_t} E\{(\alpha X_t - \epsilon|X_t - Z_t|)\delta^t + V_{t+1}(K_{t+1})\}$$

$$(3)\quad = \max_{Z_t} W_t(K_t, Z_t)$$

The above equations are found by simple application of the optimality principle of dynamic programming and conditional expectation properties.

II. The Manager's Optimal Program

The approach in finding the optimal policy must be one of induction backwards from time T. Policy at time T will depend critically on the valuation of postterminal capital stock: $V_{T+1}(K_{T+1})$. It is much more important to focus on the features of policy which are independent of V_{T+1}. Therefore, a description of policy at time T is given in the Appendix, where an additional assumption is placed on V_{T+1}. It is assumed that V_{T+1} is such that it will not be optimal to have $Z_T = X_T = 0$. This assumption solely implies that valuation of terminal capital stock must be heuristically derived from the incentive system. Having $Z_T = X_T = 0$ implies that postterminal valuation of inventory is much greater than the valuation derived from the incentive system. Hence, it is only proper to rule out the case $Z_T = X_T = 0$.[6]

Given the foregoing assumptions on V_{T+1}, it is shown in the Appendix that

$$\frac{dV_T(K_T)}{dK_T} = V'_T(K_T) = \alpha\delta^T$$

One only needs the value of $V'_T(K_T)$ in order to describe policies in previous time periods. Thus, to some extent, optimal policies for t = 1, . . . , T − 1 are independent of the particular form of V_{T+1}. As V_{T+1} can be expected to vary between different managers, I will focus on the policies that one can expect to be common to all managers. Therefore, in this section, attention is restricted to time periods 1 to T − 1.

Optimal policies are of two distinct kinds, depending on the size of the manager's time preference. The following two theorems show the relationship between the type of optimal policy and the relative sizes of the three parameters α, δ, and ϵ. Proofs are contained in the Appendix.

THEOREM 1: *If* $\alpha\delta \geqq \alpha - \epsilon$, *then for* t = 1, . . . , T − 1, (i) $V'_t(K_t) = \alpha\delta^t$ *and* (ii) *the optimal policy is given by* $X_t = K_t + Y_t$ *for* $K_t + Y_t \leqq Z_t$, $X_t = Z_t$ *for* $K_t + Y_t \geqq Z_t$, *and* Z_t *is fixed by*

$$Prob\{Y_t < Z_t - K_t\} = \frac{\alpha(1-\delta)}{\alpha(1-\delta)+\epsilon}$$

THEOREM 2: *If* $\alpha\delta \leqq \alpha - \epsilon$, *then for* t = 1, . . . , T − 1, (i) $V'_t(K_t) = \alpha\delta^t$ *and* (ii) *optimum policy is such that* $X_t = K_t + Y_t$ *and* Z_t *is given by*

$$Prob\{Y_t < Z_t - K_t\} = 1/2$$

Thus, it is observed that when the

[5]The manager is assumed to be risk neutral.

[6]In fact it can be shown that if $Z_T = X_T = 0$, the optimal policy is very similar to the optimal policy delineated in Section II. If $Z_T = X_T = 0$, then one can find an S such that T ≧ S ≧ 2, $Z_{T-1} = X_{T-1} = Z_{T-2} = X_{T-2} = \ldots = Z_{T-S} = X_{T-S} = 0$, and policies in t = 1, . . . , t − S − 1 are given by Theorems 1 and 2. Therefore, the assumption that $Z_T \neq 0 \neq X_T$ solely restricts S to be zero, which seems to be intuitively reasonable.

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manager's rate of time discount is high ($\alpha\delta \leq \alpha - \epsilon$), the incentive scheme has the properties which have been observed in the single period case. In each time period, the achieved production level is reported correctly. No inventories are carried over from one time period to another. The target value is fixed such that the enterprise has a probability of one-half of exceeding it. Therefore, the enterprise manager gives an accurate representation of production possibilities to the planners.

The performance of the incentive scheme changes drastically for those managers with a low rate of time discount ($\delta < 1, \alpha\delta \geqq \alpha - \epsilon$). The value of inventories is now greater than the value of declaring output achievements which are above the target level. Thus the manager will not report any output above the target level, but rather keep such output as inventory. The increase in inventory will cause an increase in the target level chosen for the next time period. However, the target will be conservative compared to production possibilities. The manager will choose a target which has probability of fulfillment of over one-half.

The disadvantages inherent in the incentive system are most clearly seen when one examines the policy undertaken by an enterprise manager who has no time discount ($\delta = 1$). Such a manager will undertake a riskless policy, setting a production target equal to inventory level and keeping all new output as inventory to be reported as output in the next time period. Effectively, the incentive scheme induces a manager to keep a high level of inventory and, one period later, report this inventory to the planning authorities. The target level given to the planners is not a target in the true sense, but rather a report of past accomplishments.

III. Conclusion

The foregoing analysis shows that the incentive scheme will not function in a multiperiod framework. The planner cannot in general induce managers to report a reasonable target, nor an honest level for production accomplishments. Hence, one may conclude that we are still far away from the design of an incentive scheme which will obviate the need for the imposition of targets by planners in a Soviet-type economy.

One may argue that, as the incentive system works well for managers with a discount rate such that $\alpha\delta < \alpha - \epsilon$, the planners can improve performance by reducing ϵ/α. However, since it is absolutely necessary that $\epsilon > 0$, the incentive scheme will always fail for some managers whose rate of time discount is low (δ close to 1). Thus, whatever the relative sizes of α and ϵ, there will always be the likelihood that some managers will not react to the scheme in the way in which the planners would desire.

A type of incentive scheme similar to the one discussed above has been introduced in the Soviet Union. It is fitting to end on the importance of the above results to the performance of the scheme in that country. Soviet policymakers have advocated setting ϵ at a level greater than $3\alpha/10$ (see Weitzman for details). Thus, the incentive scheme will not produce the desired results for those Soviet managers whose rate of time discount is such that $\delta > 7/10$. Even if the planning period is one-year long, one would think that $7/10$ is an unreasonably low value of δ. Thus, the scheme will fail for most managers. When it fails, the scheme would encourage conservative target setting and the buildup of excess inventories. Thus, the incentive scheme would only exacerbate trends which are already well-known in the Soviet Union.

Appendix

A result which is necessary for the proofs of Theorems 1 and 2 is that if $V_{T+1}(\cdot)$ satisfies the assumptions given in the text, then $V'_T(K_T) = \alpha\delta^T$. Thus this result is proven first.

X_T is chosen to maximize $\delta^T\{\alpha X_T - \epsilon|X_T - Z_T| + V_{T+1}(K_{T+1})|Z_T, Y_T\}$

Given the restrictions on V'_{T+1} an optimal policy is

If $K_T + Y_T \leq Z_T$, then $X_T = K_T + Y_T$

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If $K_T + Y_T > Z_T$, then

$X_T = K_T + Y_T$ if $V'_{T+1}(0) \leqq \alpha - \epsilon$

$X_T = Z_T$ if $V'_{T+1}(K_T + Y_T - Z_T) \geqq \alpha - \epsilon$

$X_T = Y_T - M(K_T)$ otherwise,

where $M(K_T)$ is defined by $V'_{T+1}[K_T + M(K_T)] = \alpha - \epsilon$. Therefore

$$W_T(K_T, Z_T) = \delta^T \int_0^{Z_T - K_T} \{(K_T + Y_T)(\alpha + \epsilon) - \epsilon Z_T\} f(Y_T)dY_t + A\delta^T$$

where $A = \int_{Z_T - K_T}^{\infty} \{(\alpha - \epsilon)(K_T + Y_T) + \epsilon Z_T\} f(Y_T)dY_T$ if $V'_{T+1}(0) \leqq \alpha - \epsilon$

or $= \int_{Z_T - K_T}^{\infty} \{\alpha Z_T + V_{T+1}(K_T + Y_T - Z_T)\} f(Y_T)dY_T$ if $V'_{T+1}(K_T + Y_T - Z_T) \geqq \alpha - \epsilon$ for all Y_T

or $= \int_{Z_T - K_T}^{Z_T + M(K_T)} \{\alpha Z_T + V_{T+1}(K_T + Y_T - Z_T)\} f(Y_T)dY_T + \int_{Z_T + M(K_T)}^{\infty} \{(\alpha - \epsilon)[Y_T - M(K_T)] + \epsilon Z_T + V_{T+1}[K_T + M(K_T)]\} f(Y_T)dY_T$ otherwise.

Z_T is chosen to maximize $W_T(K_T, Z_T)$. Therefore,

$$V'_T(K_T) = \frac{\partial W_T(K_T, Z_T)}{\partial K_T}$$

when the latter term is evaluated at the maximizing value of Z_T (see Samuelson, p. 34). Therefore,

$$\text{(A1)} \quad V'_T(K_T) = \delta^T \int_0^{Z_T - K_T} (\alpha + \epsilon) \cdot f(Y_T)dY_T + B\delta^T$$

where $B = \int_{Z_T - K_T}^{\infty} (\alpha - \epsilon) f(Y_T)dY_T$ if $V'_{T+1}(0) \leqq \alpha - \epsilon$

or $= \int_{Z_T - K_T}^{\infty} V'_{T+1}(K_T + Y_T - Z_T) \cdot f(Y_T)dY_T$ if $V'_{T+1}(K_T + Y_T - Z_T) \geqq \alpha - \epsilon$ for all Y_T

or $= \int_{Z_T - K_T}^{Z_T + M(K_T)} V'_{T+1} \cdot (K_T + Y_T - Z_T) f(Y_T)dY_T + \int_{Z_T + M(K_T)}^{\infty} (\alpha - \epsilon) f(Y_T)dY_T$ otherwise.[7]

Now

$$\text{(A2)} \quad \frac{\partial W_T(K_T, Z_T)}{\partial Z_T} = \delta^T \int_0^{Z_T - K_T} - \epsilon f(Y_T)dY_T + C\delta^T = 0$$

where $C = \int_{Z_T - K_T}^{\infty} \epsilon f(Y_T)dY_T$ if $V'_{T+1}(0) \leqq \alpha - \epsilon$

or $= \int_{Z_T - K_T}^{\infty} \{\alpha - V'_{T+1}(K_T + Y_T - Z_T)\} f(Y_T)dY_T$ if $V'_{T+1}(K_T + Y_T - Z_T) \geqq \alpha - \epsilon$ for all Y_T,

or $= \int_{Z_T - K_T}^{Z_T + M(K_T)} \{\alpha - V'_{T+1}(K_T + Y_T - Z_T)\} f(Y_T)dY_T + \int_{Z_T + M(K_T)}^{\infty} \epsilon f(Y_T)\, dY_T$ otherwise.

Now let us evaluate $V'_T(K_T)$ in each of the three cases.

Case 1: If $V'(0) \leqq \alpha - \epsilon$, then from (A2)

$$\int_{Z_T - K_T}^{\infty} \epsilon f(Y_T)dY_T = \int_0^{Z_T - K_T} \epsilon f(Y_T)dY_T$$

Substituting in (A1), we have $V'_T(K_T) = \alpha\delta^T$.

Case 2: If $V'_{T+1}(K_T + Y_T - Z_T) \geqq \alpha - \epsilon$ for all Y_T there are two subcases:

$$\text{(i)} \quad \int_0^{\infty} V'_{T+1}(Y_T) f(Y_T)dY_T \leqq \alpha$$

in which case $Z_T \geqq K_T$. Then

$$\int_{Z_T - K_T}^{\infty} \alpha f(Y_T)dY_T - \int_0^{Z_T - K_T} \epsilon f(Y_T)dY_T = \int_{Z_T - K_T}^{\infty} V'_{T+1}(K_T + Y_T - Z_T) f(Y_T)dY_T$$

[7]Which of the latter two formulas for B is applicable will depend on the value of K_T. Thus, when K_T is such that we are on the boundary between the two regions, one must understand by $V'_T(K_T)$ the left or right derivative which is applicable. Since $V'_T(K_T)$ is found not to depend on K_T, then the left and right derivatives will be the same in all cases.

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Thus, substituting into (A1), one obtains $V'_T(K_T) = \alpha\delta^T$.

(ii) If $\int_0^\infty V'_{T+1}(Y_T)f(Y_T)dY_T > \alpha$

then $Z_T < K_T$.

The cases when $Z_T = 0$ have been excluded by assumption. Thus, from (A2),

$$\int_0^\infty V'_{T+1}(K_T + Y_T - Z_T)f(Y_T)dY_T = \alpha$$

Substituting into (A1), one obtains $V'_T(K_T) = \alpha\delta^T$.

Case 3: There are two subcases when C in (A2) takes on the third possibility.

(i) If $$\int_0^{M(K_T)} V'_{T+1}(Y_T)f(Y_T)dY_T \leq \int_{M(K_T)}^\infty \epsilon f(Y_T)dY_T + \int_0^{M(K_T)} \alpha f(Y_T)dY_T$$

then $Z_T \geq K_T$ and

$$\int_{Z_T-K_T}^{Z_T+M(K_T)} V'_{T+1}(K_T + Y_T - Z_T)f(Y_T)dY_T = \int_0^{Z_T-K_T} -\epsilon f(Y_T)dY_T + \int_{M(K_T)+Z_T}^\infty \epsilon f(Y_T)dY_T + \int_{Z_T-K_T}^{M(K_T)+Z_T} \alpha f(Y_T)dY_T$$

Substituting into (A1) gives $V'_T(K_T) = \alpha\delta^T$

(ii) If $$\int_0^{M(K_T)} V'_{T+1}(Y_T)f(Y_T)dY_T > \int_{M(K_T)}^\infty \epsilon f(Y_T)\,dY_T + \int_0^{M(K_T)} \alpha f(Y_T)\,dY_T$$

then $Z_T < K_T$ and

$$\int_0^{Z_T+M(K_T)} \alpha f(Y_T)\,dY_T + \int_{Z_T+M(K_T)}^\infty \epsilon f(Y_T)\,dY_T = \int_0^{Z_T+M(K_T)} V'_{T+1}(K_T + Y_T - Z_T)f(Y_T)dY_T$$

Substituting into (A1) gives $V'_T(K_T) = \alpha\delta^T$.

Thus, in all cases $V'_T(K_T) = \alpha\delta^T$, which is the result necessary for the proofs of Theorems 1 and 2.

PROOF of Theorem 1:

The proof will be one of induction backwards from $t = T - 1$. It is known that $V'_T(K_T) = \alpha\delta^T$ therefore if one can prove the theorem is true at time t using this information, then the theorem must be true at all previous time periods. X_{T-1} is chosen to maximize $\{\delta^{T-1}(\alpha X_{T-1} - \epsilon|X_{T-1} - Z_{T-1}|) + V_T(K_T)\,|\,Y_{T-1}, Z_{T-1}\}$. Given that $V'_T(K_T) = \alpha\delta^T$ and that $\alpha\delta \gtrless \alpha - \epsilon$ then optimal policy is

$$X_{T-1} = K_{T-1} + Y_{T-1} \text{ for } K_{T-1} + Y_{T-1} \leq Z_{T-1}$$
$$X_{T-1} = Z_{T-1} \qquad \text{for } K_{T-1} + Y_{T-1} \geq Z_{T-1}$$

Hence, $$W_{T-1}(K_{T-1}, Z_{T-1}) = \int_0^{Z_{T-1}K_{T-1}} \{[(\alpha + \epsilon)(K_{T-1} + Y_{T-1}) - \epsilon Z_{T-1}]\delta^{T-1} + V_T(0)\}f(Y_{T-1})\,dY_{T-1} + \int_{Z_{T-1}K_{T-1}}^\infty \{\delta^{T-1}\alpha Z_{T-1} + V_T(K_{T-1} + Y_{T-1} - Z_{T-1})\}f(Y_{T-1})\,dY_{T-1}$$

In order to find the maximizing value of Z_{T-1} set

$$\frac{\partial W_{T-1}(K_{T-1}, Z_{T-1})}{\partial Z_{T-1}} = 0$$

(A3) $$\frac{\partial W_{T-1}(K_{T-1}, Z_{T-1})}{\partial Z_{T-1}} = -\int_0^{Z_{T-1}-K_{T-1}} \delta^{T-1}\epsilon f(Y_{T-1})\,dY_{T-1} + \int_{Z_{T-1}-K_{T-1}}^\infty \{\alpha\delta^{T-1} - V'_T(K_{T-1} + Y_{T-1} - Z_{T-1})\}f(Y_{T-1})\,dY_{T-1}$$
$$= -\delta^{T-1}\int_0^{Z_{T-1}-K_{T-1}} \epsilon f(Y_{T-1})\,dY_{T-1} + \delta^{T-1}\int_{Z_{T-1}-K_{T-1}}^\infty \alpha(1-\delta)f(Y_{T-1})\,dY_{T-1} = 0$$

Since *Prob* $[Y_{T-1} < Z_{T-1} - K_{T-1}]$

$$= \int_0^{Z_{T-1}-K_{T-1}} f(Y_{T-1})\,dY_{T-1} = 1 - \int_{Z_{T-1}-K_{T-1}}^\infty f(Y_{T-1})\,dY_{T-1}$$

solving equation (A3) gives

$$Prob\ [Y_{T-1} < Z_{T-1} - K_{T-1}] = \frac{\alpha(1-\delta)}{\alpha(1-\delta) + \epsilon} \leq \frac{1}{2}$$

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When $\delta = 1$ (no time discount),

$$Z_{T-1} = K_{T-1}$$

Because Z_{T-1} is picked to maximize $W_{T-1}(K_{T-1}, Z_{T-1})$,

$$V'_{T-1}(K_{T-1}) = \frac{\partial W_{T-1}(K_{T-1}, Z_{T-1})}{\partial K_{T-1}}$$

where the latter term is evaluated at the maximizing value of Z_{T-1} (see Samuelson, p. 34). Hence,

$$V'_{T-1}(K_{T-1}) = \delta^{T-1} \int_0^{Z_{T-1}-K_{T-1}} (\alpha + \epsilon) f(Y_{T-1})\, dY_{T-1} + \int_{Z_{T-1}-K_{T-1}}^{\infty} V'_T(K_{T-1} + Y_{T-1} - Z_{T-1}) f(Y_{T-1})\, dY_{T-1}$$

Substituting from equation (A3):

$$V'_{T-1}(K_{T-1}) = \delta^{T-1} \int_0^{Z_{T-1}-K_{T-1}} (\alpha + \epsilon) f(Y_{T-1})\, dY_{T-1} - \delta^{T-1} \int_0^{Z_{T-1}-K_{T-1}} \epsilon f(Y_{T-1})\, dY_{T-1} + \int_{Z_{T-1}-K_{T-1}}^{\infty} \delta^{T-1} \alpha f(Y_{T-1})\, dY_{T-1} = \alpha\delta^{T-1}$$

Thus, by induction the theorem is proved.

PROOF of Theorem 2:

Again an induction proof is used with exactly the same method as in the previous proof. Using $V'_T(K_T) = \alpha\delta^T$, one can prove that the theorem is true at $T - 1$ and by induction true at all previous time periods. X_{T-1} is chosen to maximize $\{\delta^{T-1}[\alpha X_{T-1} - \epsilon|X_{T-1} - Z_{T-1}|] + V_T(K_T)|Y_{T-1}, Z_{T-1}\}$. Given that $V'_T(K_T) = \alpha\delta^T$ and $\alpha\delta \leqq \alpha - \epsilon$ the optimal policy is: $X_{T-1} = K_{T-1} + Y_{T-1}$.

Hence, $W_{T-1}(K_{T-1}, Z_{T-1})$

$$= \int_0^{Z_{T-1}-K_{T-1}} \delta^{T-1}\{(K_{T-1} + Y_{T-1})(\alpha + \epsilon) - \epsilon Z_{T-1} + V_T(0)\} f(Y_{T-1})\, dY_{T-1} + \int_{Z_{T-1}-K_{T-1}}^{\infty} \delta^{T-1}\{(K_{T-1} + Y_{T-1})(\alpha - \epsilon) + \epsilon Z_{T-1} + V_T(0)\} f(Y_{T-1})\, dY_{T-1}$$

At the maximizing value of Z_{T-1}, it is required that:

$$\text{(A4)} \quad \frac{\partial W_{T-1}(K_{T-1}, Z_{T-1})}{\partial Z_{T-1}} = -\int_0^{Z_{T-1}-K_{T-1}} \delta^{T-1} \epsilon f(Y_{T-1})\, dY_{T-1} + \int_{Z_{T-1}-K_{T-1}}^{\infty} \epsilon\delta^{T-1} f(Y_{T-1})\, dY_{T-1} = 0$$

Thus, *Prob* $[Y_{T-1} < Z_{T-1} - K_{T-1}) = 1/2$, and

$$V'_{T-1}(K_{T-1}, Z_{T-1}) = \frac{\partial W_{T-1}(K_{T-1}, Z_{T-1})}{\partial K_{T-1}} = \int_0^{Z_{T-1}-K_{T-1}} \delta^{T-1}(\alpha + \epsilon) f(Y_{T-1})\, dY_{T-1} + \int_{Z_{T-1}-K_{T-1}}^{\infty} \delta^{T-1}(\alpha - \epsilon) f(Y_{T-1})\, dY_{T-1}$$

By substituting in equation (A4),

$$V'_{T-1}(K_{T-1}) = \alpha\delta^{T-1}$$

Hence, the theorem is proved.

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