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The Applicability of Information-Revealing Incentive Schemes in Economic Organizations[1]

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The applicability in centrally planned economies of bonus functions that are designed to encourage subordinates to report information honestly is evaluated. If central planners seek Pareto-efficient outcomes, a unique bonus function and therefore a unique distribution of managerial incomes results. Hence, income-distribution considerations must be embodied directly in the objective function. However, once such a change in the objective function is introduced, an optimal bonus function can no longer be found. *J. Comp. Econ.*, September 1984, **8**(3), pp. 277–289. University of Maryland, College Park, Maryland 20742, and University of Delaware, Newark, Delaware 19711.

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1. INTRODUCTION

Studies of organizational behavior, whether of large capitalist corporations or of state planning systems, invariably emphasize the dysfunctional behavior that results when superiors must elicit information from subordinates. Thus, a great deal of interest has centered on a new set of results that show that there are circumstances under which it is possible for superiors to construct

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a bonus function that encourages subordinates to report information honestly. Using such a bonus function, the superiors can obtain the full-information maximum value of their objective function, even if information is initially decentralized (Groves, 1973; Loeb and Magat, 1978; and Conn, 1979). Not surprisingly, it has been suggested that these results may be of direct practical relevance, especially for centrally planned economies.[2] Before these claims of relevance can be fully believable, however, it is necessary to examine whether results on the performance of the bonus functions are robust with respect to changes in assumptions about the environment in which these functions are to be applied. Such an examination was the primary aim of our previous paper (Miller and Murrell, 1981).

One important element of the environment assumed by Groves (1973) and Loeb and Magat (1978) was the lack of conflict between the goals of the superiors and the goals of the subordinates. Intuitively, one might define such "conflict" as a situation where a variable affects the objective functions of both superior and subordinate, but with marginal effects opposite in sign. In previous proofs of the optimality of "Groves-type" incentive schemes such conflict had been assumed away: subordinates were assumed to have no direct preferences over the inputs or outputs of production and superiors no direct preferences over the size of bonuses.[3] The aim of Miller and Murrell (1981) was to examine whether the incentive scheme could work in a situation where such conflict existed. Implicit in the argument of that paper was the assumption that conflict would be a standard feature of the situations in which such incentive functions would be most beneficial.[4]

There are two obvious ways in which conflict between superior and subordinate can arise. First, when effort is a factor of production, extra effort by the subordinate decreases subordinate welfare but increases the welfare of the superior through the effect on output. Second, when the superior wants to maximize net surplus, a larger bonus increases the welfare of the subordinate while reducing the welfare of the superior. Miller and Murrell (1981) showed that, in each of these situations, an optimal incentive scheme did not exist: the full-information optimum was not achievable. Thus, when conflict between superior and subordinate takes either of these two forms, informational problems affect economic outcomes.

In an extended comment on our article, David Conn (1982) has shown that, if the objective function of the superior includes the welfare of the subordinate, then an optimal bonus function exists. The objective function

[2] See, for example, the remarks in Loeb and Magat (1978).

[3] A direct preference is one that exists before the arguments of the bonus function are chosen.

[4] Unfortunately, we did not make this point as clear as we should have.

introduced by Conn, however, removes the element of conflict between superior and subordinate. There need be no disagreement over either the use of effort or the payment of bonuses because neither effort nor bonuses produce opposite effects on the welfares of the two participants.

Although we have no quarrels concerning the correctness of Conn's results, we do question the degree to which they are relevant to real-world problems. Our aim in this paper is to distinguish between those situations in which the Miller–Murrell (1981) results are applicable and those in which Conn's (1982) are relevant. In Section 2, we adopt Conn's perspective and examine the consequences of assuming that Pareto efficiency is an appropriate performance criterion for an economy. Proving a new result, we show that when efficiency is the sole objective of superiors, a unique bonus function exists and a unique distribution of managerial incomes results. We use this result to argue that Pareto efficiency cannot be the sole performance criterion: income-distribution considerations must also be taken into account. Then in Section 3 we investigate whether optimal bonus functions exist when the objectives of superiors include distributional concerns. Our results are largely negative, indicating that informational problems will affect economic outcomes even when the performance criterion is formulated on the basis of normative criteria.

2. THE BONUS FUNCTION UNDER NORMATIVE CONSIDERATIONS

The formulation of the superior's objective function in Conn's analysis is based on the presumption that Pareto efficiency will be the sole criterion of choice in society. We agree that efficiency is an important criterion but feel that income distribution should also be a criterion. Hence, we examine in this section whether the pursuit of efficiency is likely to conflict with the pursuit of distributional goals.

Conn (1982, p. 230) states that: "It should be noted that the A_i [lump-sum] terms can be set a priori so as to achieve any desired expected level of managerial bonuses." One might infer from this statement that lump-sum transfers can be used to accomplish distributional goals without affecting efficiency. In the present section we shall present new results showing that, if economy-wide (rather than partial equilibrium) efficiency conditions are imposed, the A_i cannot be set a priori. Thus, in general, distributional considerations must conflict with efficiency considerations.

Since we use essentially the same notation as Miller and Murrell (1981) and Conn (1982), we shall simply list that notation without describing the model in detail: the n subordinates are subscripted $i = 1, \ldots, n$;

k_i is the allocation by the center of the scarce resource to subunit i;
e_i is i's effort input;
$Q_i(k_i, e_i)$ is i's performance function, which we will call "profit";[5]
B_i is the bonus paid to the ith subordinate; each subordinate has a utility function $U_i(B_i, e_i) = B_i - V_i(e_i)$;

a superscript "F" on either Q_i or V_i indicates that the value of the function is the one sent in a message from subordinate to superior;

$\bar{K}$ is the total amount of resource available;
y_0 is the information known by the superior before the resource is allocated: $y_0 = (Q_1^F, \cdots Q_n^F, V_1^F \cdots, V_n^F)$.

The Paretian objective function is:

$$W(k, e) = \sum_i (Q_i - B_i) + \sum_i U_i$$
$$= \sum_i Q_i - \sum_i V_i. \tag{1}$$

Conn shows that (1) is maximized by implementing an allocation rule k^* and a bonus rule B^* that satisfy the following:

(A) $k_i^*(y_0)$ and $e_i^*(y_0)$ for $i = 1, \ldots, n$ are the values that maximize

$$\sum_i Q_i^F(k_i, e_i) - \sum_i V_i^F(e_i). \tag{2}$$

(B) The resource is allocated according to the values k_i^*.
(C) The bonus function is of the form:

$$B_i^*(Q_i, y_0) = Q_i(k_i^*(y_0), e_i) + \sum_{j \neq i} Q_j^F(k_j^*(y_0), e_j^*(y_0)) - \sum_{j \neq i} V_j^F(e_j^*(y_0)) - A_i \tag{3}$$

where A_i is any term independent of Q_i^F and V_i^F but may depend on Q_j^F and V_j^F for $j \neq i$.

Obviously the income distribution consequent on (3) is indeterminate as the amount A_i is not uniquely determined. However, as we will show, the reason why previous studies have been able to claim that the absolute level of the bonuses can be shown independently of efficiency considerations is that these studies have omitted an important element of the economic process from their models. These studies have ignored the fact that there is an opportunity cost in employing a person to manage an enterprise.[6] If the benefits produced by the enterprise do not exceed that opportunity cost, then inefficiency results. Thus, this opportunity cost must be introduced explicitly into the analysis.

The introduction into the model of the opportunity cost to society of employing the manager in a particular enterprise brings to the fore the distributional consequences of imposing the requirement of efficiency. That opportunity cost is determined by the best alternative use of managerial labor, which we will assume here to be employment as ordinary labor. The opportunity cost equals, therefore, the value of marginal product of nonmanagerial labor. However, when there is a free labor market,[7] efficiency considerations dictate that the value of marginal product is also the wage of nonmanagerial labor. It is this dual role played by the opportunity cost of managers that leads to the distributional consequences.

Efficiency dictates that the enterprise should close down if the benefits it produces do not exceed its costs. We view an enterprise as an institution where a manager organizes productive inputs in order to produce output. With this view there is no need to assume that the enterprise, as such, is productive. Thus, when the manager leaves there is zero production and the enterprise closes down. Efficiency, therefore, requires that the manager should leave the enterprise if the consequent net decrease in $\sum_{i=1}^n Q_i$ is less than the opportunity cost of managerial labor. That opportunity cost is also the manager's reservation wage. The manager will leave the enterprise if the bonus does not exceed this wage. Hence, in order that the manager's decision to leave be an appropriate one from an efficiency perspective, the absolute size of the bonus must be chosen correctly. The requirement of efficiency determines the size of the bonus and hence the distribution of income.

In order to embody these considerations in a formal model, further notation must be introduced. Let us assume that the manager can earn a

[5] We shall assume that $Q_i(\cdot)$ is defined in such a way that it is an appropriate performance function for an economy pursuing efficiency. We also assume that the manager of an enterprise knows $Q_i(\cdot)$ exactly. That is, we are considering a situation where incumbent managers are transmitting information about existing enterprises.

[6] Loeb and Magat (1978) make an opportunity-cost argument. However, their argument is only suggestive. Furthermore, their paper does not discuss managerial effort, which plays a crucial role in the present analysis. The structure of the "Clarke [1971] tax" is similar to what we propose, but is applied in a different context. See also Green and Laffont, 1979.

[7] There are two ways in which this assumption can be justified. First, one can look upon the assumption of a free labor market as an assumption based on normative principles: a "just" society will allow free movement of labor. Second, it can be noted that planned economies such as the one in the Soviet Union, as well as market economies, use markets to allocate labor.

fixed wage, $\bar{U}$, outside the enterprise without using any effort.[8] Thus, $\bar{U}$ will also be the manager's value of marginal product outside the enterprise. Let $k_j^{*i}(y_0)$ and $e_j^{*i}(y_0)$ be the solutions to the maximum problem at (2) for subunit j, when enterprise i is not allocated any of the resource: they are the maximizing values when enterprise i is closed down.[9] It is now possible to prove the following (the proof is relegated to the Appendix):

PROPOSITION 1. *The bonus function, which is optimal with respect to criterion (1), is unique and has the following form:*

$$B_i^*(Q_i, y_0) = Q_i(k_i^*(y_0), e_i) + \sum_{j\neq i} Q_j^{\mathrm{F}}(k_j^*(y_0), e_j^*(y_0)) - \sum_{j\neq i} V_j^{\mathrm{F}}(e_j^*(y_0))$$
$$- \sum_{j\neq i} Q_j^{\mathrm{F}}(k_j^{*i}(y_0), e_j^{*i}(y_0)) + \sum_{j\neq i} V_j^{\mathrm{F}}(e_j^{*i}(y_0)),$$

where k^* and e^* are as defined in (A) and (B) above.

Before commenting on the implications of this bonus function, it is necessary to show that implementation of such a function is feasible. As the absolute size of the bonuses are determined by the production system itself and are not themselves an object of choice, it must be shown that the total size of the bonuses will not exceed the center's capacity to pay them. Since Q_i is net profits, $\sum_i Q_i$ is the total amount of funds available to pay bonuses.

Thus, the surplus available after the payment of bonuses is:

$$\sum_{i=1}^{n} \{\sum_{j\neq i} [Q_j^{\mathrm{F}}(k_j^{*i}, e_j^{*i}) - V_j^{\mathrm{F}}(e_j^{*i})]\} - \sum_{i=1}^{n} \{\sum_{j\neq i} [Q_j^{\mathrm{F}}(k_j^{*}, e_j^{*}) - V_j^{\mathrm{F}}(e_j^{*})]\}.$$

It is easy to see that this expression is positive. The first term is the welfare produced by $n - 1$ firms when the remaining firm receives no resource. The second term is the welfare produced by the same $n - 1$ firms when the remaining firm receives some of the resource. Since more of the resource is shared out between the $n - 1$ firms in the first term than in the second term, the first term must be greater than the second. The bonuses will not exhaust the funds available for their payment.

[8] Assuming no effort exertion in the ordinary labor market simplifies the analysis without affecting the conclusions. The use of a minimum utility level in such a way is common in the principal–agent literature (see, for example, Holmström, 1979).

[9] The process can be thought of in the following way. The center announces the manner in which bonuses will be calculated. Messages are then sent to the center. The center proposes an input allocation and communicates this to the enterprises. At this time, managers who want to leave enterprises tell the center. A new proposed allocation is then communicated to the remaining enterprises. This process continues until all managers would choose to remain. Inputs are then actually allocated.

We do not claim to have shown that such a process will actually be stable and lead to an equilibrium. Rather, we have shown that, if there is an equilibrium, the bonus must conform to the one derived in Proposition 1.

To summarize, we have shown that the bonus function, including the element A_i, is unique and we have identified the functional form of A_i. These results show that, if efficiency is required, the size of managerial bonuses cannot be an instrument of government policy. There is a unique distribution of managerial incomes that results once the criterion of efficiency is specified.

We believe that it would be inappropriate to conclude from the above analysis that the center cannot pursue its distributional goals. Recognition of the result that the requirement of efficiency leads to uniquely determined managerial bonuses does not lead to the conclusion that the center should not have preferences over the size of those bonuses. Normative considerations would seem to dictate that the center should have distributional as well as efficiency goals. The appropriate conclusion to be drawn from the above results is that distributional goals cannot be pursued separately because there is no degree of freedom in the specification of the bonus function. Hence, all goals must be embodied directly in the objective function of the center. It is to a consideration of the consequences of the inclusion of distributional goals in the objective function that we now turn.

3. THE BONUS FUNCTION WHEN THE CENTER HAS PREFERENCES OVER THE DISTRIBUTION OF INCOME

In taking into account distributional considerations, we will preserve the linearity of the objective function (in order to change the original problem as little as possible) but discount the utilities of managers by a parameter $\phi < 1$:[10]

$$W_\phi(k, e) = \sum_i (Q_i - B_i) + \phi \sum_i U_i = \sum_i Q_i - \phi \sum_i V_i - (1 - \phi) \sum_i B_i. \quad (4)$$

The simple modification of the welfare function using the parameter ϕ is a natural way to proceed since, as was shown in the previous section, managers will receive bonuses higher than their opportunity wage.

In our previous analysis (Miller and Murrell, 1981) we examined a criterion similar to (4), except that effort was not a variable and $\phi = 0$. We

[10] To avoid needless notational complexity, we will not redefine the objective function so that it allows for the possibility that one of the enterprises shuts down and the manager works in the ordinary labor force.

proved that the center could not construct a bonus scheme that would lead to maximization of their objective function. A cursory inspection of our proof (Miller and Murrell, 1981, pp. 268, 269) shows that it applies when effort is a variable and for all values of ϕ such that $0 \leqq \phi < 1$.[11] Therefore, the same proof applies here, and little would be gained by repeating it. We simply invoke that result to establish that the center cannot find a bonus function that will allow it to obtain the full information value of the objective function (4).

Conn (1982), however, has presented an alternative form of the objective function. He assumes that the center does not want to economize on effort as much as the subordinates themselves do. The new objective function is

$$W_\alpha(k, e) = \sum_i Q_i(k_i, e_i) - \sum_i \alpha V_i(e_i), \qquad \text{where} \qquad 0 < \alpha < 1. \tag{5}$$

Conn (1982, p. 230) claims that an optimal bonus function exists for this objective function. We shall examine this claim in the light of the analysis of Section 2.

Using Conn's analysis and the proof of our Proposition 1, it is easy to show that if an optimal bonus function exists for the new objective function (5), it must be the form:

$$B^*_{i\alpha} = (1/\alpha)[Q_i(k^*_i, e_i) + \sum_{j \neq i} \{Q^F_j(k^*_j, e^*_j) - \alpha V^F_j(e^*_j)\} - \sum_{j \neq i} \{Q^F_j(k^{*i}_j, e^{*i}_j) - \alpha V^F_j(e^{*i}_j)\}] \tag{6}$$

[11] In explanation of this point, let us focus first on the role of effort. The proof in our previous paper showed that the full-information maximum value of the objective function could not be obtained for the case in which effort was not variable. Constant effort is just a special case of the present model. A necessary condition for the existence of an optimal bonus in the situation depicted in the present paper is that there exists a bonus that is optimal when effort is constant. Therefore, adding effort to the model of our previous paper cannot invalidate a proof showing that a bonus function does not exist. Second, when $0 \leqq \phi < 1$, attainment of the full-information maximum of value of (4) requires that managers receive $\bar{U}$. Thus, $\bar{U}$ in the present model plays a role equivalent to that of the minimum bonus payments, $\bar{B}_i$, in the model of our previous paper. Maximization of (4) with $0 \leqq \phi < 1$ is equivalent to maximization of (4) with $\phi = 0$ subject to the constraint that managers earn at least $\bar{U}$. The latter maximization is the one analyzed in our previous paper. Thus, having $\phi \neq 0$ does not invalidate the proof of Theorem 3 if $\phi < 1$. Third, although we allow in the present model the possibility that enterprises can close, a necessary condition for an optimal bonus system is that the bonus works when production conditions are such that all enterprises remain open. In the previous paper, we assumed enterprises did not close down. Thus, a proof that an optimal bonus system does not exist for the present model with the objective function (4) follows exactly the proof of Theorem 3 in our previous paper.

where k^*_j and e^*_j are determined by maximization of (5) with Q_j and V_j replaced by Q^F_j and V^F_j, and k^{*i}_j and e^{*i}_j are determined by the same process except that firm i receives none of the resource.[12]

However, as already explained in the previous section, before one can assume that such a bonus is implementable one must prove that the center will have enough funds to pay all bonuses. In fact such a proof is impossible. The bonus system may be unimplementable, as the following shows:

PROPOSITION 2. *Given the bonus function (6), there always exist values of the parameters of production and utility functions such that the total size of bonuses exceeds the center's capacity to pay bonuses (proof in Appendix).*

The importance of Proposition 2 is that it shows that it would be impossible for the center to use the bonus function (6). The center does not know the values of the relevant parameters before it sets the bonus function and thus can never guarantee that bonuses will conform to (6). Hence, the center would need to specify how bonus payments would be modified if those promised in (6) exceeded the center's ability to pay. However, such a specification amounts to a fundamental modification of the bonus system. Since we know that (6) is the *only* possible optimal bonus function, the modified function could not be optimal. Hence, an immediate corollary of Proposition 2 is the following.

PROPOSITION 3. *There exists no optimal bonus function that would allow the center to obtain the full-information maximum value of the objective function (5).*

Thus, we have shown that when income-distribution considerations are introduced directly into the objective function of superiors, there exists no optimal bonus function. Since we argue in Section 2 that normative principles imply that the objective function should be formulated in such a way, the present section's results are consistent with the general conclusion

[12] In deriving the bonus formula further assumptions were needed concerning the way in which the value of marginal product of nonmanagers should be counted in the objective function. There are a variety of ways in which these assumptions based on different arguments about the objectives of the center could be made. Here it is not necessary to delve into extended discussion of these arguments because the final conclusions reached are independent of the particular formulation adopted. However, since details in the proofs depend upon the assumptions, we will state our assumptions explicitly. We interpret the objective function as indicating that the center wants to obtain greater output even at the cost of above-Pareto-efficient levels of effort. Hence, we assume that output outside the enterprise counts equally with output in the enterprise in the center's objective function and that nonmanagers are paid $1/\alpha$ times their value of marginal product (to encourage appropriate effort levels).

reached in Miller and Murrell (1981). This conclusion is that informational problems cannot be overcome solely with the use of bonus functions.

4. CONCLUSION

We have argued in this paper that, when Pareto efficiency is required, there is a unique bonus function and therefore a unique distribution of managerial incomes. Hence, we argue that income-distribution considerations must be embodied directly in the objective function. However, in Section 3 we show that, once such a change in the objective function is introduced, an optimal bonus function can no longer be found.

The importance of the results in this paper and in Miller and Murrell (1981) can only be understood when contrasted with those of Groves (1973), Loeb and Magat (1978), and Conn (1979, 1982). The latter set of papers prove that in some circumstances the problems associated with initial decentralization of information can be overcome. In contrast, we have argued here that the results contained in that set of papers are not likely to be relevant in any practical situation. We argue that, in fact, the objectives of superiors will be such that an optimal bonus function cannot be found. This argument implies that the initial decentralization of information will affect economic outcomes in significant ways.

APPENDIX: PROOFS OF PROPOSITIONS

PROPOSITION 1. Define a managerial-choice variable, θ_i:

$$\theta_i = 1, \quad \text{if manager decides to stay in the enterprise.}$$
$$= 0, \quad \text{if manager decides to leave the enterprise.}$$

Define the following new functions:

$$G_i(k_i, e_i, \theta_i) = \theta_i Q_i(K_i, e_i) + (1 - \theta_i)\bar{U}_i,$$
$$Z_i(e_i, \theta_i) = \theta_i V_i(e_i).$$

In order to obtain efficiency, the center must maximize:

$$\sum_i G_i - \sum_i Z_i.$$

Thus, solely by defining these new functions, we have made the problem equivalent to the incentive problem discussed by Conn (1982). Hence, the bonus-function and resource-allocation procedures must be as defined in the text by (A), (B), and (C), with now Q_i replaced by G_i, V_i replaced by Z_i and the decision variable e_i replaced by the vector (e_i, θ_i). Here, it is

important to note that the additive element A_i must be independent of G_i^F and Z_i^F but can depend upon G_j^F and Z_j^F (for $j \neq i$). Hence, A_i cannot depend upon θ_i because the manager's choice of θ_i depends upon k_i^*, which depends upon G_i^F and Z_i^F. The importance of this point is that the functional form of B_i^* cannot change with the value of θ_i: the bonus when the manager is in the enterprise is partially determined by the payment made when the manager decides to seek employment elsewhere.

A manager working outside the enterprise earns U_i. (Note that, for efficiency reasons, it is important that the manager earn exactly this amount, so that the manager must not be paid anything by the enterprise in this situation.) Thus, treating B_i^* as the payment to the manager, whether or not the manager is in the enterprise, we know that $B_i = \bar{U}_i$ when $\theta_i = 0$. Hence, A_i can be found by using the bonus formula (3), with Q_i and V_i replaced by G_i and Z_i, and substituting in $B_i = \bar{U}_i$ when $\theta_i = 0$:

$$\bar{U}_i = G_i(k_i, e_i, 0) + \sum_{j \neq i} G_j^F(k_j^*, e_j^*, \theta_j^*) - \sum_{j \neq i} Z_j^F(e_j^*, \theta_j^*) - A_i.$$

Now $G_i(k_i, e_i, 0) = \bar{U}_i$ and when $\theta_i = 0$ the optimal values of k_i^* and e_i^* are zero. Thus, when $\theta_i = 0$, $k_j^* = k_j^{*i}$ and $e_j^{*i} = e_j^*$. Hence,

$$A_i = \sum_{j \neq i} G_j^F(k_j^{*i}, e_j^{*i}, \theta_j^*) - \sum_{j \neq i} Z_j^F(e_j^{*i}, \theta_j^*).$$

In order to present A_i in a form that is intuitively interpretable, assume that $\theta_j = 1$ for $j \neq i$ and then:

$$A_i = \sum_{j \neq i} Q_j^F(k_j^{*i}, e_j^{*i}) - \sum_{j \neq i} V_j^F(e_j^{*i}).$$

The uniqueness of the bonus scheme is guaranteed by the theorems of Conn (1982). However, here this uniqueness also applies to the element A_i, as defined above.

PROPOSITION 2. In the following proof, we show that there is at least one configuration of the parameters of the problem for which the bonus function is infeasible. Thus, we shall use specific functional forms for the utility and production functions and find values for the parameters of those functions, the number of firms, and $\bar{U}$, which lead to infeasibility.

Assume that there are n identical firms. The number of firms that stay open is actually endogenous but later in the proof we shall construct a value of $\bar{U}$ such that all n firms are required to produce. Therefore, in the following example, all n firms remain open.

If the bonus function works correctly, then everybody is honest and messages are equal to the true values of the relevant functions. The total fund available for the payment of bonuses is

$$\sum_{i=1}^{n} Q_i(k_i^*, e_i^*).$$

Subtracting from this the total amount of bonuses, one obtains the surplus:

$$\frac{1}{\alpha}\sum_{i=1} [\sum_{j\neq i} \{Q_j(k_j^{*i}, e_j^{*i}) - \alpha V_j(e_j^{*i})\}$$

$$- \sum_{j\neq i} \{Q_j(k_j^*, e_j^*) - \alpha V_j(e_j^*)\}] - \left(\frac{1}{\alpha} - 1\right) \sum_{i=1}^{n} Q_i(k_i^*, e_i^*).$$

Assume all enterprises have the same net profit function, $k^\gamma e^\beta$, and all managers have the same effort-aversion function $V(e) = e$. Then when the bonus function is (6), managers, given a value of k, maximize their utility by choosing a value of e. Thus, e^* will be a function of k^*. Using this function to substitute for e^* in $Q_i - \alpha V_i$ and in the expression for managerial utility, the maximizing behavior of managers implies that (using the fact that $k^* = \bar{K}/n$):

$$Q_i(k_i^*, e_i^*) - \alpha V_i(e_i^*) = A\left(\frac{\bar{K}}{n}\right)^{\gamma/1-\beta}, \tag{A.1}$$

where

$$A = \left\{\left(\frac{\beta}{\alpha}\right)^{\beta/1-\beta} - \alpha\left(\frac{\beta}{\alpha}\right)^{1/1-\beta}\right\},$$

and

$$B_i^*(Q_i, y_0) - V(e_i^*) = \frac{A}{\alpha} \bar{k}^{\gamma/1-\beta}\{n^{1-\gamma-\beta/1-\beta} - (n-1)^{1-\gamma-\beta/1-\beta}\}$$

$$+ \left(\frac{1}{\alpha} - 1\right)\left\{\left(\frac{\beta}{\alpha}\right)^{1/1-\beta} \bar{K}^{\gamma/1-\beta}\right\}. \tag{A.2}$$

Therefore:

$$\text{Surplus} < \frac{1}{\alpha}\sum_{i=1}^{n}\left[\sum_{j\neq i} A\left(\frac{\bar{K}}{n-1}\right)^{\gamma/1-\beta} - \sum_{j\neq i} A\left(\frac{\bar{K}}{n}\right)^{\gamma/1-\beta}\right]$$

$$- \left(\frac{1}{\alpha} - 1\right)\left[\sum_{i} A \cdot \left(\frac{\bar{K}}{n}\right)^{\gamma/1-\beta}\right]$$

$$= An\bar{K}^{\gamma/1-\beta}\left[\left(\frac{n-1}{\alpha}\right)\cdot(n-1)^{\gamma/\beta-1} - \left(\frac{n-1}{\alpha}\right)\cdot n^{\gamma/\beta-1} - \left(\frac{1}{\alpha} - 1\right)n^{\gamma/\beta-1}\right].$$

Thus, the surplus is less than zero when:

$$\left(\frac{n}{n-1}\right)^{\gamma/1-\beta} < \left\{\frac{n-\alpha}{n-1}\right\} \tag{A.3}$$

and each manager's welfare is greater than zero when:

$$\gamma + \beta < 1. \tag{A.4}$$

Thus, if $n \geqslant 2$, then for every $0 < \alpha < 1$, there is an infinite number of values of α and β that lead to a surplus less than zero. At the same time, these values of α and β can satisfy (A.4), so that there must exist a $\bar{U} > 0$ such that every firm does, and should, stay open and produce.

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