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# Optimal growth models as Economic Planning Tools*

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## 1. Introduction

In the last 15 years, much effort has been devoted to the theoretical analysis of optimal growth models. Yet, little attention has been paid to the applicability of these models. The natural domain of application would be in planning (see, for example, Gale and Cass). However, no study has been made to investigate directly the relevance of optimal growth models as planning tools. In this paper, I undertake such an investigation.

Before proceeding, it is necessary to introduce some terminology. The nature of planning will depend on the 'economic structure' of a country. The economic structure is the set of institutions, both tangible and intangible, which influence the nature of economic activity. The 'controlled' sector of an economy is that sphere which the government directs. In formulating a planning model, one can assume that the controlled sector follows direct orders. There is no need to specify the instruments of direction.[1] In contrast, economic units within the 'uncontrolled' sector are free to pursue their own goals subject to legal limits and the influence of government instruments. Those limits and the nature of the instruments will be defined before plan construction. Thus, the distinguishing feature of the uncontrolled sector is that the extent to which policy-makers can influence this sector is circumscribed by law.[2]

Abstract mathematical models usually ignore the economic structure of any particular economy. This is a valid procedure in some contexts, but does obscure the use of the model. Thus, in this paper, I investigate which economic structures could possibly be relevant to optimal growth models. By undertaking such an investigation, it is possible to show that these models are inapplicable, as presently formulated. Therefore, I present a method of plan construction, using optimal growth methods, which confronts the issue of making the plan suitable for a given economic structure. In presenting this general method, mention is made of new issues which arise when one takes due account of the economic structure during plan construction.

## 2. The Interpretation of Planning Models

The objective of plan construction is to find the values of M endogenous variables in each time period. Information will be derived from one of three sources:

(a) Behavior of uncontrolled agents (N items of information).

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[1]The theory of this paper is consistent with the notion that the controlled sector is non-existent because a government is unable to control its nominal subordinates.

[2]As noted in a later section, all economies will have an uncontrolled sector.

(b) Economy-wide identities (R items).
(c) Values of government instruments and strategies of the controlled sector (S items).

ne information must be sufficient to enable planners to build a determinate forecast. 'ith M endogenous variables, one needs at least M sources of information: $\leqq N + R + S$.

In using optimizing models, it is easy to forget that $M \leqq N + R + S$ must be satisfied. ecause these models produce a determinate solution, any user will be implicitly suming that $S \leqq M - N - R$ or equivalently that government controls enough of the onomy to make the optimal plan implementable. However, assumptions on government control should not arise due to the nature of the modelling method chosen. ather those assumptions should be formulated before a model is constructed. Planners ould describe the economic structure before building a model. In contrast, optimal owth models are formulated with no particular economic structure in mind. In the llowing subsections, I investigate which economic structures are relevant for optimal owth models by examining a simple model.

## (a). The Optimal Growth Model

The two sector model is presented in order to use it as an example of optimal growth odels in general.[3] Two goods are produced in separate sectors: capital and consumpon. Capital once installed in one sector cannot be removed from that sector. Subscript refers to the consumption sector, 2 to the capital sector. The following variables are sed: $L$ = constant population (equal to the labor force), $c$ = per capita consumption, = society's time preference rate, $\delta$ = capital depreciation rate, $L_i$ = labor allocated to e ith sector, $K_i$ = capital stock in the ith sector, $Y_i$ = investment flow to the ith sector, $_i(K_i, L_i)$ = production function in ith sector. Society's objective is to maximize the nctional:

$$\int_0^\infty e^{-\rho t} L(t)u(c(t))dt. \qquad (1)$$

'aking into account the usual constraints and introducing the artificial variables $\phi_i$, $\eta_i$, $_i$, and $\theta$, one can apply the Maximum Principle to obtain the following necessary onditions:

$$u'(c) = \phi_1 \qquad (2)$$
$$\phi_i F_{iL} = \theta \qquad i = 1, 2 \qquad (3)$$
$$\pi_i + \eta_i = \phi_2 \qquad i = 1, 2 \qquad (4)$$
$$\eta_i Y_i = 0, \eta_i \geqq 0, Y_i \geqq 0 \qquad i = 1, 2 \qquad (5)$$
$$L = L_1 + L_2 \qquad (6)$$
$$F_1(K_1, L_1) = Lc \qquad (7)$$
$$F_2(K_2, L_2) = Y_1 + Y_2 \qquad (8)$$
$$\dot{\pi} = (\rho + \delta)\pi_i - \phi_i F_{ik} \qquad i = 1, 2 \qquad (9)$$
$$\dot{K}_i = Y_i - \delta K_i \qquad i = 1, 2 \qquad (10)$$

'hus, one has a 14-equation system in 14 variables: $(c, Y_i, L_i, \phi_i, \dot{K}_i, \dot{\pi}_i, \theta)$. ($\pi_i$ and $K_i$ re determined by the previous behavior of the system.) There are problems in btaining a solution to this model which need not be discussed here. Ryder has btained a solution for a somewhat simpler model.

[3]For more details on this model see Ryder. All variables are functions of time but the time argument will be nored when no ambiguity results (e.g. $Y \equiv Y(t)$). A dot above a variable represents rate of change (e.g. $\dot{Y} = dy(t)/dt$).

## 2 (b) A Perfectly Competitive System

In the presentation of the two sector model, there has been no discussion concerning the type of economy to which the model could be applied. Thus, one must examine the results of the model in order to obtain some clue about the domain of its applicability. One standard result is that $\theta$, $\pi_i$, and $\phi_i$ can be viewed as shadow prices (see Arrow and Kurz)[4]. Then (3) simply says that the marginal value product of labor must equal the wage rate. This condition obtains in perfectly competitive markets but not in imperfectly competitive markets. Thus, in looking for an economic system which is not totally controlled and which may behave as described by the optimal growth model, one must examine a perfectly competitive system.

A description of a perfectly competitive system will now be given. Assumptions are made in order to make that system as similar as possible to the optimal growth model. Thus, no mention will be made of distributional issues and monetary flows are ignored. Let $P_i$ = price of the ith good, $a_i$ = discount rate of ith producer, and w = wage rate.[5] The planning period for each firm is [0, T].[6] Each producer will maximize:

$$\int_0^T e^{-a_i t} (P_i F_i (K_i, L_i) - wL_i - P_2 Y_i) dt. \qquad (11)$$

To apply the Maximum Principle to (11), use $q_i$ and $S_i$ as artificial multiplier variables. After obtaining a system of equations by maximizing (11) for each producer, there is one degree of freedom due to the indeterminancy of absolute prices. Thus, one can choose a numeraire. Let $P_1 = u'(c)$ at each instant. Then the perfectly competitive system follows the following set of equations:

$$u'(c) = P_1 \qquad (12)$$
$$P_i F_{iL} = w \qquad i = 1, 2 \qquad (13)$$
$$S_i + q_i = P_2 \qquad i = 1, 2 \qquad (14)$$
$$S_i Y_i = 0 \qquad S_i \geqq 0,\ Y_i \geqq 0,\ i = 1, 2 \qquad (15)$$
$$L = L_1 + L_2 \qquad (16)$$
$$F_1 (K_1, L_1) = Lc \qquad (17)$$
$$F_2 (K_2, L_2) = Y_1 + Y_2 \qquad (18)$$
$$\dot{q}_i = (a_i - \delta_i) q_i - P_i F_{ik} \qquad i = 1, 2 \qquad (19)$$
$$\dot{K}_i = Y_i - \delta K_i \qquad i = 1, 2 \qquad (20)$$

## 2 (c). Comparison of the Two Systems

The systems of equations are nearly identical. The difference occurs between equations (9) and (19). Society's discount rate ($\rho$) is not necessarily equal to the discount rates ($a_i$) of individuals in that society. In order to compare the two systems analytically, it is necessary to introduce notation to distinguish the two systems. Label with a superscript$^0$ (e.g. $K_1^0$) real variables along the optimal growth path and with a superscript$^1$ the same variables along the perfectly competitive path. Assume that the two systems start at the same place: $K_i^0(0) = K_i^1(0)$. The comparison is crystallized in the following theorem:

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[4]See Arrow and Kurz Chapter 2 for the method used in deriving the necessary conditions. The derivation is spelled out in detail in Murrell.

[5]To keep the notation simple, real variables will be given the same symbols as in the optimal growth model.

[6]At this stage, terminal conditions can be ignored.

*heorem*

The behavior of the optimal growth system (2)-(10) will differ from that of the erfectly competitive system (12)-(20) in the paths of real variables. This result is ndependent of the terminal conditions for the two systems.

*roof*

The proof will analyze behaviour at the initial instant. Equations (4), (8), (14), and 18) give: $\phi_2 = \max(\pi_i)$ and $P_2 = \max(q_i)$. The two subsystems [(2), (3), (6), and (7)] and (12), (13), (16), and (17)] are identical. Totally differentiating these two sub-systems, ne can show that $L_i^0$ is a unique function of $(\phi_2, K_1^0, K_2^0)$ and that $L_i^1$ is the same unique unction of $(P_2, K_1^1, K_2^1)$.[7]

By the same method,

$$\frac{\partial L_1^0}{\partial \phi_2} < 0, \frac{\partial L_2^0}{\partial \phi_2} > 0 \text{ and } \frac{\partial L_1^1}{\partial P_2} < 0, \frac{\partial L_2^1}{\partial P_2} > 0.$$

There are two possible alternatives for the initial conditions: (i) $\phi_2(0) \neq P_2(0)$ and (ii) $\phi_2$ 0) $= P_2(0)$. Each alternative will be examined separately. Under (i) $[K_1^0(o), K_2^0(o)] =$ $K_1^1(o), K_2^1(o)]$ but $\phi_2(o) \neq P_2(o)$. Hence, using the results stated for the two sub-systems, $L_i^0(o) \neq L_i^1(o)$. Hence, $c^0(o) \neq c^1(o)$. The two systems display different behaviors if (i) is he initial condition.

Under (ii): $[P_2(o), K_1^1(o), K_2^1(o)] = [\phi_2(o), K_1^0(o), K_2^0(o)]$. (For brevity's sake, the zero ime argument is omitted in the rest of the proof; all variable values are initial values.) Previously stated results imply that $L_i^1 = L_i^0$,

$\frac{\partial L_i^0}{\partial \phi_2} = \frac{\partial L_i^1}{\partial P_2}, \frac{\partial L_i^0}{\partial K_j^0} = \frac{\partial L_i^1}{\partial K_j^1}$ for i = 1, 2. Totally differentiating (7) and (17):

$$L\dot{c}^i = F_{1K}^i \dot{K}_1^i + F_{1L}^i \dot{L}_1^i$$

$$\text{Thus, } L\dot{c}^0 = F_{1L}^0\left(\frac{\partial L_1^0}{\partial \phi_2}\dot{\phi}_2 + \frac{\partial L_1^0}{\partial K_2^0}\dot{K}_2^0 + \frac{\partial L_1^0}{\partial K_1^0}\dot{K}_1^0\right) + F_{1k}^0 \dot{K}_1^0$$

$$\text{and } L\dot{c}^1 = F_{1L}^1\left(\frac{\partial L_1^1}{\partial P_2}\dot{P}_2 + \frac{\partial L_1^1}{\partial K_2^1}\dot{K}_2^1 + \frac{\partial L_1^1}{\partial K_1^1}\dot{K}_1^1\right) + F_{1K}^1 \dot{K}_1^1.$$

$$\text{Let } A = F_{1L}^0\frac{\partial L_1^0}{\partial \phi_2} = F_{1L}^1\frac{\partial L_1^1}{\partial P_2}, B = F_{1L}^0\frac{\partial L_1^0}{\partial K_2^0} = F_{1L}^1\frac{\partial L_1^1}{\partial K_2^1}$$

$$\text{and } C = F_{1L}^0\frac{\partial L^0}{\partial K_1^0} + F_{1K}^0 = F_{1L}^1\frac{\partial L^1}{\partial K_1^1} + F_{1K}^1.$$

$$\text{Thus, } L(\dot{c}^1 - \dot{c}^0) = A(\dot{P}_2 - \dot{\phi}_2) + B(\dot{K}_2^1 - \dot{K}_2^0) + C(\dot{K}_1^1 - \dot{K}_2^0)$$

Comparing (9) and (19), if $a_i \neq \rho$, then $\dot{P}_2 \neq \dot{\phi}_2$. Therefore, $\dot{c}^1 - \dot{c}^0 = 0$, $\dot{K}_1^1 - \dot{K}_1^0 = 0$, and $\dot{K}_2^1 = \dot{K}_2^0 = 0$ cannot hold simultaneously since A, B, C and $L \neq 0$. Therefore, when $\phi_2(0) = P_2(0)$ the two systems will proceed on differing paths.

[7]Using the implicit function theorem, see Apostol p. 147.

The theorem has shown that the optimal growth model cannot be used for planning purposes in a perfectly competitive economy. The shadow prices and interest rates of the optimal growth model, and its intersectoral allocation of capital, will differ from those of a perfectly competitive economy. A government which did use the optimal growth model's results in a competitive context would have no theoretical justification; less still could they describe such use as 'optimal'. It is not possible to prove that, in general, the optimal growth model will have no relevance to any other economy than a totally controlled one. It is always feasible that some other type of economy will have an identical path of development. Yet such will only happen adventitiously because all economies will have different equations of motion from those of the optimal growth model. Thus, one may conclude that optimal growth models, as presently formulated, will not be relevant to any economy other than ones with a totally controlled economic structure.

## 3. The Consumption Sector and Optimal Growth

The previous model ignored choice by consumers. Yet most economies are likely to have some freedom of choice by consumers. A completely controlled economy is unlikely ever to exist. Therefore, one must examine the conditions which must be placed on optimal growth models because consumer decisions are not completely controlled. Given that consumers make free decisions, efficiency requires that prices reflect consumer tastes. The formation of consumption good prices can be represented formally. Let $(c_1 \ldots c_n) = c$ be a vector of consumption goods, $(P_1 \ldots P_n) = P$ be a vector of consumption prices and $M$ = aggregate money income. The demand for goods can be represented by:

$$c_i = d_i(P, M) \qquad i = 1, \ldots, n \tag{21}$$

Recognizing the homogeneity of degree zero of the demand functions and assuming that inverse functions exist one can write:

$$P_i = Mg_i(c) \tag{22}$$

The functions $g_i(\cdot)$ give the market-clearing prices which would be formed if aggregate money income was M and consumer goods equal to c were supplied to the market. The prices represent consumer valuations of the goods. Thus, one condition for efficiency is that the value of the marginal product of labor when evaluated at these prices should be equal in all sectors:

$$\frac{P_i F_{iL}}{P_j F_{jL}} = 1 \text{ where } F_{iL} = \frac{\partial F_i(K_i, L_i)}{\partial L_i} \qquad i, j = 1, \ldots, n.$$

Thus, efficiency requires $$\frac{g_i(c)\, F_{iL}}{g_j(c)\, F_{jL}} = 1 \quad i, j = 1, \ldots, n \tag{23}.$$

Analogous conditions can be derived from optimal growth models. In those models, one maximizes a utility function subject to production constraints. By solving the first-order necessary conditions one can derive the following equations:

$$\frac{u_i(c)F_{iL}}{u_j(c)F_{jL}} = 1 \qquad i, j = 1, \ldots, n. \tag{24}$$

hus, if an optimal growth plan is to be efficient and reflect the development of a ociety with freedom of choice for consumer goods:

$$\frac{u_i(c)}{u_j(c)} = \frac{g_i(c)}{g_i(c)} \qquad i, j = \ldots, n \tag{25}$$

ust be satisfied for all feasible c. Objective functions constructed according to the references of governments will not, in general, satisfy (25). The requirement imposes e condition that the economy-wide instantaneous objective function should embody onsumer preferences. If the objective function does not satisfy (25), then the optimal rowth plan will only be relevant for an economy in which either there is central llocation of consumer goods or production decisions do not reflect consumer tastes.

## . Planning an Economy with an Uncontrolled Sector

Optimal growth theorists have avoided specification of the economic structures of he economies to which their models are to be applied. When formulating a model in uch a way, it will only be applicable to a totally controlled economy. Thus, planning xercises must always begin by reviewing the salient characteristics of the economy eing planned. The planners' model can then be based on these characteristics, implifying where necessary for practical reasons. In this section, the correct method of roceeding to build a plan is exemplified by describing a simple economy. Then the ole of planners can be made clear and the place of optimal growth methods in plan onstruction can be clarified.

### 1(a). A Description of an Economy

It is not necessary to describe a specific economy. Assumptions will be made to acilitate discussion. However, it must be realized that, where an assumption is made, n practice planners should be prepared to argue that the assumption is valid given the conomy being planned. The economy is a closed four sector economy.[8] Sector 1 roduces certain consumption goods, sector 2 all other consumption goods, sector 3 apital goods and sector 4 public goods. Define the following variables: $D_i$ = interest earing assets (debts) of ith producer, S = interest bearing assets of consumers, $P_i$ = price of ith good, r = interest rate, M = disposable income, $K_i$ = capital stock in ith sector, $Y_i$ = investment in the ith sector, $L_i$ = labor used in ith sector, L = total labor supply (exogenous), $\delta$ = capital depreciation rate, $c_i$ = production of ith good, T = tax collections, and w = wage rate. The following definitional relationships will hold:

Changes in producers' debt[9]: $\dot{D}_i = p_i c_i - wL_i - p_3 Y_i + rD_i \qquad i = 1, 2, 3.$ (26)

Change in national debt: $\dot{D}_4 = T - wL_4 - p_3 Y_4 + rD_4.$ (27)

Consumer income $= M = \sum_{i=1}^{4} wL_i + rS - T.$ (28)

Clearing of capital goods markets: $c_3 = \sum_{i=1}^{4} Y_i.$ (29)

[8]In practice, the assumption of a small number of sectors should be justified on the basis of an appropriate aggregation procedure.

[9]In order to avoid extended discussion of dividend policy, it is assumed that all earnings are retained by producers.

Having noted definitions, planners should now proceed to analyze the behaviour of uncontrolled economic agents. This analysis is, of course, the most difficult of the planners' tasks. This task will not only require much gathering of information but also some judicious simplifying assumptions based on heuristic evaluation of the most important features of the economy. In the present exercise, because the aim is to examine a methodology, the assumptions can be made in a somewhat more cavalier manner. Let us assume that consumers can be treated as a single homogeneous group and that their consumption and savings functions are the following:

$$\text{Clearing of labor market: } L = \sum_{i=1}^{4} L_i. \tag{30}$$

$$\dot{S} = H_3(p_1, p_2, M, r) \tag{31}$$

$$c_1 = H_1(p_1, p_2, M, r) \tag{32}$$

$$c_2 = H_2(p_1, p_2, M, r) \tag{33}$$

These equations will be presumed to satisfy the budget equation:

$$H_3(p_1, p_2, M, r) + \sum_{i=1}^{2} p_i H_i(p_1, p_2, M, r) = M$$

for any values of $(p_1, p_2, M, r)$.[10]

Let us suppose that producers act as perfect competitors. They face the following physical constraints on their activities[11]:

$$c_i = F_i(K_i, L_i) \qquad i = 1, \ldots, 4 \tag{34}$$

$$\dot{K}_i = Y_i - \delta K_i \qquad i = 1, \ldots, 4 \tag{35}$$

Let us assume that producers maximize the following:

$$\int_0^T e^{-ait}[p_i F_i(K_i, L_i) - wL_i + rD_i - p_3 Y_i]dt \; i = 1, 2, 3 \tag{36}$$

subject to constraints (26), (34), and (36). (At the present juncture, terminal objectives are ignored.) Then, using the Maximum Principle, producers will find that they must satisfy the following conditions (using artificial multiplier variables $q_{1i}$, $q_{2i}$, and $m_i$):

$$p_i F_{iL} = w \tag{37}$$

$$q_{2i} + m_i = (1 + q_{1i})p_3 \tag{38}$$

$$m_i Y_i = 0, \; m_i \gtreqless 0, \; Y_i \lesseqgtr 0 \tag{39}$$

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[10] The definitional equation $\dot{S} + \sum_{i=1} D_i = 0$ has been omitted from the list of definitions because it is immediately implied by the other definitions and the budget equation for consumers.

[11] The physical constraints on the public sector (subscript 4) have been included here for the sake of convenience.

$$\dot{q}_{1i} = a_i q_{1i} - r - q_{1i} r \quad (40)$$

$$\dot{q}_{2i} = (a_i + \delta)\, q_{2i} - (1 + q_{1i}) p_i F_{iK} \quad (41)$$

### 4(b) Government Behavior and the Planners' Task

Given that initial values of the state variables are available, the economic system described above has 37 unknowns at each instant of time: $\dot{D}_i$, $P_i$, $C_i$, W, $L_i$, r, T, S, M, $\dot{K}_i$, $\dot{q}_{2i}$, m, and $\dot{q}_{1i}$. Equations (26)–(35) and (37)–(41) give 33 relationships. Thus, there is at least four degrees of freedom. One degree of freedom will be due to the indeterminacy of the absolute level of prices. If the planners decide that consideration of the monetary sector is not important, this one indeterminacy can be removed simply by choosing a numeraire price. However, if an understanding of monetary behavior is crucial to an understanding of the system as a whole, then this indeterminacy must be removed by including monetary relationships in the model. For present purposes, it will be assumed that the absolute level of prices is determined and that there are three degrees of freedom remaining.

The remaining degrees of freedom will be removed when the government chooses the levels of the three instruments under its control: the income tax, the level of employment in the public goods industry, and the level of capital accumulation in the public goods industry. The government's ability to use these instruments has, of course, been determined before the beginning of model construction. Their use may be subject to some constraints: for example, the size of the public debt may be limited by law. Such constraints could be easily incorporated into the planning process, in the same way that the more usual constraints (e.g. (26) to (30)) are embodied.

The values chosen for the government instruments will affect the behavior of the whole of the economy. For example, the direct result of increasing the demand for labor in the public goods sector is an increase in the output of public goods. Indirectly, the amount of labor employed in other sectors, and hence output, will be reduced. Increased taxes to pay for the higher employment will reduce saving and consumption, and so on. Thus, the choice of each instrument value must be evaluated within an economy-wide framework and the effects of all instruments must be analyzed simultaneously. At this stage of the analysis, optimal growth methods become useful. The government can supply planners with an objective function. One such function would be, for example[12]:

$$\int_0^T e^{-et} u(c_1, c_2, c_4)\, dt \quad (42)$$

The task of the planners is now clear. They should maximize the functional (42) subject to the constraints (26)–(35) and (37)–(41). Optimal growth methods can be used to solve this maximization problem. The planners will then be able to calculate a path of development for all thirty-seven variables. This calculation will essentially be a forecast of the thirty-three variables not under the government's control plus determination of the values which the government will use for its instruments.

The planning process can now be summarized. First, planners formulate theories of consumer and producer behavior. Then using these theories to formulate constraints on the planning process, a plan is calculated. Before implementation can occur, the

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[12]Again, terminal capital conditions are ignored, to be discussed at a later juncture.

[13]Government will also want to ensure that society's capital stock does not get too small. However, as will be made clear, they will be severely constrained in bringing this preference to fruition because the planners cannot over-ride the preferences of the uncontrolled sector during plan construction.

uncontrolled sector must become aware of the time paths of equilibrium prices. Either there will be a tatonnement process, or planners can provide the information using plan calculations. Plan implementation in the uncontrolled sector consists of producers and consumers bringing their strategies to fruition. Implementation in the controlled sector entails government departments using the values of government instruments sent to them by planners. The influence of the plan will be threefold. First, planners may help the uncontrolled sector to improve their strategies by providing accurate information on the values of variables which affect those strategies. Secondly, planning may provide a surrogate for the tatonnement process which is necessary to find equilibrium prices in a perfectly competitive economy. Lastly and most important, the planning process allows government policy to be formulated within a framework which views an economy as an integrated system rather than a collection of autonomous parts. It is this last aspect which is the stamp of the planning approach.

## 4(c). The Influence of the Uncontrolled Sector on Planners' Behavior: An Example

No mention has yet been made about terminal capital conditions. Producers will want to ensure that capital stock at the end of the planning horizon will not be too small.[13] Thus, they will impose conditions of the following form:

$$K_i(T) = K_i^* \qquad i = 1, 2, 3. \tag{43}$$

where $K_i^*$ is an exogenous value. Condition (43) will therefore be a constraint on the maximization of (36). This condition will be imposed as a pragmatic response by producers to the difficulties, both computational and informational, of computing a plan for an infinite time period. It is quite possible that the time horizon, T, and the capital value, $K_i^*$, will be chosen somewhat arbitrarily because the information needed for choice on a rational maximizing basis does not exist.

The theory of previous sections has shown us that planners must also accept conditions (43) as constraints on the plan. Planners must essentially choose T as their time horizon and (43) as terminal capital conditions. This choice must be made even though the planners themselves regard it as much more appropriate to plan over the period $[0, \infty)$ so that the welfare of all future generations can be taken into account. *Pragmatic behavior by entrepreneurs will necessitate pragmatic behavior by planners.* Because society as a whole has decided there should be an uncontrolled sector, planners must accept the fact that their behavior will be constrained by that of the rest of society. Planners must accept a constrained optimality rather than attempting to construct 'optimal' plans which cannot be implemented. The discussion of terminal capital conditions makes this point clear because that discussion shows that planners must adopt an approach which is far from the rational maximizing approach which is the standard fare of planning textbooks and articles.

## 5. Conclusion

In the present paper, I have carried out an investigation into the form of models which would be suitable for economy-wide planning. Optimal growth models have usually been constructed in an abstract framework. Therefore, the economies for which optimal growth models would be suitable are not identified. I have shown that, because these models only embody purely physical constraints, they will, in general, only be suitable for totally controlled economies. The conclusion, establishing the general inapplicability of optimal growth models to non-controlled economies, is dependent on the view of economic planning which regards the economic structure, particularly the scope of government policy, as fixed prior to plan construction.

The second half of the paper elucidates the nature of the planning process when lanners take into account the economic structure of the economy which they are lanning. I have deliberately chosen to show the planning process in an abstract ›ntext for a simple economy, because the object is not to describe how to plan a articular type of economy, but to describe a methodology of planning. In that ıethodology, there is a place for optimal growth methods but only after the planners ave given utmost consideration to the particular features of the economy they are lanning. In using that methodology, one finds that problems must be confronted hich are hidden when optimal growth models are formulated in the usual manner. hus, planning for an infinite horizon may be the planners' preferred method on some asic philosophical grounds but in fact in order to construct an implementable plan hey may need to follow the uncontrolled sector and adopt a pragmatic finite horizon pproach.

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