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NON-PRICE RATIONING OF INTERMEDIATE GOODS IN CENTRALLY PLANNED ECONOMIES: A COMMENT

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This paper investigates an operational inconsistency between the constrained priority rationing scheme introduced by Manove in 1973 and the criterion used to determine an "optimal" priority matrix. A truncation of the optimality criterion is suggested in order to resolve this inconsistency and also reduce a potential source of bias against final demand.

1. INTRODUCTION

IN A RECENT PAPER, Manove [1] has presented a feasible rationing system for intermediate goods in a centrally planned economy as well as a method for setting rationing priorities in an "optimal" way. This is a major contribution to the planning literature since problems of plan implementation have received scant theoretical attention in comparison with the problems of plan construction. However, there remain certain inconsistencies in Manove's analysis which we propose to demonstrate and partially resolve in this comment.[2]

The major problem with the paper lies in the inconsistency between the operational procedure suggested for the constrained priority rationing system (CPRS) and the optimality criterion used to determine the optimal rationing matrices. Furthermore, the assumption of "additivity" introduced for analytical tractability and elegance seriously weakens the significance of Manove's empirical conclusions based upon the 1966 Soviet input-output table. In order that the CPRS may indeed have normative significance for existing centrally planned economies, we propose a small change in the optimality criterion used in the solution algorithm, a change which also avoids all problems of matrix convergence.

2. THE OPERATION OF THE CPRS: CONTINUOUS OR SEQUENTIAL

Section 2 of Manove's paper suggests that a rationing decision within any sector would be made only once within a production period (one month within an annual plan). At the decision point (perhaps the first day of the month), the distribution manager for each sector would know the size of the anticipated shortage for his commodity. If there were such a shortage he would apply the CPRS procedure to determine the rationed allotments to all sectors during the current month. This interpretation is most consistent, we feel, with the author's claim to bureaucratic simplicity and with his simple two-commodity example used for exposition.

[1] This paper benefited from suggestions by participants in the Workshop in Economic Planning, University of Pennsylvania. The authors would also like to acknowledge the suggestions made by Professor J. Michael Montias.

[2] There are other objections that might be raised in regard to Manove's model, e.g., the absence of input substitution or the use of exogenous constraints on the rationing procedure. However, we feel that these assumptions are the kind of assumptions which can be regarded as realistic for an analysis of short-run plan implementation.

In contrast, the optimality criterion introduced in Section 3 of his paper is appropriate for a continuous application of the CPRS, an infinite number of iterations. Clearly, multiple iteration of the CPRS within a month is infeasible in any bureaucratic procedure. Furthermore, the presence of stochastic elements in both production and distribution would suggest a sequential rationing procedure even in a fully-automated system of management. Consequently, the higher-order iterations, whose losses are represented in the calculation of eventual values, will be iterations made operationally in subsequent months. Derived shortages can only be carried forward to later months through inventory decumulation, a process left quite vague in Manove's presentation. Subsequent iterations, even so, would not be continued beyond the duration of the annual plan. Consequently, the priority and rationing matrices determined by Manove's procedure have little claim to "optimality" in a bureaucratic procedure constrained to a limited number of iterations.

Manove presents his rationing procedure as applicable to monthly plan implementation in an economy with a Leontief technology. The monthly time frame justifies Manove's concern for a rationing scheme which: "can be applied by a decentralized bureaucracy to a wide variety of rationing situations within a given economy" [**1**, p. 835]. However, he also conceives of the plan implementation problem within the month as a simultaneous system, i.e., this month's production will be the source of intermediate inputs this month. The simultaneity assumption is a reasonable approximation for longer planning periods, such as yearly plans, but as the planning period shortens the assumption becomes less reasonable. Manove assumes both that the monthly plan is "taut" in terms of the stock/flow ratio for intermediate goods, and that "production processes are very fast" [**1**, p. 834]. Implicitly, the integrity of inventory/production ratios becomes the highest priority use of sectoral output in this peculiar industrial system.

Alternatively, the rationing problem might have been presented in the context of a sequential production system, where all inputs are delivered and used during the month but no output appears before the end of the month. The sector manager, according to this scheme, would distribute last month's sector output during the current month, following the allocation row of the plan; this allocation row would be adjusted by the CPRS, if there were a shortage. Given the actual lags observed in industrial production and distribution, such a point-output sequential model would be a closer representation of reality for the monthly rationing problem than the simultaneous model. However, inventories would require more explicit treatment than afforded in Manove's model.

This distinction between simultaneous and sequential production becomes crucial when one tries to interpret Manove's two-commodity example [**1**, pp. 835–837]. Consider the matrix of rationed allotments when shortages of 1 unit of coal and 2 units of textiles are distributed by the CPRS. If the Leontief technology is represented by the "Demanded Allotments of Inputs" matrix [**1**, p. 837], how should we interpret the post-rationing situation in the coal industry? Given inputs of 7.9 units of coal and 2.0 units of textiles, it should be feasible for the coal industry to produce 19.75 units of coal, rather than the 19 rationed during the month. Is this month's shortage of coal 0.25, 1.00, or 1.25 units?

In a taut simultaneous system, a shortage might be regarded as an inventory deficiency which the sectoral manager must replenish out of this month's production. Then, one unit of the 19.75 of coal produced would be designated for inventory, leaving only 18.75 units for use during the current month. With 19 units to be allocated, sector managers will exercise discretion in allocating the 0.25 shortage either to inventories or to deliveries. In general, one cannot rely on inventories being large enough to absorb the secondary shortages, except for a limited amount of time.[3] Reducing deliveries still further would involve re-application of the CPRS, a process which would lead us into new current month shortages. As we noted earlier, that interpretation of the CPRS procedure abandons any claim to bureaucratic feasibility.

Alternatively, one might construe the shortage in Manove's example as an impairment of sector capacity, e.g., because of the flooding of a coal mine. In this case, one must make a further assumption about the rigidity of the flow matrix. Are all rationed inputs consumed in the production process even though the feasible output (19.75 units of coal) cannot be attained? With suitable intra-industry distribution of inputs, e.g., non-delivery to the flooded mine, this month's output of coal could equal this month's rationed allotment of coal and no secondary shortages of coal need result. Only inputs technologically required to produce the total allotment might be consumed, with excess inputs accumulating as input inventories.

The only interpretation of Manove's example which is consistent with his optimality criterion is one in which the production system is simultaneous with "shortages" known ex ante by sector managers. Such deliveries might be directed to restore the required inventory level, sent abroad to a needy ally, or dropped in the Arctic Ocean. Thus, Manove's procedure is applicable to a simultaneous model, but with certain sequential characteristics. The outputs in any particular month are used as sectoral inputs in the same month. However, inventories are large enough so that a shortage can be carried over into the next time period, as is usually necessary after single application of the CPRS. (Note that this is just the case in Manove's example, where there are secondary shortages of 0.25 units of coal and 2 units of textiles passed on to the next time period.) Therefore, the economy to which the CPRS is applicable is not as taut as Manove would have us believe.

Our interpretation of Manove's example raises a final observation. If a sector manager reduces his own ration, he knows that he might face a shortage next month regardless of rationing decisions made in other sectors. Given the difficulties any central authority faces in monitoring the own-allocation of a sector's output,

[3] Manove [1, p. 840, Footnote 9] indicates how the calculation of eventual values could be modified when stocks of commodities exist. However, the central planning organization will not know of the existence of supra-normal stocks when the CPRS rationing matrices are calculated. At this time, the plan will, presumably, try to ensure that industries only have a normal level of inventories. Therefore the distribution-of-marginal-shortage matrix would be computed under the assumption of tautness, but the implementation of that matrix could vary from month to month depending on the relation between actual inventories and their normal level. Each sector manager will be acting independently because in a rationing situation there would, by definition, be no time to co-ordinate information from each sector.

in practice one might expect the inter-industry section of the distribution-of-marginal-shortage matrix to be zero on the diagonal.

3. OPTIMALITY OF THE PRIORITY RANKING AND DISTRIBUTION-OF-MARGINAL-SHORTAGE MATRICES

In his third section, Manove proves the existence of an optimal pair of priority-ranking and distribution-of-marginal-shortage matrices, $(\tilde{R}, \tilde{H})$, and presents an algorithm for their determination. For each commodity, $(\tilde{R}, \tilde{H})$ minimizes the loss in the value of final demand resulting from a shortage which has been distributed by application of the CPRS an infinite number of times. As we noted in the previous section, one should not expect a large number of CPRS iterations, certainly not an infinite number of iterations, either within a single month or over successive months.

The "optimal" matrices, $(\tilde{R}, \tilde{H})$, found by Manove, will not in general be the best ones to use when the CPRS is applied a finite number of times. In order to find optimal matrices, one must consider the valuation of secondary shortages. The most suitable choice would be to include in the algorithm for finding the optimal matrix the procedure for allocating secondary shortages which will be adopted in the economy. In this way, one would obtain consistency between the use for which the optimal matrix is optimal, and the use to which it will be applied in practice.

In a simultaneous scheme with no slack, all shortages at some stage will have to be allocated to final demand. It might be reasonable to assign all secondary shortages to final demand. The rationing procedure then might be envisioned as one in which initial shortages are assigned by the CPRS and then sector managers are told to allocate any secondary shortages to final demand.[4] This scheme would be operable in a simultaneous environment with no slack and with initial shortages known ex ante.

By using such a procedure, planning authorities have a means of ameliorating the effect on final demand of a shortage. For example, consider the case of a shortage in coal, for which household demand may be highly inelastic. Allocating all of this shortage directly to final demand would be unnecessarily harsh. However, a single application of the CPRS might allocate the shortage to luxury industries which use coal as a productive input. Then, the secondary shortages in luxuries could be immediately allocated to final demand. Assessing all secondary shortages against final demand has no claim to perfection. However, such a method does seem a reasonable compromise between the inflexibility of assigning initial shortages to final demand, and introducing the bureaucratic complexities of higher order iterations of the CPRS.

[4] There is a limitation to our scheme when a sector has zero or negligible deliveries to final demand so that secondary shortages in that sector cannot be assigned to final demand. With a highly disaggregated rationing scheme, this may be a serious limitation. In a taut system, a practical resolution to this problem might be to prohibit the reduction of allocations to such sectors which do not deliver to final demand.

The objective of the algorithm would then be to minimize each component of the truncated vector

$$V^T = PF + (Q \times M)PE$$

subject to (i) $\underline{H} \leqslant H \leqslant \overline{H}$ and (ii) for each i, $\Sigma_{j=1}^{n+1} h_{ij} = 1$, where P is the diagonal matrix of final output prices; H is the distribution-of-marginal-shortage matrix (DMS matrix); $\overline{H}$ is the maximum-allotment-reduction matrix; $\underline{H}$ is the minimum-allotment-reduction matrix; M is the inter-industry section of the DMS matrix, H; F is the final demand column of the DMS matrix, H; Q is the matrix of direct productivities of inputs; n is the number of industries; $E' = (1, 1, \ldots, 1)$; V is the vector of eventual values; and superscript T refers to matrices and vectors calculated using our objective function.

Manove's theorem and algorithm would be applicable to this problem, leading to optimal ranking matrices, say (R^T, H^T). Indeed, our case is much simpler. Manove had to assume the existence of an R_0 such that V_0 exists, and his algorithm requires finding such an R_0; any ranking matrix, however, will serve to start the algorithm to find our V^T, since V_0^T exists for any R_0. The matrices (R^T, H^T) are then optimal under the assumption that secondary shortages will be allocated to final demand.

4. ON CONCLUSIONS FROM APPLICATION OF THE CPRS TO SOVIET DATA

In the calculation of the ($\tilde{R}, \tilde{H}$) matrices, Manove makes: "the mathematically tractable assumption that the total loss of output will be the sum of the losses that would be caused by each of the shortages separately" [**1**, p. 839]. In an application of the CPRS Manove draws several conclusions which we will show are direct, almost tautological, results of this additivity assumption.

In order to show the importance of the assumption of additivity, let us examine the process of calculation of eventual values. Suppose, without loss of generality, that there is a shortage of the first good. The CPRS will, in general, reduce intermediate inputs into other sectors causing secondary shortages. Suppose that there are shortages of two goods, i and j, which are distributed by a secondary application of the CPRS. In general, i and j will be inputs into the production of some good k. The loss in output of good k, the tertiary shortage, is calculated, using the additivity assumption, as the sum of the loss in output of good k due to shortage of input i only *plus* the loss in output of good k due to a shortage of input j only. The usual calculation of loss in output would entail finding the maximum of these two individual losses. Therefore, the additivity assumption overestimates the loss in output. Thus, *goods will seem inherently more valuable as intermediate inputs than as constituents of final demand.*

The foregoing description indicates the conditions under which use of the additivity assumption will give a reasonable approximation for the vector of shortages remaining after a single application of the CPRS. The calculation of new shortages will be the more accurate the greater are the number of zero elements in the original vector of shortages. Two particular cases markedly violate this condition: (i) there are originally shortages in many industries; and (ii) there is an initial shortage of only one commodity, but the CPRS is iterated many times.

If the CPRS causes shortages to be assigned to intermediate demand at each stage, then as one proceeds to higher order iterations more and more sectors will be affected. When the vector of shortages has many non-zero elements, Manove's rationalizations [**1**, pp. 838–839] for the additivity assumption lose their force Shortages allocated from different sectors or from previous iterations are much more likely to coincide at the same manufacturing establishments. It is very likely that the vector of shortages will converge to zero in reality, although application of the additivity assumption might indicate exactly the opposite conclusion. The more we iterate the CPRS, the less tenable becomes the additivity assumption. In particular, calculations of losses based on infinite iteration of the CPRS will be meaningless.

The optimal ranking matrix, $\tilde{R}$, will be chosen from the admissible set, i.e., those ranking matrices for which the corresponding vector of eventual values exists. As we have seen, the additivity assumption causes goods to be spuriously overvalued in their use as intermediate inputs relative to their use in final demand. Any R matrix which gives high priority to final demand will be less likely to generate an eventual value vector which exists. We have said nothing about prices; existence of the eventual value vector is solely dependent on the relation between value in final demand or in intermediate use, for any good. Prices only play a role in the selection of an optimal R matrix from those in the admissible set, most of which will assign low priority to final demand.

Manove found "that the structure of the priority ranking matrices turned out to be remarkably insensitive to the prices used to compute GNP" [**1**, p. 844]. Our analysis has shown that this is not in the least surprising. Insensitivity to prices indicates that the admissible set of R matrices is small and relatively homogeneous, as a result of the method of calculating eventual values. In contrast, using our V^T, reducing intersectoral deliveries will seem less costly than when one uses Manove's V. V includes the pyramiding of losses through many iterations, both in a real sense, due to the high productivity of some inputs, and in a spurious sense, due to the double-counting (or n-counting) of value caused by the additivity assumption. The truncated criterion, V^T, would be more sensitive to relative prices because such pyramiding does not occur.

Manove claims that "a second conspicuous result of this example is that the final product sector (consumption and investment) is invariably assigned the lowest priorities for the acquisition of goods" [**1**, pp. 851–852]. This is, again, a consequence of the additivity assumption which causes calculations of the value of intermediate goods to be overestimated.

How, under any circumstances, would final demand receive top priority in the ranking matrix for a single commodity? This would occur if the value of a commodity unit in final use exceeded the direct and indirect losses in GNP resulting from reducing the allotment to any other sector by one unit. With Manove's untruncated criterion, this would require a very high relative price for the commodity and little feedback (few inputs from other sectors). For certain Soviet consumers, direct consumption of agricultural output may be much more highly valued per ruble unit than indirect consumption. With our truncated criterion, such cases would certainly be more numerous so that final demand would not invariably be the lowest priority for "reasonable" price vectors.

We seem to have reached a somewhat paradoxical conclusion. Our truncated criterion assumes all secondary shortages are assigned to final demand. Under Manove's procedure only part of the derived shortages are assigned to final demand at each stage. Yet, our criterion might well cause final demand to have a higher priority than Manove's. The paradox lies in the incompatibility of assuming an infinite number of CPRS applications and then calculating the loss at each stage using the additivity assumption.

5. CONCLUSION

Manove's rationing scheme has intrinsic difficulties of interpretation, as we have shown in our Section 2. Also, two assumptions seem to limit the usefulness of his scheme due to their practical inconsistency. Infinite iterations are rarely possible in a "bureaucratic" world, and especially undesirable when one is searching for simplicity. Combining the assumption of an infinity of iterations with the additivity assumption diminishes greatly the plausibility of the latter. Thus the normative and descriptive significance of Manove's scheme is impaired.

Adopting Manove's model, we have modified his optimality criterion to be more consistent with the operational context in which the CPRS might be applied. Recognizing that only a finite number of iterations will be possible, we have assigned the burden of all secondary shortages to fall on final demand. In this truncated version, the "additivity" assumption becomes a much more acceptable mathematical approximation. Furthermore, Manove's algorithm will still represent an efficient procedure for the determination of optimal priority and rationing matrices.

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REFERENCE

[1] MANOVE, MICHAEL: "Non-Rationing of Intermediate Goods in Centrally Planned Economies," *Econometrica*, 41 (1973), 829–852.