Economics Letters 32 (1990) 307–311
North-Holland

A MODEL OF ELECTORAL COMPETITION WITH INTEREST GROUPS

Peter J. COUGHLIN, Dennis C. MUELLER and Peter MURRELL
University of Maryland, College Park, MD 20742, USA

Received 8 June 1989
Accepted 23 August 1989

This letter specifies and analyzes a model of electoral competition with interest groups. The assumptions used in the model are drawn from the existing literature. We (i) show that certain assumptions that have been used in empirical analyses of voting provide a sufficient condition for the existence of an electoral equilibrium, and (ii) characterize the equilibrium strategies. Embodied in the characterization is a set of parameters that can be said to measure the electoral strengths of the interest groups. We also state the appropriate definition of the Pareto relation for our model and note that each of the equilibrium strategies is Pareto optimal.

1. Introduction

This letter specifies and analyzes a model of electoral competition with interest groups. We prove that the political parties in the model have equilibrium strategies that can be viewed as maximizing a social objective function. The strength of an interest group can be defined in terms of a politician's perception of a group's reliability in delivering the votes of its members – and, therefore, corresponds to one of the parameters of the model. Our results reveal that the electoral strength parameters are also parameters of the social welfare function that is implicitly maximized. We also state the appropriate definition of Pareto optimality for our model and observe that the chosen policies are Pareto optimal.

The model presented here is a useful tool for examining the relationship between the strengths of interest groups and the nature of government policies. Our equilibrium existence theorem establishes sufficient conditions for using the model. Our characterization of the implicit social objective function provides the basic information needed to apply traditional comparative statics methods to analysis of the effects of interest groups. As an example of how our model can be used: In Coughlin, Mueller and Murrell (1989) we have applied these results to identify the relationship between the size of government and shifting patterns of interest group influence.

Finally, it should be noted that this letter builds upon two earlier references that modeled interest groups in the same basic way: (i) Borooah and Van der Ploeg (1983), which considered the distributional assumption that is made in this paper – but did not address the questions of whether an electoral equilibrium exists, where such equilibria will be located, and how the location is related to the strengths of the interest groups (viz., because their goal was to develop an econometric model); (ii) Enelow and Hinich (1984), which established an equilibrium existence theorem and characterized the nature of the equilibrium strategies – but only considered unidimensional strategy spaces, and made a different distributional assumption and stronger assumptions about voters' utility functions.

2. The electoral competition

As in Downs (1957, p. 54): The government, g, faces a single rival – the opposition, o. As in Fair (1978, p. 161): All voters use the same measure, m_g, of g's performance prior to the current period – and also use a common measure of past performance, m_o, for the o. The set of possible values for m_g is denoted by $\mathscr{M}_g$, the set for m_o by $\mathscr{M}_o$. It is assumed that, in a given election, g has a fixed $m_g \in \mathscr{M}_g$ and o has a fixed $m_o \in \mathscr{M}_o$. As in most public choice models of elections [see Mueller (1989, Ch. 10)]: The set of feasible alternatives for g in a given election (denoted by S_g) is a set of possible current policies and/or policy positions. The set of feasible alternatives for the opposition, S_o, is a set of possible policy positions. S_g and S_o are assumed to be non-empty, compact subsets of Euclidean spaces. Their elements are denoted by $s_g \in S_g$ and $s_o \in S_o$, respectively. We use A to denote the set $(\mathscr{M}_g \times S_g) \cup (\mathscr{M}_o \times S_o)$. As in Borooah and Van der Ploeg (1983) and Enelow and Hinich (1984), interest groups are modeled by assuming (i) the electorate can be partitioned into N groups of voters with common interests on policy-related matters, and (ii) U_i: $A \to E^1$ is the utility function for all members of group i for the purpose of evaluating policy-related matters. U_i is assumed to be upper semi-continuous in s_g and s_o. [Note: We assume upper semi-continuity but not lower semi-continuity because of Denzau and Parks' (1979, pp. 341–343) result that public sector preferences generally inherit closed upper contour sets but do not generally inherit closed lower contour sets.]

The probability that an individual votes for g depends upon two factors. The first factor is his evaluation of g and o on policy-related matters [i.e., $U_i(m_g, s_g)$ and $U_i(m_o, s_o)$]. The second factor is his evaluation of g and o on non-policy matters, such as ideology or the personal characteristics of a particular candidate. As in Fair (1978), Borooah and Van der Ploeg (1983), and Enelow and Hinich (1984), these evaluations are summarized by (i) a number, ξ^g_{ij}, which is the non-policy value that the jth member of interest group i attaches to having g re-elected, and (ii) an analogous value, ξ^o_{ij}, for o winning. Thus a voter has an expected utility bias in favor of g, $b_{ij} = \xi^g_{ij} - \xi^o_{ij}$. For voter ij, the conditional probability for the event 'ij will vote for g, given that the measure of performance for g is m_g and g has chosen s_g *and* the measure of performance for o is m_o and o has chosen s_o' is

$$P^g_{ij}(m_g, s_g; m_o, s_o) = \begin{cases} 1 & \text{if} \quad U_i(m_g, s_g) + \xi^g_{ij} > U_i(m_o, s_o) + \xi^o_{ij}, \\ 0 & \text{if} \quad U_i(m_g, s_g) + \xi^g_{ij} \le U_i(m_o, s_o) + \xi^o_{ij}. \end{cases} \tag{1}$$

Using the bias notation, (1) can be rewritten as

$$P^g_{ij}(m_g, s_g; m_o, s_o) = \begin{cases} 1 & \text{if} \quad U_i(m_o, s_o) - U_i(m_g, s_g) < b_{ij}, \\ 0 & \text{if} \quad U_i(m_o, s_o) - U_i(m_g, s_g) \ge b_{ij}, \end{cases} \tag{2}$$

as in Fair (1978, p. 161, eq. 1′). The probability of ij voting for the opposition, $P^o_{ij}(m_g, s_g; m_o, s_o)$, is similarly defined (with g and o interchanged). Each party is assumed to have the objective of maximizing its expected plurality in the current election [denoted by $P\ell^g(m_g, s_g; m_o, s_o)$ and $P\ell^o(m_g, s_g; m_o, s_o)$, for the government and opposition, respectively].

The assumptions made up to this point clearly imply that, for any given m_g and m_o, the decisions of the two parties constitute a two-person game:

$$\Gamma(m_g, m_o) = \left(S_g, S_o; P\ell^g(m_g, s_g; m_o, s_o), P\ell^o(m_g, s_g; m_o, s_o)\right). \tag{3}$$

Since the assumptions that have been made also imply that this game is zero-sum: For given m_g and m_o, (s_g^*, s_o^*) is an 'equilibrium in $\Gamma(m_g, m_o)$' if and only if it is a saddle point for the payoff function $\phi(s_g, s_o) = P\ell^g(m_g, s_g; m_o, s_o)$.

3. Sufficient conditions for existence and optimality

The following conditions are based on assumptions that have been used in empirical work [see Fair (1978) Borooah and Van der Ploeg (1983)] – but not, heretofore, in analyses of electoral equilibria.

Let I be an index set for the individual voters. We assume that each $ij \in I$ (i.e., each voter) has one vote. $\mathscr{I}$ denotes a σ-algebra of subsets of I. For each interest group i, we assume that $I_i = \{ij: ij$ is in interest group $i\} \in \mathscr{I}$. Finally, let μ be a probability measure on the measurable space $(I, \mathscr{I})$ such that, for each set $J \in \mathscr{I}$, $\mu(J)$ is the proportion of the total vote which is in the set J. We will also use the shorter notation, n_i, in place of $\mu(I_i)$ when this is convenient. We assume $n_i > 0$, $\forall i \in N$.

We assume that one of the following statements about the bias terms holds:

Assumption 1.a. g and o believe that, *for each interest group i*, the bias term, b_{ij}, (a) is a random variable with respect to the probability measure space $(I_i, \mathscr{I}_i, \mu_i)$ [where $\mathscr{I}_i = \{J_i \subseteq I_i: J_i = I_i \cap J$, for some $J \in \mathscr{I}\}$ and $\mu_i(J_i) = \mu(J_i)/\mu(I_i)$, $\forall J_i \in \mathscr{I}_i$], and (b) has a uniform distribution over a real interval $(\ell_i, \imath_i)$.

Assumption 1.b. g and o believe that, *for each voter ij*, the bias term, b_{ij}, is a random variable with a uniform distribution over a real interval $(\ell_i, \imath_i)$.

If, as in Borooah and Van der Ploeg's (1983, Section 6.4) generalization of Fair (1978), one uses Assumption 1.a, there is an uncountably infinite number of voters. In contrast, since each of the uniform distributions referred to in Assumption 1.b is for an individual voter, having either a finite or infinite number of voters is consistent with the latter alternative.

Our final condition corresponds to Fair's assumption (1978, p. 162) [also in Borooah and Van der Ploeg (1983, Section 6.4)] that each voter's utility difference (if the bias term is ignored) is in the interval on which the distribution for the bias term is defined. More specifically,

Assumption 2. For any given $(m_g, m_o) \in \mathscr{M}_g \times \mathscr{M}_o$ and interest group i,

$$\ell_i < U_i(m_g, s_g) - U_i(m_o, s_o) < \imath_i, \forall (s_g, s_o) \in S_g \times S_o. \tag{4}$$

To simplify our notation in what follows, we will let $\alpha_i = 1/(\imath_i - \ell_i)$.

Theorem 1. For any given $m_g \in \mathscr{M}_g$ and $m_o \in \mathscr{M}_o$, (i) there exists an equilibrium in $\Gamma(m_g, m_o)$, and (ii) (s_g^*, s_o^*) is an equilibrium in $\Gamma(m_g, m_o)$ if and only if s_g^* maximizes $\omega_g(s_g | m_g) = \sum_{i=1}^N n_i \cdot \alpha_i \cdot U_i(m_g, s_g)$ over S_g and s_o^* maximizes $\omega_o(s_o | m_o) = \sum_{i=1}^N n_i \cdot \alpha_i \cdot U_i(m_o, s_o)$ over S_o.

Lemma 1. For any given $m_g \in \mathscr{M}_g$ and $m_o \in \mathscr{M}_o$,

$$P\ell^g(m_g, s_g; m_o, s_o) = 2 \cdot \sum_{i=1}^N n_i \cdot \alpha_i \cdot U_i(m_g, s_g) - 2 \cdot \sum_{i=1}^N n_i \cdot \alpha_i \cdot U_i(m_o, s_o) + \sum_{i=1}^N n_i \cdot \alpha_i \cdot (\imath_i + \ell_i). \tag{5}$$

Proof of Lemma 1. For any $s_g \in S_g$ and $s_o \in S_o$, the expected vote for o is

$$EV^o(m_g, s_g; m_o, s_o) = \sum_{i=1}^{N} \int_{I_i} P_{ij}^o(m_g, s_g; m_o, s_o) \cdot d\mu(ij). \tag{6}$$

If Assumption 1.a holds, then

$$EV^o(m_g, s_g; m_o, s_o) = \sum_{i=1}^{N} \mu\big(\{ij \in I_i : U_i(m_o, s_o) - U_i(m_g, s_g) > b_{ij}\}\big), \tag{7}$$

and the cumulative distribution function for b_{ij} across the voters in i is

$$F_{ij}(y) = \Pr(b_{ij} < y) = \begin{cases} 1 & \text{if} \quad y \geq r_i \\ \alpha_1 \cdot (y - \ell_i) & \text{if} \quad \ell_i < y < r_i \\ 0 & \text{if} \quad y \leq \ell_i. \end{cases} \tag{8}$$

By (8) and Assumption 2, the proportion of the voters in I_i who are in $\{ij \in I_i : U_i(m_o, s_o) - U_i(m_g, s_g) > b_{ij}\}$ is

$$\alpha_i \cdot \big(U_i(m_o, s_o) - U_i(m_g, s_g) - \ell_i\big). \tag{9}$$

Therefore, by (7),

$$EV^o(m_g, s_g; m_o, s_o) = \sum_{i=1}^{N} n_i \cdot \alpha_i \cdot \big(U_i(m_o, s_o) - U_i(m_g, s_g) - \ell_i\big). \tag{10}$$

Suppose that Assumption 1.b holds. Then, for any given voter ij, (i) $\Pr\{ij$ votes for o$\} = \Pr\{b_{ij} < U_i(m_o, s_o) - U_i(m_g, s_g)\}$ and (ii) the cumulative distribution function for b_{ij} is (8). Therefore, using (8) and Assumption 2, $\Pr\{ij$ votes for o$\}$ is (9). Therefore, by (6),

$$EV^o(m_g, s_g; m_o, s_o) = \sum_{i=1}^{N} \alpha_i \cdot \big(U_i(m_o, s_o) - U_i(m_g, s_g) - \ell_i\big) \cdot \mu(I_i). \tag{11}$$

Since $\mu(I_i) = n_i$ for each interest group i, (10) (once again) holds.

Finally, since $\Pr\{U_i(m_o, s_o) - U_i(m_g, s_g) = b_{ij}\} = 0$, we also have

$$P\ell^g(m_g, s_g; m_o, s_o) = \sum_{i=1}^{N} n_i - 2 \cdot EV^o(m_g, s_g; m_o, s_o). \tag{12}$$

Therefore [using (10), (12), and $\alpha_i = 1/(r_i - \ell_i)$] e.q. (5) follows. Q.E.D.

Proof of Theorem 1. The separable payoff function derived in Lemma 1 immediately implies that (s_g^*, s_o^*) is a saddle point in $\Gamma(m_g, m_o)$ if and only if s_g^* maximizes $\omega_g(s_g \mid m_g)$ and s_o^* maximizes $\omega_o(s_o \mid m_o)$. For each i, the function $U_i(m_g, s_g)$ is an upper semi-continuous function of s_g. Hence $\omega_g(s_g \mid m_g)$ is an upper semi-continuous function of s_g. In addition, S_g is a compact. Hence there exists an s_g^* that maximizes $\omega_g(s_g \mid m_g)$. By a similar argument, there exists an s_o^* that maximizes $\omega_o(s_o \mid m_0)$. Hence a saddle point exists. Q.E.D.

Our next theorem establishes an important normative property of the equilibria in any given $\Gamma(m_g, m_o)$. To state the property precisely, we use the following definitions, based on Hildenbrand (1974, p. 230). The Pareto relation, $R_g(m_g)$, on S_g for the utility functions $\{U_{ij} = U_i;\ ij \in I\}$ is defined by: $(x, y) \in R_g(m_g) \Leftrightarrow$ (i) $x, y \in S_g$ *and* (ii) $U_{ij}(m_g, x) \geq U_{ij}(m_g, y)$ almost everywhere, with respect to $(I, \mathcal{I}, \mu)$. Using this notation: y is in the Pareto optimal set in S_g if and only if (i) $y \in S_g$ *and* (ii) there is no $x \in S_g$ such that $(x, y) \in R_g$ and $(y, x) \notin R_g$. The Pareto relation, $R_o(m_o)$, on S_o and the Pareto optimal set in S_o are similarly defined (with o replacing g). A straightforward implication of Theorem 1 and the above definitions is

Theorem 2. For any given $m_g \in \mathcal{M}_g$ and $m_o \in \mathcal{M}_o$, each equilibrium, (s_g^*, s_o^*), in $\Gamma(m_g, m_o)$ is such that (i) s_g^* is in the Pareto optimal set in S_g, and (ii) s_o^* is in the Pareto optimal set in S_o.

References

Borooah, V. and F. Van der Ploeg, 1983, Political aspects of the economy (Cambridge University Press, Cambridge).

Coughlin, P., D. Mueller and P. Murrell, 1989, Electoral politics, interest groups, and the size of government, Working paper.

Denzau, A. and R. Parks, 1979, Deriving public sector preferences, Journal of Public Economics 11, 335–354.

Downs, A., 1957, An economic theory of democracy (Harper and Row, New York).

Enelow, J. and M. Hinich, 1984, The spatial theory of voting (Cambridge University Press, Cambridge).

Fair, R., 1978, The effect of economic events on votes for president, Review of Economics and Statistics 60, 159–173.

Hildenbrand, W., 1974, Core and equilibria of a large economy (Princeton University Press, Princeton, NJ).

Mueller, D., n.d., Public choice Vol. II (Cambridge University Press, Cambridge) forthcoming.