Random versus Directed Search for Scarce Resources*

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Abstract

In this paper, I study the welfare effects of adverse selection and congestion in search markets. There is a large mass of agents (e.g., firms) with homogeneous preferences and a relatively smaller mass of objects (e.g., workers) that differ in quality. Agents search for objects either through random or directed search. Random search—agents are randomly matched to an object of any quality—gives rise to adverse selection, while directed search—agents choose with which quality types to match—gives rise to congestion. When utility is either non-transferable or transferable through Nash bargaining, I show that random search dominates directed search in terms of welfare, even though each agent would prefer to be able to direct her search.

1 Introduction

In search markets with frictions, how agents search and form matches can have significant welfare implications. For example, consider an agent who wants to make an online dinner reservation on Yelp. One possible search procedure would be for the agent to lookup the keyword "dinner," and then to randomly browse through her search results until she finds a restaurant that meets her specifications. A more sophisticated procedure would be for the agent to further filter her search results by considering only the "highest rated" restaurants. From this one agent's perspective, the latter search procedure would save time and may result in

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a better match. However, if all agents on Yelp filter their search results, they could end up competing for the limited number of reservations at the best restaurants in town, leading to congestion and long wait times. In this paper, I show that the congestion that arises when agents filter their search can have a significantly negative impact on welfare.

In particular, I study how different search procedures affect welfare through congestion and adverse selection. I consider a search model with objects that differ in quality on one side of the market and strategic agents who want a good/high-quality object on the other side. The search market is marked by scarcity: there are relatively more agents than objects. The agents search either by randomly sampling the objects from the entire search pool (random search) or by filtering their search pool based on quality (directed search).

In random search, each agent is paired randomly with an object of any quality type. The agent either accepts the object and leaves the market, or rejects it and searches again at a cost. When the search cost is low, the agent is strategically "picky," i.e., she would rather continue searching than accept a match with a bad object. Yet, in the aggregate, the agents' strategic pickiness generates negative externalities: some of the agents remain in the search market for too long, matching with bad objects until they find a good one, and their pickiness gives rise to adverse selection—the search pool will have more rejected bad objects than good ones.

The inefficiencies inherent in random search are a consequence of the matching procedure which, with positive probability, pairs agents to low quality objects they would rather avoid. Can we therefore improve welfare by allowing the agents to direct their search? After all, when an agent can direct her search, she no longer wastes her time matching with bad objects. Additionally, directed search offers each agent more flexibility than random search. Hence, it may seem that directed search could overcome the negative externalities that are generated by random search.

However, the main result of this paper shows that when objects are scarce, welfare under random search is higher than welfare under directed search, even though each agent would prefer to be able to direct her search.

Despite providing the agents more flexibility in how they search, directed search gives rise to new negative externalities. While an agent who directs her search no longer wastes time matching with bad objects, she now competes with other agents who also seek to match with the scarce number of good objects. This gives rise to congestion, i.e., the agent must wait until a good object is rationed to her. Once again, the agent remains in the market for too long.

Furthermore, the agents are pickier when they can direct their search than when they search randomly, which makes the negative externalities they impose on the search market more severe. Under random search, an agent's pickiness is characterized by an intertemporal trade-off between the object she is matched to *today* and the possibly better objects she may get in the *future* if she continues to search at a cost. I show that directed search can be thought of as random search but with a contemporaneous trade-off between the object she can get *today* with certainty and the possibly better objects she may get *today* through rationing. As the latter trade-off does not involve the search cost, the agents under directed search can afford to be pickier, which makes the negative externalities they impose on the search market more severe. Ultimately, the additional flexibility under directed search leads to lower welfare.

Let me now give more details about the model. I first study a search market with non-transferable utility, i.e., there are no prices. I consider a steady state search market which is composed of agents with homogeneous preferences and objects of heterogeneous quality types. The agents are strategically looking to match with high-quality objects. The two key assumptions in the paper are that (i) there are (weakly) fewer objects than agents, and (ii) matching is not frictional, i.e., all possible matches that can occur, do occur. The latter assumption implies that any inefficiencies that arise from strategic behavior is not "cooked in" exogenously. In principle, the setup allows each agent the best chance to match with one object and get a higher payoff than her outside option.

Under random search, an agent is randomly paired to an object. Once matched, the agent observes the object's quality. If the agent accepts her match, she exits the market with the object. If the agent declines the match, both the object and the agent return to the search market. A fraction $\gamma \in (0, 1)$ of the agents and objects left in the search market "die". Thus, γ is the cost of searching. Finally, the market is kept in a steady state by an inflow of new agents and objects, where the newly arriving objects have quality types distributed according to some exogenous absolutely continuous distribution function. Then the process repeats itself.

To model directed search, the search market is further divided into sub-markets. All objects contained in a sub-market have the same quality, and all objects of the same quality are in the same sub-market. The agents can distinguish between the different sub-markets, i.e., the agents can use sub-markets to filter the objects by quality. For example, suppose there are two types of objects: high quality objects h and low quality objects l. Assume there is a unit mass of objects with a mass m of type h and a mass $1 - m$ of type l. Then there is a sub-market H of size m that contains all the type h objects and a sub-market L of size $1 - m$ that contains all the type l objects.

In each period under directed search, an agent chooses which sub-market to enter. The agent is then matched to an object within that sub-market with the short-side of the sub-market being rationed to the long-side. For example, assume there is also a unit mass of agents. If $n > m$ agents enter sub-market H, each agent in sub-market H gets matched to a type h object with probability m/n, because the agents are on the long-side (the objects must be rationed). In contrast, each agent in sub-market L gets matched to a type l object with probability 1, because the agents are on the short-side of the sub-market. If an agent gets matched to an object, she exits the market. If she did not get a match, she returns to the search market. Once again, before the period ends, a fraction γ of the agents and objects left unmatched in the search market "die," and a mass of new agents and objects replace all the ones that exited the market. Then the process repeats itself.

The main result of this paper shows that for any $\gamma \in (0, 1]$, welfare under random search is strictly higher than welfare under directed search. In fact, the main result is stronger: It shows that (*i*) an agent's equilibrium payoff under random search is higher than her payoff under directed search,[1] and (*ii*) more agents leave the market in every period under random search. In other words, random search is good at clearing the market which makes room for new agents, each of whom enjoys a high equilibrium payoff. In contrast, directed search creates a congested market (because all the agents go after the best quality objects) and results in low equilibrium payoffs (because the likelihood of getting matched in a congested market is low).

I also show that in the limit as the cost of searching γ goes to zero, so does the welfare gap between the two search procedures. Intuitively, when the cost of searching disappears, a random searcher is equivalent to a directed searcher; she can keep searching until she gets her desired quality without incurring any loss. Thus, both welfare and the agents' equilibrium payoff under random and directed search coincide.

The parsimonious model in this paper permits a tractable way of comparing different search procedures without changing the underlying environment. While it is not tailored to a specific application, the model can also be seen as a reduced-form labor market search model in which *firms* play the role of the agents, and *workers* of differing productivity play the role of the objects. For example, consider the labor market consisting of all graduating college students. Under random search, a firm simply posts an open call to fill a vacancy. Once a student applies and has been interviewed (random match arrives and productivity is observed), the firm decides whether or not to hire the student. There is a possibility that the vacancy remains unfilled if the student's productivity is too low. On the other hand, if a student's productivity is correlated

[1] While this seemingly contradicts the agent's preference for directed search, that is not the case; a higher equilibrium payoff under random search implies that an agent would prefer to live in a world where everyone is forced to use random search instead of directed search. However, holding all other agents' search procedure fixed, the same agent would prefer to be able to direct her search.

to the quality of the university she attends, then each university can be seen as a sub-market. Under directed search, a firm can choose to recruit only the students of a specific university (e.g. campus career fairs). However, there may be a possibility that a vacancy goes unfilled because too many other firms also recruit at the same university.

In order to accommodate such a labor search market, I extend the non-transferable utility model in two dimensions: First, both the firms and the workers are strategic with each side of the market looking to maximize payoffs. Second, utility is transferable in the form of wages that are endogenously determined through ex-post Nash Bargaining as in Mortensen (1982), Diamond (1982), and Pissarides (1985). Everything else in the model, such as the inflow and outflow dynamics, is the same as the model with non-transferable utility.

Similar to the case with non-transferable utility, when utility is transferable through wages determined by the Nash bargaining solution, I show that there exist equilibrium outcomes in directed search markets with a lower social welfare than in random search markets. The result stands in stark contrast to the literature on the efficiency of directed search with posted prices, which I discuss further in the related literature. I also show that if the firms have all the bargaining power, the equilibrium outcomes of random and directed search in the transferable utility model coincide to the corresponding outcomes of the non-transferable utility model. Essentially, when the firms have all the bargaining power, they can extract the full surplus which reduces the workers into non-strategic objects. Furthermore, I show that if the firms do not have all the bargaining power, then equilibrium outcomes in both random and directed search markets with transferable utility converge to a competitive market outcome as the cost of searching vanishes. The competitive market limit result is in line with Lauermann (2013) who provides necessary and sufficient conditions under which the equilibrium outcomes of search and bargaining models converge to a Walrasian outcome as search frictions disappear.

1.1 Related Literature

In contrast to the main result of this paper, there is a large literature in search theory that shows that directed search is more efficient than random search. In the context of exchange economies, Butters (1977) shows that search directed through price-advertising leads to socially efficient outcomes, and Peters (1991) shows that directed search yields a more efficient outcome than random search. Additionally, Peters (1997, 2000) shows that the equilibrium allocations in finite markets with directed search converge to some competitive market outcome as the number of buyers and sellers grows large. In the context of labor markets, Montgomery (1991) and Moen (1997) show that the equilibrium outcomes under directed search correspond to the socially efficient outcomes of a competitive labor market. Similarly, Mortensen and Wright (2002) show that directed search always leads to socially efficient outcomes whereas random search is efficient only when search frictions disappear. The efficiency of directed search also holds in labor search markets with ex-ante firm investments (Acemoglu and Shimer, 1999a) and with risk-averse workers (Acemoglu and Shimer, 1999b), and in markets with two-sided heterogeneity (Shi, 2002; Shimer, 2005; Eeckhout and Kircher, 2010).

The sharp contrast between the previous literature and the results of this paper stem from the fact that the prior papers study search markets in which prices/wages are posted ex-ante while my paper studies a market either without transfers or one where prices are determined ex-post. All the papers listed consider a directed search markets in which agents on Side "A" of the market post and commit to prices ex-ante while agents on Side "B" of the market direct their search based on the observed prices. In equilibrium, Side A agents take into account how the prices they post affect the search strategy of the Side B agents and the subsequent externalities searching may generate. Thus, the ex-ante posted prices internalize the search externalities the agents create, resulting in an efficient outcome. Even without posted prices, Kim and Kircher (2015) show that directed search based on cheap talk can lead to socially efficient outcomes if the trading mechanism is an efficient first-price auction.

In my paper, utility is either non-transferable or is transferable through prices that are determined after a Side A and Side B agent match. Since the bargaining occurs ex-post, the congestion and adverse selection externalities generated in the search markets cannot be fully off-set by changing the ex-post value of a match. Hence, one can interpret the results in this paper as showing that without the Side A agents' ability to commit to their posted prices or another form of ex-ante efficient trading mechanism, directed search is not only inefficient but is less efficient than random search.

More generally, we can interpret the differences between random and directed search as a difference in the timing of information. Agents have more ex-ante information about their match payoffs under directed search than under random search. As such, the adverse effects of ex-ante information in search markets studied in this paper are in the same spirit to the Hirshleifer effect in exchange economies (Hirshleifer, 1971). More recently, Lester et al. (2016) show that increasing transparency in a search market for "lemons" is sometimes detrimental.

This paper is also related to the literature on adverse selection in search markets. The effect of adverse selection has been studied by Burdett and Coles (1997) in marriage matching markets, Davis (2001) in labor markets, Inderst (2005) in insurance markets, and Lauermann

(2012) in exchange economies. However, all these papers focus only on random search markets.

While not directly related to this paper, the literature on market and search platform design has pointed out that limiting choices (Halaburda et al., 2017), limiting the number of potential partners (Arnosti et al., 2015), limiting preference signaling (Coles et al., 2013), or preventing one side of the market from initiating contact with potential partners (Kanoria and Saban, 2017) could all be welfare improving.

The parsimonious search model in this paper can be seen as a variant of Gale (1987) and Satterthwaite and Shneyerov (2007, 2008). Recently, Lauermann (2013) uses a similar model of search to study how limits of equilibrium allocations relate to competitive market outcomes when search frictions disappears. Similarly, Olszewski and Wolinsky (2016) use a similar model to study search markets for objects with multiple attributes.

The remainder of the paper is structured as follows: In Section 2, I describe the underlying steady state search framework with non-transferable utility, derive the unique steady state equilibrium outcomes under random and directed search, and present the main result that random search is more efficient than directed search in markets with scarcity. In Section 3, I extend the model to accommodate transfers and derive the equilibrium outcomes in a labor search market with ex-post Nash bargaining. Section 4 concludes.

2 Non-transferable Utility

2.1 Model

I consider a search market with agents on one side and objects on the other side. The objects are of differing quality, denoted by $\theta \in \Theta \triangleq [0,1]$. I assume that the objects are non-strategic, i.e., they make no decisions and have no preferences.[2] In contrast, the agents are strategically searching with the hopes of being matched to a high quality object.

Time is discrete. In each period, the agents and the objects in the search market are paired together. The specific details of how they match is discussed in the subsequent sections. If an agent has been matched to an object, she observes the object's type θ and decides to either accept or decline the match. If the agent accepts, she exits the market with the object and gets a utility of $u(\theta) = \theta$. I will refer to an accepted match as *successful*. If the agent declines the match, both the object and the agent return to the search market.

[2] In Section 3, I consider a search market with strategic players on both sides: firms (instead of agents) and workers (instead of objects) who differ in their productivity.

Before the period ends, a fraction $\gamma \in (0,1)$ of the agents and objects left in the search market "die". An agent that exits the market through death gets a payoff of 0. Finally, a mass k_a of new agents and a mass k_o of new objects flow into the market. An important feature of the search model is scarcity; (weakly) fewer objects than agents flow into the market with $k_a \geq k_o > 0$. The newly arriving objects have quality distributed according to an absolutely continuous distribution function $F : \Theta \to [0,1]$ with a bounded and positive density function f.[3]

The search market in each period t is characterized by the tuple $\Psi_t \triangleq \langle G_t, M_t^a, M_t^o \rangle$ where $G_t : \Theta \to [0,1]$ is an endogenously determined market quality distribution, M_t^a is the total mass of agents in the market, and M_t^o is the total mass of objects in the market. Thus, $M_t^o G_t(\theta)$ represents the total mass of objects available in the search market in period t whose quality is at most θ. I will refer to Ψ_t as the market composition in period t. The strategic decisions made by the agents in period t determine the market composition in $t+1$, and so on.

2.2 Steady State Markets

Throughout the paper, the focus is on steady state Markov outcomes: an agent's decision in period t depends only on the current market composition Ψ_t. Furthermore, the market composition is stationary with $\Psi_t = \Psi_{t+1}$. Henceforth, I will omit all time indices.

Fix some steady state outcome. Let $\beta : \Theta \to [0,1]$ be a measurable function with $\beta(\theta)$ representing the probability that a type θ object exits the market through a successful match. Let $\boldsymbol{G}(\cdot;\beta) : \Theta \to [0,1]$ be the market distribution that arises in such a steady state, and let $\boldsymbol{M}^a(\beta)$ and $\boldsymbol{M}^o(\beta)$ be the corresponding market population of agents and objects respectively.

The steady state market is characterized by the balance of the outflow and the inflow of objects and agents. In each period, there is a mass k_o of objects flowing into the market, so an equal mass of objects must be leaving the market. The inflow-outflow dynamics for objects is given by

$$\underbrace{k_o}_{\substack{\text{inflow of}\\ \text{objects}}} = \underbrace{\boldsymbol{M}^o(\beta) \int_\Theta \Big(\underbrace{\beta(\theta)}_{\substack{\text{exit via}\\ \text{successful}\\ \text{match}}} + \underbrace{(1-\beta(\theta))\gamma}_{\substack{\text{exit via}\\ \text{death}}} \Big) d\boldsymbol{G}(\theta;\beta)}_{\substack{\text{outflow of}\\ \text{objects}}}. \tag{1}$$

[3]Specifically, there exist constants $0 < \underline{f} < \bar{f} < \infty$ such that $f(\theta) \in (\underline{f}, \bar{f})$ for all $\theta \in \Theta$.

Additionally, the inflow-outflow dynamics for objects of quality $\theta \in [\theta', \theta'']$ must satisfy

$$\underbrace{k_o\big(F(\theta'') - F(\theta')\big)}_{\text{inflow}} = \underbrace{\boldsymbol{M}^o(\beta) \int_{\theta'}^{\theta''} \Big(\beta(\theta) + (1 - \beta(\theta))\gamma\Big) d\boldsymbol{G}(\theta; \beta)}_{\text{outflow}} . \tag{2}$$

Similarly, there is a mass k_a of agents flowing into the market, so an equal mass of agents must be leaving the market. The inflow-outflow dynamics for agents is given by

$$\underbrace{k_a}_{\substack{\text{inflow of} \\ \text{agents}}} = \underbrace{\boldsymbol{M}^o(\beta) \int\limits_{\Theta} \beta(\theta) d\boldsymbol{G}(\theta; \beta)}_{\substack{\text{outflow via} \\ \text{successful} \\ \text{match}}} + \underbrace{\gamma \left(\boldsymbol{M}^a(\beta) - \boldsymbol{M}^o(\beta) \int\limits_{\Theta} \beta(\theta) d\boldsymbol{G}(\theta; \beta) \right)}_{\substack{\text{outflow via} \\ \text{death}}} . \tag{3}$$

Since a successful match involves an object-agent pair, the mass of agents that leave the market through a successful match is equal to the mass of objects that leave the market through a successful match, given by the first term on the right-hand side of (3). Of the remaining agents, a fraction γ leave through death, given by second term on the right-hand side of (3).

Prior to describing the search models, it is helpful to characterize a general property of the steady state market.

Lemma 1

Let $\beta : \Theta \to [0, 1]$ be some measurable function representing the probability that an object leaves the market through a successful match. For any fixed $\gamma > 0$, there exists a unique steady state market composition $\Psi = \langle \boldsymbol{G}(\cdot; \beta), \boldsymbol{M}^a(\beta), \boldsymbol{M}^o(\beta) \rangle$. The market distribution $\boldsymbol{G}(\cdot; \beta) : \Theta \to [0, 1]$ is absolutely continuous with a positive and bounded density function $\boldsymbol{g}(\cdot; \beta)$. Furthermore, $\boldsymbol{M}^a(\beta) \geq (=) \boldsymbol{M}^o(\beta)$ if, and only if, $k_a \geq (=) k_o$.

Proof: Appendix.

2.3 Random Search

In a random search market, an agent observes an object's type only after being matched to it. Prior to a match, the agent only knows the prevailing market composition $\Psi \triangleq \langle G, M^a, M^o \rangle$. In each period, agents and objects are randomly paired so that the maximal number of matches is achieved. As there are (weakly) more agents than objects, each object gets matched to a single agent with probability 1 whereas each agent gets matched to a single object with probability M^o/M^a.

The agent can choose to accept the match and exit the market or can decline and search again next period with probability $1-\gamma$. For a given market composition Ψ, let $U^r(\Psi)$ be an agent's value from randomly searching, defined by

$$U^r(\Psi) = \frac{M^o}{M^a}\int_{\Theta} \max\{\theta, (1-\gamma)U^r(\Psi)\}dG(\theta) + \left(1 - \frac{M^o}{M^a}\right)(1-\gamma)U^r(\Psi).$$

An agent's optimal strategy is given by a cutoff $\bar{\theta}(\Psi)$ such that the agent accepts any object of quality $\theta > \bar{\theta}(\Psi)$ and rejects any object of quality $\theta < \bar{\theta}(\Psi)$. The cutoff is characterized by the indifference condition

$$\bar{\theta}(\Psi) = (1-\gamma)U^r(\Psi),$$

the marginal quality at which the agent is indifferent between accepting her current match and continuing to search for a possibly better match.[4]

Each agent's cutoff strategy depends on the market composition Ψ which she takes as a given. However, the market composition is itself an endogenous variable that depends on the aggregate effect of the agents' strategies.

When all agents follow a cutoff strategy, the inflow-outflow equations take a particularly simple form. With some abuse of notation, let $\boldsymbol{G}(\cdot;\hat{\theta}) : \Theta \to [0,1]$ be the steady state market distribution that arises endogenously when all agents use the cutoff $\hat{\theta} \in \Theta$.[5] Similarly define $\boldsymbol{M}^a(\hat{\theta})$ and $\boldsymbol{M}^o(\hat{\theta})$. Objects with quality below $\hat{\theta}$ exit the market only through death while objects with quality above $\hat{\theta}$ exit the market only through a successful match. Thus, the inflow-outflow dynamics for objects given in (1) can be expressed as

$$k_o = \boldsymbol{M}^o(\hat{\theta})\Big(\boldsymbol{G}(\hat{\theta};\hat{\theta})\gamma + 1 - \boldsymbol{G}(\hat{\theta};\hat{\theta})\Big). \tag{4}$$

Additionally, for any $\theta < \hat{\theta}$, the inflow-outflow equations given in (2) are now simplified to

$$k_o F(\theta) = \boldsymbol{M}^o(\hat{\theta})\boldsymbol{G}(\theta;\hat{\theta})\gamma, \tag{5}$$

[4] Since $\Theta = [0,1]$ and $\gamma \in (0,1)$, it is never the case that $\bar{\theta}(\Psi) = 0$; the agent can always do better by continuing to search and accepting any quality strictly greater than her outside option. The results in the paper easily generalize to the case where $\Theta = [\theta_l, \theta_u]$ where the indifference condition satisfies $\bar{\theta}(\Psi) \geq (1-\gamma)U^r(\Psi)$, with equality if $\bar{\theta}(\Psi) > \theta_l$.

[5] Specifically, $\boldsymbol{G}(\cdot;\hat{\theta}) = \boldsymbol{G}(\cdot;\beta)$ with $\beta(\theta) = 0$ if $\theta < \hat{\theta}$ and $\beta(\theta) = 1$ if $\theta > \hat{\theta}$.

and for any $\theta > \hat{\theta}$, the inflow-outflow equations are given by

$$k_o\Big(F(\theta) - F(\hat{\theta})\Big) = \boldsymbol{M}^o(\hat{\theta})\Big(\boldsymbol{G}(\theta;\hat{\theta}) - \boldsymbol{G}(\hat{\theta};\hat{\theta})\Big). \tag{6}$$

Solving the system of equations (4)-(6) yields a solution for the steady state market composition given by

$$\boldsymbol{G}(\theta;\hat{\theta}) = \begin{cases} \frac{F(\theta)}{\gamma+(1-\gamma)F(\hat{\theta})} & \text{for} \quad \theta \leq \hat{\theta} \\ \frac{\gamma F(\theta)+(1-\gamma)F(\hat{\theta})}{\gamma+(1-\gamma)F(\hat{\theta})} & \text{for} \quad \theta \geq \hat{\theta} \end{cases}, \tag{7}$$

$$\boldsymbol{M}^o(\hat{\theta}) = \gamma^{-1}k_o\Big(\gamma + (1-\gamma)F(\hat{\theta})\Big), \tag{8}$$

$$\boldsymbol{M}^a(\hat{\theta}) = \gamma^{-1}\Big(k_a - k_o(1-\gamma)(1-F(\hat{\theta}))\Big). \tag{9}$$

Definition 1 *A random search stationary Markov Equilibrium (SME) is given by a cutoff $\theta_r \in \Theta$ and a market composition Ψ_r such that*

(i) $\theta_r = \bar{\theta}(\Psi_r)$, *and*

(ii) $\Psi_r = \langle \boldsymbol{G}(\cdot;\theta_r), \boldsymbol{M}^a(\theta_r), \boldsymbol{M}^o(\theta_r)\rangle$.

In other words, the pair (θ_r, Ψ_r) constitute an equilibrium if (i) given the market composition Ψ_r, the cutoff θ_r is optimal for each agent, and (ii) the market composition Ψ_r is consistent with a steady state outcome as given by (7)-(9) when all agents use the cutoff θ_r.

Proposition 1
There exists a unique random search SME (θ_r^*, Ψ_r^*) such that

$$\theta_r^* = (1-\gamma)\frac{k_o}{k_a}\int_{\theta_r^*}^{1} \theta dF(\theta). \tag{10}$$

Proof. Fix some market composition $\Psi \triangleq \langle G, M^a, M^o\rangle$. An agent's optimal cutoff is $\bar{\theta}(\Psi)$.

Hence, the value from random search is given by

$$U^r(\Psi) = \frac{M^o}{M^a}\int_{\bar{\theta}(\Psi)}^1 \theta dG(\theta) + (1-\gamma)U^r(\Psi_r)\left(\frac{M^o}{M^a}G\big(\bar{\theta}(\Psi)\big) + 1 - \frac{M^o}{M^a}\right)$$

$$= \frac{\frac{M^o}{M^a}\int_{\bar{\theta}(\Psi)}^1 \theta dG(\theta)}{\gamma + \frac{M^o}{M^a}(1-\gamma)\Big(1 - G\big(\bar{\theta}(\Psi)\big)\Big)}.$$

The pair (θ_r, Ψ_r) constitutes an SME if $\theta_r = \bar{\theta}(\Psi_r)$ and $\Psi_r = \langle \boldsymbol{G}(\cdot;\theta_r), \boldsymbol{M}^a(\theta_r), \boldsymbol{M}^o(\theta_r)\rangle$. Substituting in the market composition expressions in (7)-(9) simplifies the value function to

$$U^r(\Psi_r) = \frac{k_o}{k_a}\int_{\theta_r}^1 \theta dF(\theta).$$

Recall that the agent's cutoff strategy for a given market composition Ψ is characterized by the indifference condition $\bar{\theta}(\Psi) = (1-\gamma)U^r(\Psi)$. Thus, $\theta_r = \bar{\theta}(\Psi_r)$ if, and only if,

$$\theta_r = (1-\gamma)\frac{k_o}{k_a}\int_{\theta_r}^1 \theta dF(\theta).$$

Define $\mathbb{U}(\hat{\theta}) = \int_{\hat{\theta}}^1 \theta dF(\theta)$ which (i) is strictly decreasing in $\hat{\theta}$, (ii) $\mathbb{U}(0) = \mathbb{E}_F[\theta]$, and (iii) $\mathbb{U}(1) = 0$. Hence, there is a unique fixed point to the function $\frac{k_o}{k_a}(1-\gamma)\mathbb{U}$ given by θ_r^*, as depicted in Figure 1. The market composition Ψ_r^* is then derived from the cutoff θ_r^* using (7)-(9) ■

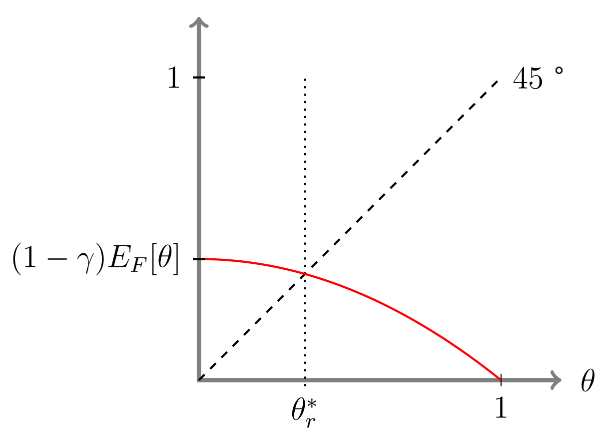

Figure 1: The dashed black line is the 45° line while the solid red curve plots $(1-\gamma)\mathbb{U}$. $F = Unif[0,1]$, $k_a = k_o$, and $\gamma = 0.2$.

Intuitively, an agent who searches randomly faces an inter-temporal trade off; she can either settle for the object she currently has matched to or she can continue randomly searching for an object with a higher quality. However, an agent who foregoes her match today can continue to search only if she survives into the next period. Thus, her continuation value of search must be scaled down by her "effective discount factor" $1-\gamma$. The indifference condition (10) captures the resolution of this inter-temporal trade off. The cutoff θ_r^*, which I refer to as the agent's *pickiness*, is the lowest quality the agent is willing to settle for today after accounting for her discounted net present value of searching in the future, given by $(1-\gamma)\frac{k_o}{k_a}\int_{\theta_r^*}^1 \theta dF(\theta)$.

For a given $\gamma > 0$, let $\theta_r^*(\gamma)$ be the unique equilibrium cutoff satisfying the indifference condition (10). Let $\boldsymbol{U}_\gamma^r = \frac{\theta_r^*(\gamma)}{1-\gamma}$ be the agent's unique equilibrium value of random search.

As γ decreases, the agent's effective discount factor increases which in turn increases her discounted value of search. Thus, the agent becomes less willing to settle for low quality objects. However, as γ decreases, the low quality objects that the agents do not want are more likely to remain in the search market. This leads to an increase in the likelihood of matching with an object below the agent's cutoff, thereby reducing the *undiscounted* value of search. In other words, a decrease in γ reduces the agent's value of search but increases her pickiness.

Corollary 1

(i) $\theta_r^*(\gamma)$ *is decreasing in* γ*, i.e., the agent becomes less picky as* γ *increases, and*

(ii) $\boldsymbol{U}_\gamma^r$ *is increasing in* γ*, i.e., the agent's value of search increases as* γ *increases.*

The unique equilibrium outcome in a random search market is inefficient. The efficient outcome would be for all agents to accept all matches which would clear the market as much as possible (taking into account that there are weakly more agents than objects) and would yield a higher average payoff of $\frac{k_o}{k_a}\mathbb{E}_F[\theta]$. However, accepting all matches is not payoff maximizing for any individual agent. In equilibrium, an agent instead chooses to accept all types above $\theta_r^*(\gamma)$ after taking into account the negative externalities that other agents impose on the market composition. However, agents do not fully internalize the negative externalities of their own strategic choices. Hence, the random search SME is inefficient.

A key source of the inefficiency in random search markets is that agents are forced to match with positive probability to objects they would rather avoid. As a result, the agents remain in the market for too long. In the following section, I consider a directed search model in which the agents are able to filter their search based on the quality of objects. Hence, each agent has more information ex-ante that she can use to only match with the objects she deems acceptable.

2.4 Directed Search

In this section, the search market is further subdivided into a continuum of sub-markets. A sub-market $S(\theta)$ exclusively contains all objects in the market with quality θ. Given a market composition $\Psi \triangleq \langle G, M^a, M^o \rangle$, sub-market $S(\theta)$ has a "mass" of $M^o g(\theta)$ objects, where g is the density function of G. The object-side of the entire search market is an aggregation of sub-makets, $\cup_{\theta \in \Theta} S(\theta)$, with size $\int_\Theta M^o g(\theta) d\theta = M^o$.

Agents can distinguish between all the sub-markets. Therefore, each agent knows not only the market composition but also the quality of an object prior to matching. Additionally, agents can choose which sub-market to search in for a given period. Let $Q(\theta)$ be the fraction of agents entering sub-markets $\cup_{\theta' \leq \theta} S(\theta')$. I will refer to Q as the *queuing CDF*, even though there is no actual queue in the formal sense of the word.

I assume that Q is absolutely continuous with respect to the market distribution G, i.e., agents cannot enter into a sub-market that does not exist. By Lemma 1, Q is also absolutely continuous with respect to the Lebesgue measure and admits a density function q. I will refer to $M^a q(\theta)$ as the queue length in sub-market $S(\theta)$.

In each period, agents and objects within a sub-market are paired randomly so that the maximal number of matches is achieved. The long-side of the sub-market is rationed while the short-side is guaranteed a match. Let $\alpha(\theta; Q, \Psi) = \min \left\{ \frac{M^o g(\theta)}{M^a q(\theta)}, 1 \right\}$ be the *agent match probability*, the probability that an agent in sub-market $S(\theta)$ gets a match in a given period. Similarly, let $\beta(\theta; Q, \Psi) = \min \left\{ \frac{M^a q(\theta)}{M^o g(\theta)}, 1 \right\}$ be the *object match probability*, the probability that an object in sub-market $S(\theta)$ gets matched in a given period.

To simplify exposition, assume that agents remain in the first sub-market they enter. Additionally, assume that once an agent receives a match, she accepts and leaves the market, i.e., all matches are successful. Both of these assumptions are without loss of generality. In equilibrium, agents must be indifferent among all of the sub-markets they enter. Whether or not an agent remains in the first sub-market she enters is irrelevant for her payoff. Moreover, it is never rational for an agent to enter a sub-market and then decline a match. After all, the agent cannot get matched to a higher quality object by waiting in the same sub-market and could have done better by choosing a different sub-market to enter in the first place.

Given a queuing CDF Q and a market composition Ψ, let $U^d(\theta; Q, \Psi)$ be the agent's value from entering sub-market $S(\theta)$, given by

$$\begin{aligned} U^d(\theta; Q, \Psi) &= \alpha(\theta; Q, \Psi)\theta + (1-\gamma)\big(1-\alpha(\theta; Q, \Psi)\big)U^d(\theta; Q, \Psi) \\ &= \frac{\alpha(\theta; Q, \Psi)\theta}{\gamma + \alpha(\theta; Q, \Psi)(1-\gamma)}. \end{aligned}$$

An agent's decision of which sub-market to enter is affected by Q and Ψ which she takes as a given. However, both Q and Ψ themselves depends on the aggregate effect of the agents' entry strategies.

Definition 2 *A direct search SME is given by a pair* (Q_d, Ψ_d) *such that*

(i) $Q_d(\theta) = \int_0^\theta q_d(\omega)d\omega$ *for all* $\theta \in \Theta$ *with* $Q_d(1) = 1$,

(ii) $q_d(\theta) > 0$ *implies* $\theta \in \arg\max_{\theta' \in \Theta} U^d(\theta'; Q_d, \Psi_d)$, *and*

(iii) $\Psi_d = \langle \boldsymbol{G}\big(\cdot; \beta(\cdot; Q_d, \Psi_d)\big), \boldsymbol{M}^a\big(\beta(\cdot; Q_d, \Psi_d)\big), \boldsymbol{M}^o\big(\beta(\cdot; Q_d, \Psi_d)\big)\rangle$ *as given by* (1)-(3).

In other words, the pair (Q_d, Ψ_d) constitute an equilibrium if *(i)* Q_d is an absolutely continuous queuing CDF, *(ii)* given agent match probabilities $\alpha(\cdot; Q_d, \Psi_d)$, agents enter only the sub-markets that maximize their payoff, and *(iii)* given object match probabilities $\beta(\cdot; Q_d, \Psi_d)$, the market distribution and size is characterized by (1)-(3).[6]

In order to characterize the equilibrium, I make the following observation that simplifies the problem.

Lemma 2
If (Q_d, Ψ_d) *is a direct search SME, there exists a cutoff* $\theta_d \in \Theta$ *such that*

$$q_d(\theta) \begin{cases} = 0 & \text{for} \quad \theta < \theta_d \\ > 0 & \text{for} \quad \theta > \theta_d \end{cases}$$

and $\Psi_d = \langle \boldsymbol{G}\big(\cdot; \theta_d\big), \boldsymbol{M}^a\big(\theta_d\big), \boldsymbol{M}^o\big(\theta_d\big)\rangle$ *as given by* (7)-(9).

If agents enter a sub-market $S(\theta')$ in equilibrium, then they must enter all sub-markets containing objects of quality $\theta > \theta'$. Otherwise, if the queue length at $S(\theta'')$ for some $\theta'' > \theta'$

[6] Recall that all matches are assumed to be successful without loss of generality.

were 0, the agent could enter $S(\theta'')$ and get matched to an object of a higher quality with probability 1.

Yet, agents must be indifferent among all the sub-markets they enter in equilibrium. Consequently, sub-markets that contain objects of higher quality must also have longer queue lengths (lower agent match probabilities). In other words, for almost all sub-markets the agents enter, the agents must be on the long-side while the objects are on the short-side.

Combining these two observations, there exists some cutoff θ_d such that objects with quality above θ_d leave the market only through a successful match while objects with quality below θ_d leave the market only through death. The inflow and outflow of objects now closely resemble the dynamics in the random search model, yielding a similar market composition.

Proof.
Let (Q_d, Ψ_d) be a direct search SME. By definition, $q_d(\theta) > 0$ only if $\theta \in \arg\max_{\theta'} U^d(\theta'; Q_d, \Psi_d)$. Assume $q_d(\theta') > 0 = q_d(\theta'')$ for some $\theta' < \theta''$. Then, $\alpha(\theta''; Q_d, \Psi_d) = \min\left\{\frac{M^o g_d(\theta'')}{M^a q_d(\theta'')}, 1\right\} = 1$ and $U^d(\theta''; Q_d, \Psi_d) = \theta'' > \theta' \geq U^d(\theta'; Q_d, \Psi_d)$, which is a contradiction.

Therefore, there is a cutoff θ_d such that $q_d(\theta) > 0$ for $\theta > \theta_d$ and $q_d(\theta) = 0$ for $\theta < \theta_d$. Additionally, $U^d(\theta''; Q_d, \Psi_d) = U^d(\theta'; Q_d, \Psi_d)$ for all $\theta'' > \theta' \geq \theta_d$, which requires $\alpha(\theta; Q_d, \Psi_d)$ to be strictly decreasing in θ over $[\theta_d, 1]$. As $\alpha(\theta; Q_d, \Psi_d) < 1$ for all $\theta > \theta_d$, by definition, $\beta(\theta; Q_d, \Psi_d) = 1$. Thus, all objects in sub-markets $S(\theta)$ with $\theta > \theta_d$ exit the search market only through a successful match.

Furthermore, $q_d(\theta) = 0$ for all $\theta < \theta_d$ implies that $\beta(\theta; Q_d, \Psi_d) = \min\left\{\frac{M^a q_d(\theta)}{M^o g_d(\theta)}, 1\right\} = 0$. Thus, objects in sub-markets $S(\theta)$ with $\theta < \theta_d$ exit only through death. We can now replace the complicated inflow-outflow equations (1)-(3) by the simpler inflow-outflow equations (4)-(6) with the cutoff $\hat{\theta} = \theta_d$. The resulting market composition has the closed-form solution given by (7)-(9). ■

The following proposition characterizes the direct search SME and shows that it is uniquely pinned down by the cutoff θ_d.

Proposition 2
There exists a unique direct search SME (Q_d^*, Ψ_d^*) such that

(i) $\Psi_d^* = \langle \boldsymbol{G}\big(\cdot; \theta_d^*\big), \boldsymbol{M}^a\big(\theta_d^*\big), \boldsymbol{M}^o\big(\theta_d^*\big)\rangle$,

(ii) $Q_d^*(\theta) = \begin{cases} 0 & \textit{if} \quad \theta \le \theta_d^* \\ \int_{\theta_d^*}^{\theta} \frac{k_o(\omega-(1-\gamma)\theta_d^*)}{\left(k_a-k_o(1-\gamma)(1-F(\theta_d^*))\right)\theta_d^*} dF(\omega) & \textit{for} \quad \theta \ge \theta_d^* \end{cases},$

(iii) $U^d(\theta; Q_d^*, \Psi_d^*) = \begin{cases} \theta & \textit{for} \quad \theta \le \theta_d^* \\ \theta_d^* & \textit{for} \quad \theta \ge \theta_d^* \end{cases}.$

The cutoff $\theta_d^* \in \Theta$ *is characterized by*

$$\theta_d^* = \frac{k_o}{k_a} \int_{\theta_d^*}^{1} \theta dF(\theta). \tag{11}$$

Proof. Let (Q_d, Ψ_d) be a direct search SME with $\Psi_d \triangleq \langle G_d, M_d^a, M_d^o \rangle$. From Lemma 2, there exists a cutoff $\theta_d \in \Theta$ such that $q_d(\theta) > (=) 0$ for all $\theta > (<) \theta_d$.

For all $\theta < \theta_d$, the agent match probability $\alpha(\theta; Q_d, \Psi_d) = 1$ as the queue length is 0 below the cutoff. Hence, $U^d(\theta; Q_d, \Psi_d) = \theta$ for all $\theta < \theta_d$. Additionally, $\lim_{\theta \uparrow \theta_d} U^d(\theta; Q_d, \Psi_d) = \theta_d$.

As the agents must be indifferent to all the sub-markets they enter, $U^d(\theta''; Q_d, \Psi_d) = U^d(\theta'; Q_d, \Psi_d)$ for all $\theta'' > \theta' > \theta_d$. Moreover, these sub-markets must be payoff-maximizing: $U^d(\theta; Q_d, \Psi_d) \ge \theta_d$ for all $\theta > \theta_d$. However, $U^d(\theta; Q_d, \Psi_d) \le \theta$ for all $\theta \in \Theta$, which implies $\lim_{\theta \downarrow \theta_d} U^d(\theta; Q_d, \Psi_d) \le \lim_{\theta \downarrow \theta_d} \theta = \theta_d$. Thus, $U^d(\theta; Q_d, \Psi_d) = \theta_d$ for all $\theta \ge \theta_d$.

The indifference condition $U^d(\theta; Q_d, \Psi_d) = \theta_d$ for all $\theta > \theta_d$ necessitates that

$$\alpha(\theta; Q_d, \Psi_d) = \begin{cases} 1 & \text{for} \quad \theta \le \theta_d \\ \frac{\gamma \theta_d}{\theta - (1-\gamma)\theta_d} & \text{for} \quad \theta \ge \theta_d \end{cases}.$$

Because $\alpha(\theta; Q_d, \Psi_d) < 1$ for all $\theta > \theta_d$, agents must be on the long-side of each sub-market above the cutoff. By definition, we also have that $\alpha(\theta; Q_d, \Psi_d) = \frac{g_d(\theta)}{q_d(\theta)} \zeta_d$ for all $\theta > \theta_d$, where

$\zeta_d = \frac{M_d^o}{M_d^a}$. Equating these two expressions for the agent match probability and integrating over the interval $[\theta_d, 1]$ gives

$$\gamma\theta_d \int_{\theta_d}^{1} q_d(\theta)d\theta = \zeta_d \int_{\theta_d}^{1} \theta dG_d(\theta) - \zeta_d(1-\gamma)\theta_d(1 - G_d(\theta_d)).$$

Since $q_d(\theta) = 0$ for all $\theta < \theta_d$, we can rewrite $\int_{\theta_d}^1 q_d(\theta)d\theta$ as $\int_\Theta q_d(\theta)d\theta$ which equals 1 (because Q is a well-defined CDF). Hence, the expression simplifies to

$$\theta_d = \frac{\zeta_d \int_{\theta_d}^1 \theta dG_d(\theta)}{\gamma + \zeta_d(1-\gamma)\big(1 - G_d(\theta_d)\big)}$$

From Lemma 2, the market composition Ψ_d is consistent with $\langle \boldsymbol{G}\big(\cdot;\theta_d\big), \boldsymbol{M}^a\big(\theta_d\big), \boldsymbol{M}^o\big(\theta_d\big)\rangle$ as given by (7)-(9). Substituting in these expressions for G_d, M_d^a, and M_d^o gives

$$\theta_d = \frac{k_o}{k_a}\int_{\theta_d}^{1} \theta dF(\theta).$$

Recall the function $\mathbb{U}$ defined by $\mathbb{U}(\hat{\theta}) = \int_{\hat{\theta}}^1 \theta dF(\theta)$ which (i) is strictly decreasing in $\hat{\theta}$, (ii) $\mathbb{U}(0) = \mathbb{E}_F[\theta]$, and (iii) $\mathbb{U}(1) = 0$. Hence, there is a unique fixed point to $\frac{k_o}{k_a}\mathbb{U}$ given by θ_d^*, as shown in Figure 2. The market composition Ψ_d^* is then pinned down from the cutoff θ_d^* using (7)-(9). Similarly, the queuing CDF is pinned down from the agent match probability with

$$\alpha(\theta; Q_d^*, \Psi_d^*) = \frac{\boldsymbol{M}^o(\theta_d^*)\boldsymbol{g}(\theta;\theta_d^*)}{\boldsymbol{M}^a(\theta_d^*)q_d^*(\theta)} = \frac{\gamma\theta_d^*}{\theta - (1-\gamma)\theta_d^*}$$

for all $\theta > \theta_d^*$. ■

Intuitively, an agent in a directed search market faces the same trade-offs present in the random search market, i.e., settle for a low quality object versus search for a high quality object. However, the trade-off under directed search is contemporaneous not inter-temporal. In particular, the agent can either settle for a low quality object she can get with certainty *today*, or she can queue in a congested market *today* for the possibility of getting a high quality object. The indifference condition (11) captures the resolution of this contemporaneous trade off. The cutoff θ_d^* is the highest quality the agent can get with certainty while $\frac{k_o}{k_a}\int_{\theta_d^*}^1 \theta dF(\theta)$ is her value of directed search from using Q as her searching strategy.

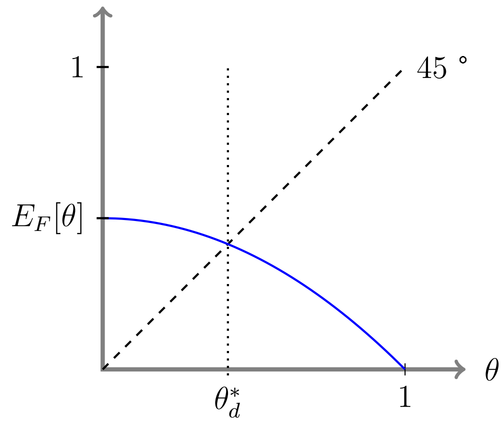

Figure 2: The dashed black line is the 45° line while the solid blue curve plots $\mathbb{U}$. $F = Unif[0,1]$, $k_a = k_o$, and $\gamma = 0.2$.

For a slightly different intuition, let us re-write (11) as

$$k_a \theta_d^* = k_o \int_{\theta_d^*}^{1} \theta dF(\theta).$$

The left-hand side is the quality "demanded" by the new agents flowing into the market. Specifically, there are k_a new agents that arrive in each period, and each one expects to get a payoff of $\mathbb{E}_{Q_d^*}[U^d(\theta; Q_d^*, \Psi_d^*)] = \theta_d^*$. The right-hand side is the total quality "supplied" to the agents. In each period, high quality objects in sub-markets $\cup_{\theta > \theta_d^*} S(\theta)$ are depleted through successful matches. Hence, the equality between the left and right-hand side captures the idea that we can deliver a total expected utility of $k_a \theta_d^*$ to the agents only through the newly arriving objects that replenish the sub-markets $\cup_{\theta > \theta_d^*} S(\theta)$.

For a given $\gamma > 0$, let $\theta_d^*(\gamma)$ be the unique equilibrium cutoff satisfying (11). Let $\boldsymbol{U}_\gamma^d = \theta_d^*(\gamma)$ be the agent's unique equilibrium value of directed search.

Interestingly, the cutoff $\theta_d^*(\gamma)$ (and thus, the value of directed search) is independent of the cost of searching. As γ decreases, an agent who failed to get a match today is more likely to survive into the next period. Hence, she becomes more willing to queue in congested sub-markets that contain high quality objects. However, the same is true of all agents; as γ decreases, the mass of agents in the market increases which makes the congestion in the desirable sub-markets worse. Furthermore, the supply of objects in the desirable sub-markets $\cup_{\theta > \theta_d^*(\gamma)} S(\theta)$ remains unaffected by a change in γ as all of the objects leave the market only through a successful match. In other words, while the cost of searching has decreased, the probability of being rationed a high quality object has also decreased by the same factor. Hence, a decrease in γ has no effect in a directed search market.[7]

Corollary 2 *The cutoff $\theta_d^*(\gamma)$ and the value of directed search $\boldsymbol{U}_\gamma^d$ are constant in γ.*

Similar to the random search model, the unique equilibrium outcome in a directed search market is inefficient. Agents could have queued in all the sub-markets, with $q(\theta) = g(\theta)$ for all $\theta \in \Theta$, which would clear the market as much as possible and would deliver an average payoff of $\frac{k_o}{k_a}\mathbb{E}_F[\theta]$. However, an agent queued in a sub-market with low quality objects would have an incentive to deviate and queue in a sub-market with a high quality objects. In equilibrium, an agent instead chooses to queue at sub-markets $\cup_{\theta > \theta_d^*(\gamma)} S(\theta)$ after taking into account the congestion that other agents create. However, agents do not fully internalize the congestion externalities of their own strategic choices which leads to an inefficient outcome.

In the next section, I take a closer look at the differences between random and directed search in terms of equilibrium payoffs and welfare.

2.5 Random versus Directed Search

In the previous sections, I have noted that the equilibrium outcomes of random and directed search markets are inefficient. In this section, I show that the inefficiencies created by directed search are worse than the inefficiencies arising from random search.

Since all agents have the same preferences, $k_a\boldsymbol{U}_\gamma^r$ is the total expected payoff delivered to the newly arriving agents in a random search market when the cost of searching is fixed at $\gamma > 0$. Similarly, $k_a\boldsymbol{U}_\gamma^d$ is the total expected payoff delivered to the newly arriving agents in a directed search. Yet, for welfare comparisons, focusing only on the total payoff delivered to new arrivals is misleading because the population of agents in a random search market differs from the population in a directed search market. As such, I define welfare by normalizing the total expected payoff for new arrivals by the total population of agents in the market. Let $\boldsymbol{W}_\gamma^r = \frac{k_a}{M_r^a(\gamma)}\boldsymbol{U}_\gamma^r$ be welfare in a random search market and let $\boldsymbol{W}_\gamma^r = \frac{k_a}{M_d^a(\gamma)}\boldsymbol{U}_\gamma^d$ be welfare in a directed search market.[8]

[7]Under random search, there were two additional effects: First, the mass of low quality objects increases as more of them survive in the search market. Hence, the likelihood of randomly matching with these low quality objects increases as γ decreases which lowers $\boldsymbol{U}_\gamma^r$. In directed search, the mass of low quality objects also increases but this has no effect as the agents can choose to not enter the sub-markets containing the low quality objects. Second, under random search, the effective discount factor $(1-\gamma)$ increases making the inter-temporal trade-off cheaper for an agent, which in turn made her more picky. In contrast, the trade off under directed search is contemporaneous and thus does not involve the effective discount factor.

[8]For a fixed $\gamma > 0$, $M_r^a(\gamma)$ and $M_d^a(\gamma)$ are the endogenous agent populations in a random and directed search SME respectively.

Proposition 3
For any $\gamma > 0$,

(i) $\boldsymbol{U}_\gamma^r > \boldsymbol{U}_\gamma^d$, *and*

(ii) $\boldsymbol{W}_\gamma^r > \boldsymbol{W}_\gamma^d$.

Furthermore, $\lim_{\gamma \to 0} \boldsymbol{U}_\gamma^r - \boldsymbol{U}_\gamma^d = 0$.

Proof. Recall the strictly decreasing function $\mathbb{U} : \Theta \to \Theta$ defined by $\mathbb{U}(\hat{\theta}) = \int_{\hat{\theta}}^1 \theta dF(\theta)$. The cutoff $\theta_r^*(\gamma)$ is the unique fixed point of $(1-\gamma)\frac{k_o}{k_a}\mathbb{U}$ and θ_d^* is the unique fixed point of $\frac{k_o}{k_a}\mathbb{U}$. Thus, for any $\gamma > 0$, $\theta_r^*(\gamma) < \theta_d^*(\gamma)$, and

$$\boldsymbol{U}_\gamma^d = \theta_d^*(\gamma) \underbrace{=}_{\substack{\text{by} \\ \text{Proposition 2}}} \mathbb{U}\big(\theta_d^*(\gamma)\big) < \mathbb{U}\big(\theta_r^*(\gamma)\big) \underbrace{=}_{\substack{\text{by} \\ \text{Proposition 1}}} \frac{\theta_r^*(\gamma)}{1-\gamma} = \boldsymbol{U}_\gamma^r.$$

Furthermore,

$$M_d^a(\gamma) = \frac{k_a - k_o(1-\gamma)\Big(1 - F\big(\theta_d^*(\gamma)\big)\Big)}{\gamma} > \frac{k_a - k_o(1-\gamma)\Big(1 - F\big(\theta_r^*(\gamma)\big)\Big)}{\gamma} = M_r^a(\gamma),$$

where the first and last equality follow from (9). Therefore, $\boldsymbol{W}_\gamma^r = \frac{k_a}{M_r^a(\gamma)}\boldsymbol{U}_\gamma^r > \frac{k_a}{M_d^a(\gamma)}\boldsymbol{U}_\gamma^d = \boldsymbol{W}_\gamma^d$.

Finally, as γ goes to 0, the fixed point of $\frac{k_o}{k_a}\mathbb{U}$ and $(1-\gamma)\frac{k_o}{k_a}\mathbb{U}$ converge, i.e. $\lim_{\gamma \to 0} \theta_r^*(\gamma) - \theta_d^*(\gamma) = 0$. Thus, $\lim_{\gamma \to 0} \boldsymbol{U}_\gamma^r - \boldsymbol{U}_\gamma^d = 0$. ■

Figure 3 depicts the characterization of cutoffs and equilibrium expected payoffs under random and directed search.

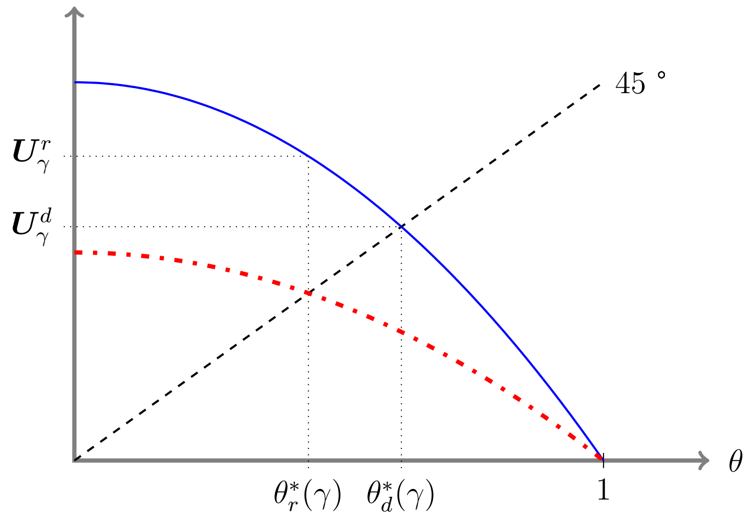

Figure 3: The dashed black line is the 45° line. The solid blue curve plots $\mathbb{U}$ while the dash-dotted red curve plots $(1-\gamma)\mathbb{U}$. $F = Unif[0,1]$, $k_a = k_o$, and $\gamma = 0.2$.

To simplify, let us assume that $k_a = k_o$ which, by Lemma 1, implies that the mass of agents and objects in the search market are equal. Hence, each agent is guaranteed a match with an object of some quality.

When search is random, each agent is paired to an object of some quality $\theta \in [0,1]$. She accepts or rejects the object by trading-off the current quality of her match against the discounted expected quality of her future matches. The cutoff $\theta_r^*(\gamma)$ is her strategically optimal level of pickiness: it is the lowest quality she is willing to settle for today in order to forsake potentially better matches in the future after taking into account the equilibrium market composition.

However, the inefficiencies in a random search market arise precisely because the agents' pickiness imposes negative externalities on the market composition. First, the agents' pickiness gives rise to adverse selection; in each period, all the high quality objects leave the search market while a fraction of the low quality objects remain. In other words, the search market over-represents the low quality objects which decreases the likelihood that the agents will match with the high quality objects they desire. Furthermore, as a positive fraction of agents are matched to low quality objects, these agents decline their matches and either remain in the market for too long or exit the market through death. However, these agents could have left the market through a successful match by settling for a low quality object.

In general, payoffs and welfare decrease when agents in a random search market become more picky. Intuitively, as agents become pickier, (i) the adverse selection problem becomes worse, and (ii) the fraction of agents who exit through a successful match decreases. Formally, consider a hypothetical search market where all the agents are paired randomly to objects, and forced to accept matches of quality above $\hat{\theta}$ and to decline matches below $\hat{\theta}$. Let $\hat{\Psi} = \langle \boldsymbol{G}(\cdot;\hat{\theta}), \boldsymbol{M}^a(\hat{\theta}), \boldsymbol{M}^o(\hat{\theta})\rangle$ be the resulting market composition as given in (7)-(9). As $\hat{\theta}$ increases, the market distribution $\boldsymbol{G}(\cdot;\hat{\theta})$ decreases in the first-order stochastic dominance sense as shown in Figure 4, capturing the worsening adverse selection problem. Furthermore, as $\hat{\theta}$ increases, the fraction of agents leaving through a successful match, $1-\boldsymbol{G}(\hat{\theta};\hat{\theta})$, decreases. In other words, the pickier the agents are, the bigger the market size because it clears too slowly.

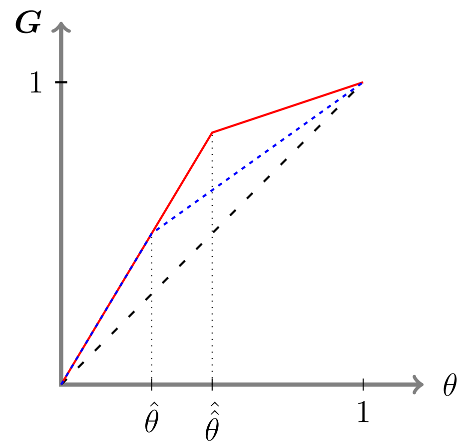

Figure 4: The loosely dashed black line is the distribution of new arrivals $F = Unif[0,1]$, the dotted blue line is $\boldsymbol{G}(\cdot;\hat{\theta})$, and the solid red line is $\boldsymbol{G}(\cdot;\hat{\hat{\theta}})$ for $\hat{\hat{\theta}} > \hat{\theta}$.

Let $\mathbb{U}(\hat{\theta})$ be an agent's payoff in this hypothetical market given by

$$\begin{aligned}\mathbb{U}(\hat{\theta}) &= \int_{\hat{\theta}}^{1} \theta d\boldsymbol{G}(\theta;\hat{\theta}) + (1-\gamma)\mathbb{U}(\hat{\theta})\boldsymbol{G}(\hat{\theta};\hat{\theta}) \\ &= \int_{\hat{\theta}}^{1} \theta dF(\theta),\end{aligned}$$

where the last equality follows from substituting the expression for $\boldsymbol{G}(\cdot;\hat{\theta})$ given by (7). As $\mathbb{U}(\hat{\theta})$ is a decreasing function of $\hat{\theta}$, a social planner who wants to maximize the agents' payoff would set $\hat{\theta} = 0$. However, when the agents can choose their pickiness, the equilibrium outcome is $\hat{\theta} = \theta^*_\gamma > 0$ and $\boldsymbol{U}^r_\gamma = \mathbb{U}(\theta^*_r(\gamma))$.

Let us now consider directed search markets. When search is directed, each agent chooses a sub-market to enter by trading-off the ex-ante probability of getting a match with the ex-post quality of her potential match. In particular, each agent is willing to forsake a guaranteed match to an object of quality $\theta \in [0, \theta^*_d(\gamma))$ for the chance to match to an object of quality $\theta \in (\theta^*_d(\gamma), 1]$

with probability less than one. Hence, directed search yields an inefficient outcome—some of the agents leave the market through death because they choose to enter a congested sub-market even though they could have left through a successful match by entering an uncongested sub-market.

The congestion externalities generated in directed search markets are seemingly different from the adverse selection externalities generated by random search. For instance, when search is directed, whether or not the market composition over-represents low quality objects does not matter because the agents can simply avoid matching with these low quality objects. Yet, we can think of directed search as random search except the likelihood that an agent is paired to an object of a specific quality is not uniformly random over Θ but determined by the queuing CDF Q_d^* and the rationing probability $\alpha(\theta; Q_d^*, \Psi_d^*) = \min\{\frac{g_d^*(\theta)}{q_d^*(\theta)}, 1\}$. Formally,

$$\begin{aligned}\boldsymbol{U}_\gamma^d &= \int_{\theta_d^*(\gamma)}^1 \theta\alpha(\theta; Q_d^*, \Psi_d^*)dQ_d^*(\theta) + (1-\gamma)\boldsymbol{U}_\gamma^d \int_{\theta_d^*(\gamma)}^1 (1-\alpha(\theta; Q_d^*, \Psi_d^*))dQ_d^*(\theta) \\ &= \int_{\theta_d^*(\gamma)}^1 \theta dG_d^*(\theta) + (1-\gamma)\boldsymbol{U}_\gamma^d G_d^*(\theta_d^*(\gamma)).\end{aligned}$$

Thus, the expected payoff in a directed search SME is equivalent to the expected payoff in the hypothetical search market with $\hat{\theta} = \theta_d^*(\gamma)$, i.e., $\boldsymbol{U}_\gamma^d = \mathbb{U}(\theta_d^*(\gamma))$.[9]

We can now compare directed search and random search markets by focusing on the hypothetical market. Since $\mathbb{U}(\hat{\theta})$ is a decreasing function, $\boldsymbol{U}_\gamma^r > \boldsymbol{U}_\gamma^d$ if, and only if, $\theta_d^*(\gamma) > \theta_r^*(\gamma)$. In other words, random search yields a higher equilibrium payoff than directed search because the agents are less picky when search is random. Consider the simplest case with $\gamma = 1$. When search is random, each agent is willing to accept any match because rejecting would yield a payoff of 0. Hence, when $\gamma = 1$, random search achieves the efficient outcome. In contrast, when search is directed, each agent is willing to forsake a guaranteed match with a low quality object for the chance to match with a high quality object with probability less than one. Hence, $0 = \theta_r^*(1) < \theta_d^*(1)$ and directed search yields a less efficient outcome than random search.

However, as γ decreases, $\theta_r^*(\gamma)$ increases (Corollary 1) while $\theta_d^*(\gamma)$ remains constant (Corollary 1). Yet, Proposition 3 implies that the agents are always less picky when search is random.

[9]This is even more evident given

$$\boldsymbol{U}_\gamma^d = \theta_d^*(\gamma) \underbrace{=}_{\text{by (11)}} \int_{\theta_d^*(\gamma)}^1 \theta dF(\theta) = \mathbb{U}(\theta_d^*(\gamma)).$$

Why?

When search is directed, an agent's pickiness is characterized by the fixed point of $(1-\gamma)\mathbb{U}$, the inter-temporal trade-off between her current (and therefore guaranteed) match and her value from searching for potentially better matches in the future. In contrast, when search is directed, an agent's pickiness is characterized by the fixed point of $\mathbb{U}$, the contemporaneous trade-off between a guaranteed match today and her value from searching for potentially better matches today. As an agent facing the former trade-off has to discount her value of search by $1-\gamma$, she is always less picky when search is random. Consequently, random search yields a higher equilibrium payoff than directed search. Additionally, when agents are less picky, more of them leave the market through a successful match. Hence, random search also yields higher welfare than directed search.

3 Transferable Utility

This section extends the search model with non-transferable utility in two dimensions: first, I now consider a model where both agents and objects are strategic. Second, I allow for transfers. However, the terminology of agents and objects is a bit cumbersome when both are strategic. Henceforth, I consider a labor market framework in which *firms* (she) play the role of agents, and *workers* (he) play the role of objects. The exposition stays as close as possible to the previous sections.

3.1 Setup

In each period, the firms on one side of the search market are matched to the workers on the other side. The workers differ in the level of output they produce. In particular, a worker of productivity type $\theta \in \Theta$ can produce θ units of output.[10] If a firm and a worker of type θ successful match at a wage of $w \in \mathbb{R}$, then the firm gets a payoff of $\theta - w$ while the worker gets w.

Once a firm and a worker match, the firm observes the worker's type θ. If both agree to accept the match, they determine wages through Nash bargaining and exit the search market. If at least one of them rejects the match, they both return to the search market. The inflow and outflow of firms and workers follows the same procedure as the respective inflow and outflow

[10] We can think of output as the lifetime revenue generated by a worker for a firm.

of agents and objects in the non-transferable utility model.[11]

3.2 Random Search with Transfers

In a random search market, a firm observes a worker's productivity only after being matched. Prior to a match, a firm only knows the prevailing market composition $\Psi \triangleq \langle G, M^a, M^o \rangle$ where G is the endogenously determined productivity distribution of the workers in the market, M^a is the total mass of firms in the market, and M^o is the total mass of workers in the market. In each period, firms and workers are randomly paired so that the maximal number of matches is achieved. As there are (weakly) more firms than workers, each worker gets matched to a single firm with probability 1 whereas each firm gets matched to a single worker with probability M^a/M^o.

Once a firm and a worker are matched, the firm observes the worker's productivity θ. If either the firm or the worker decline the match, both return to the market and search again next period with probability $1-\gamma$. Otherwise, the matched pair determine wages using a Nash bargaining solution concept. For a given market composition Ψ, let $\mathcal{U}^r(\Psi)$ be a firm's value from searching randomly. Similarly, let $\mathcal{V}^r(\theta;\Psi)$ be the value of random search for a worker with productivity $\theta \in \Theta$. Let $w^r(\theta;\Psi)$ be the Nash bargaining solution between a firm and a type θ worker, given by

$$w^r(\theta;\Psi) = (1-\rho)\Big(\theta - (1-\gamma)\mathcal{U}^r(\Psi)\Big) + \rho(1-\gamma)\mathcal{V}^r(\theta;\Psi),$$

where $\rho \in (0,1)$ is bargaining power of firms.[12]

Let $\Theta_A(\Psi)$, which I refer to as the acceptance set, be the set of worker types that form successful matches with firms. Since a worker of type $\theta \in \Theta_A(\Psi)$ exists the market only through

[11] A fraction γ of the firms and workers left in the search market die. Firms and workers that exit the market without a successful match get a payoff of 0. Finally, a mass k_a of new firms and a mass k_o of new workers flow into the market, with $k_a \geq k_o > 0$. The newly arriving workers have quality distributed according to an absolutely continuous distribution function F with a bounded and positive density function f.

[12] In particular,

$$w^r(\theta;\Psi) = \arg\max_{w\in\mathbb{R}} \Big(\theta - w - (1-\gamma)\mathcal{U}^r(\Psi)\Big)^{\rho} \Big(w - (1-\gamma)\mathcal{V}^r(\theta;\Psi)\Big)^{1-\rho},$$

a successful match, $\mathcal{V}^r(\theta;\Psi) = w^r(\theta;\Psi)$ which can be rewritten as

$$\mathcal{V}^r(\theta;\Psi) = \frac{(1-\rho)\Big(\theta - (1-\gamma)\mathcal{U}^r(\Psi)\Big)}{1-\rho(1-\gamma)}.$$

Furthermore, it must be sequentially rational for a worker of type $\theta \in \Theta_A(\Psi)$ to accept a match, i.e., $w^r(\theta;\Psi) \geq (1-\gamma)\mathcal{V}^r(\theta;\Psi)$. Similarly, it must be sequentially rational for a firm to accept a match with a worker of type $\theta \in \Theta_A(\Psi)$, i.e., $\theta - w^r(\theta;\Psi) \geq (1-\gamma)\mathcal{U}^r(\Psi)$. Using the expression for w^r and $\mathcal{V}^r$, worker and firm sequential rationality implies a *positive surplus* condition $\theta \geq (1-\gamma)\mathcal{U}^r(\Psi)$.

Let $\Theta_R(\Psi) = \Theta \backslash \Theta_A(\Psi)$, which I refer to as the rejection set, be the set of worker types that do not successfully match with a firm. Since, a worker of type $\theta \in \Theta_R(\Psi)$ exists the market only through death, $\mathcal{V}^r(\theta;\Psi) = 0$. Furthermore, it must be sequentially rational for a firm to reject a match with a worker of type $\theta \in \Theta_R(\Psi)$, i.e., $\theta - w^r(\theta;\Psi) < (1-\gamma)\mathcal{U}^r(\Psi)$. Using the expression for w^r and $\mathcal{V}^r$, firm sequential rationality implies a *negative surplus* condition $\theta < (1-\gamma)\mathcal{U}^r(\Psi)$. Therefore, we can express the firm's value from search as

$$\mathcal{U}^r(\Psi) = \frac{M^o}{M^a}\int_\Theta \max\left\{\theta - w^r(\theta;\Psi), (1-\gamma)\mathcal{U}^r(\Psi)\right\} dG(\theta) + \left(1 - \frac{M^o}{M^a}\right)(1-\gamma)\mathcal{U}^r(\Psi).$$

Furthermore, there exists a cutoff $\bar{\theta}(\Psi) \in \Theta$ such that $\theta \in \Theta_A(\Psi)$ if $\theta > \bar{\theta}(\Psi)$ and $\theta \in \Theta_R(\Psi)$ if $\theta < \bar{\theta}(\Psi)$. The cutoff is characterized by the indifference condition $\bar{\theta}(\Psi) = (1-\gamma)\mathcal{U}^r(\Psi)$.

The transferable utility framework now resembles random search without transfers except a firm's payoff is given by $\mathcal{U}^r$ instead of U^r. A stationary equilibrium for random search with transfers is therefore characterized by a pair (θ_r, Ψ_r) that satisfy Definition 1, i.e., (i) given the market composition Ψ_r, it is sequentially rational for each firm to accept a match with a worker of type $\theta > \theta_r$ at wage $w^r(\theta;\Psi_r) = \frac{(1-\rho)(\theta-\theta_r)}{1-\rho(1-\gamma)}$ and to decline a match with a worker of type $\theta < \theta_r$, and (ii) the market composition Ψ_r is consistent with a steady state outcome as given by (7)-(9) when all firms use the cutoff θ_r.

Proposition 4
There exists a unique random search SME $(\theta_r^{**}, \Psi_r^{**})$ *such that*

$$\theta_r^{**} = \frac{k_o\rho\gamma(1-\gamma)}{k_a(1-\rho(1-\gamma)) - k_o(1-\gamma)(1-\rho)(1-F(\theta_r^{**}))}\int_{\theta_r^{**}}^1 \theta dF(\theta). \tag{12}$$

Proof. Fix some market composition $\Psi \triangleq \langle G, M^a, M^o \rangle$. Given the firm's sequential rational cutoff is $\bar{\theta}(\Psi)$, her value from random search is given by

$$\begin{aligned}\mathcal{U}^r(\Psi) &= \frac{M^o}{M^a}\int_{\bar{\theta}(\Psi)}^1 \theta - w^r(\theta;\Psi)dG(\theta) + \left(1 - \frac{M^o}{M^a} + \frac{M^o}{M^a}G(\bar{\theta}(\Psi))\right)(1-\gamma)\mathcal{U}^r(\Psi) \\ &= \frac{\xi\int_{\bar{\theta}(\Psi)}^1 \theta dG(\theta)}{\gamma + (1-\gamma)\xi(1-G(\bar{\theta}(\Psi)))},\end{aligned}$$

where $\xi = \frac{M^o}{M^a}\left(\frac{\rho\gamma}{1-\rho(1-\gamma)}\right)$.

The pair (θ_r, Ψ_r) constitutes an SME if $\theta_r = \bar{\theta}(\Psi_r)$ and $\Psi_r = \langle \boldsymbol{G}(\cdot;\theta_r), \boldsymbol{M}^a(\theta_r), \boldsymbol{M}^o(\theta_r)\rangle$. Substituting in the market composition expressions in (7)-(9) simplifies the value function to

$$\mathcal{U}^r(\Psi_r) = \frac{k_o\rho\gamma}{k_a(1-\rho(1-\gamma)) - k_o(1-\gamma)(1-\rho)(1-F(\theta_r))}\int_{\theta_r}^1 \theta dF(\theta).$$

Since the firm's cutoff strategy for a given market composition Ψ is characterized by the indifference condition $\bar{\theta}(\Psi) = (1-\gamma)\mathcal{U}^r(\Psi)$,

$$\theta_r = \frac{k_o\rho\gamma(1-\gamma)}{k_a(1-\rho(1-\gamma)) - k_o(1-\gamma)(1-\rho)(1-F(\theta_r))}\int_{\theta_r}^1 \theta dF(\theta).$$

Recall the function $\mathbb{U}(\hat{\theta}) = \int_{\hat{\theta}}^1 \theta dF(\theta)$ which (i) is strictly decreasing in $\hat{\theta}$, (ii) $\mathbb{U}(0) = \mathbb{E}_F[\theta]$, and (iii) $\mathbb{U}(1) = 0$. Define the function

$$\mathbb{H}(\theta) = \frac{k_o\gamma}{k_a(1-\rho(1-\gamma)) - k_o(1-\gamma)(1-\rho)(1-F(\theta))}$$

which (i) is also strictly decreasing in θ, and (ii) $\mathbb{H}(\theta) \in (0,1]$ for all $\theta \in \Theta$. Hence, there is a unique fixed point to the function $(1-\gamma)\rho\mathbb{H}\cdot\mathbb{U}$ given by θ_r^{**}. The market composition Ψ_r^{**} is then derived from the cutoff θ_r^{**} using (7)-(9) ■

Similar to the case of random search with non-transferable utility, a firm who searches randomly faces an inter-temporal trade off; she can either settle for a worker she currently has matched to or she can continue randomly searching for a worker with a higher productivity. However, a firm who foregoes her match today can continue to search only if she survives into the next period. Thus, her continuation value of search must be discounted by $1-\gamma$. The indifference condition (12) captures the resolution of this inter-temporal trade off. The cutoff θ_r^{**} is the lowest productivity worker that firms are willing to hire.

For a given $(\rho,\gamma) \in (0,1)^2$, let $\theta_r^{**}(\rho,\gamma)$ be the unique equilibrium cutoff satisfying the indifference condition (12). Let $\boldsymbol{\mathcal{U}}_{\rho,\gamma}^r = \frac{\theta_r^{**}(\rho,\gamma)}{1-\gamma}$ be the firm's unique equilibrium value of random search.

As a firm's bargaining power increases, she extracts more of the surplus generated by a successful match. In particular, her ex-post payoff becomes more and more sensitive to the worker's productivity. Thus, as ρ increases, the firms become more picky. In the limit when the firms have all the bargaining power, the workers are effectively reduced into the non-strategic objects from the previous section. The firms extract all the surplus and the equilibrium outcomes under transferable and non-transferable utility coincide.

Corollary 3 *For any $\gamma > 0$, $\theta_r^{**}(\rho,\gamma)$ and $\boldsymbol{\mathcal{U}}_{\rho,\gamma}^r$ are increasing in ρ. Furthermore, $\lim_{\rho\to 1}\theta_r^{**}(\rho,\gamma) = \theta_r^*(\gamma)$ and $\lim_{\rho\to 1}\boldsymbol{\mathcal{U}}_{\rho,\gamma}^r = \boldsymbol{U}_\gamma^r$.*

Unlike the case of non-transferable utility, the comparative statics with respect to γ is ambiguous and non-monotone. In a Nash bargaining protocol, the discounted continuation values of the players are the disagreement points. As γ decreases, the firms disagreement point improves but the same is true for the workers. Which party comes out on top depends on the relative bargaining power of the players.

Nonetheless, the right-hand side of (12) converges to 0 as γ goes to 1 or as γ goes to 0, which implies the firms become less picky either when searching becomes costless or too costly. In particular, if the firms don't have full bargaining power, the random search equilibrium outcome converges to a competitive market outcome in which firms hire all workers at wages $w^r(\theta) = \theta$ when the cost of searching vanishes. This observation is consistent with Lauermann (2013) who shows more generally that outcomes in search and bargaining games converge to competitive market outcomes if, and only if, there is *competitive pressure* in the search market. One such competitive pressure is that both sides of the market have bargaining power.[13] In contrast, if the firms have full bargaining power, we have the non-transferable utility case: the random search equilibrium outcome converges to the inefficient directed search outcome (Proposition 3).

Corollary 4 *As $\gamma \to 0$, the random search SME converges to a competitive market outcome if, and only if, $\rho < 1$.*

[13] Other forms of competitive pressure are asymmetric information with noisy signals, and one firm bargaining with multiple workers simultaneously.

Similar to random search market with non-transferable utility, the unique equilibrium outcome in a random search market with transferable utility is inefficient. The efficient outcome would be for all firms to accept a match with any type of worker. However, accepting all matches is not sequentially rational for any single firm. In equilibrium, a firm instead chooses to accept all types above $\theta_r^{**}(\rho,\gamma)$ after taking into account the negative externalities that other firms impose on the market composition. However, firms do not fully internalize the negative externalities of their own strategic choices. Furthermore, since bargaining is over the ex-post surplus, wages also do not fully internalize the negative externalities. Hence, the random search SME with transferable utility is also inefficient.

3.3 Directed Search with Transfers

Once again, the search market is further subdivided into a continuum of sub-markets. A sub-market $S(\theta)$ exclusively contains all workers in the market with productivity θ. Firms can distinguish between all the sub-markets and can choose which sub-market to search in for a given period. For instance, we can think of $S(\theta)$ as a university that has a reputation of producing workers of type θ. Firms can then choose in which universities they recruit for employees. Let Q be the absolutely continuous queuing CDF with density function q. Given a market composition $\Psi = \langle G, M^a, M^o \rangle$, sub-market $S(\theta)$ has a "mass" of $M^o g(\theta)$ workers and a queue length of $M^a q(\theta)$.

In each period, firms and workers within a sub-market are paired randomly so that the maximal number of matches is achieved. The long-side of the sub-market is rationed while the short-side is guaranteed a match. Let $\alpha(\theta; Q, \Psi) = \min\left\{\frac{M^o g(\theta)}{M^a q(\theta)}, 1\right\}$ be the *firm match probability*, the probability that a firm in sub-market $S(\theta)$ gets a match in a given period. Similarly, let $\beta(\theta; Q, \Psi) = \min\left\{\frac{M^a q(\theta)}{M^o g(\theta)}, 1\right\}$ be the *worker match probability*, the probability that a worker in sub-market $S(\theta)$ gets matched in a given period.

For a given market composition Ψ and queuing CDF Q, let $\mathcal{U}^d(\theta; Q, \Psi)$ be a firm's value from directing her search to sub-market $S(\theta)$. Similarly, let $\mathcal{V}^d(\theta; Q, \Psi)$ be the value function for a worker in sub-market $S(\theta)$. On the equilibrium path (the sub-markets with positive queue length), a matched firm and a worker determine wages using a Nash bargaining solution concept. Let $w^d(\theta; Q, \Psi)$ be the Nash bargaining solution between a firm and a type θ worker, given by

$$w^d(\theta; Q, \Psi) = (1-\rho)\Big(\theta - (1-\gamma)\mathcal{U}^d(\theta; Q, \Psi)\Big) + \rho(1-\gamma)\mathcal{V}^d(\theta; Q, \Psi).$$

The value of search for a worker in an on-path sub-market $S(\theta)$ can then be written as

$$\begin{aligned}\mathcal{V}^d(\theta;Q,\Psi) =&\beta(\theta;Q,\Psi)w^d(\theta;Q,\Psi)+(1-\beta(\theta;Q,\Psi))(1-\gamma)\mathcal{V}^d(\theta;Q,\Psi)\\ =&\frac{\beta(\theta;Q,\Psi)(1-\rho)\Big(\theta-(1-\gamma)\mathcal{U}^d(\theta;Q,\Psi)\Big)}{\gamma+\beta(\theta;Q,\Psi)(1-\gamma)(1-\rho)}.\end{aligned}$$

Similarly, the value of search for a firm in an on-path sub-market $S(\theta)$ can be written as

$$\begin{aligned}\mathcal{U}^d(\theta;Q,\Psi) =&\alpha(\theta;Q,\Psi)(\theta-w^d(\theta;Q,\Psi))+(1-\alpha(\theta;Q,\Psi))(1-\gamma)\mathcal{U}^d(\theta;Q,\Psi)\\ =&\frac{\rho\alpha(\theta;Q,\Psi)\theta}{\gamma+(1-\gamma)\Big(\rho\alpha(\theta;Q,\Psi)+(1-\rho)\beta(\theta;Q,\Psi)\Big)}.\end{aligned}$$

I will focus on equilibrium where wages are continuous in θ. However, this could mean that off the equilibrium path (the sub-markets in which firms do not enter), wages are no longer determined by Nash bargaining. Formally, I assume that off-path wages are limit points of on-path wages. Furthermore, I assume that off-path wages do not increase too fast.[14]

Assumption 1 *Wages $w^d(\theta;Q,\Psi)$ are continuous in θ and determined by Nash bargaining if $\theta \in Supp(q)$. Furthermore, for all $\theta',\theta'' \notin Supp(q)$, $|w^d(\theta';Q,\Psi)-w^d(\theta'';Q,\Psi)|<|\theta'-\theta''|$.*[15]

The transferable utility framework now resembles directed search without transfers except a firm's payoff is given by $\mathcal{U}^d$ instead of U^d. A stationary equilibrium for directed search with transfers is therefore characterized by a pair (Q_d,Ψ_d) that satisfy Definition 2, i.e., given queuing CDF Q_d and market composition Ψ_d (which determine firm and worker match probabilities), (i) firms enter only the sub-markets that maximize their profits in a manner that is consistent

[14] In the online appendix **?**, I relax this assumption and derive the equilibrium outcome when all wages (both on and off-path) are determined by Nash bargaining. However, the latter approach makes the problem less tractable. Notice that wages depend on the worker match probability $\beta(\cdot;Q,\Psi)$. Suppose firms only enter sub-markets above a cutoff θ' in equilibrium. For a small $\epsilon>0$, the sub-market $S(\theta'-\epsilon)$ is off-path and thus $\beta(\theta'-\epsilon;Q,\Psi)=0$. If $\beta(\theta'+\epsilon;Q,\Psi)$ discretely jumps up, then so do the Nash bargaining wages, i.e., $w^d(\theta'-\epsilon;Q,\Psi)\ll w^d(\theta'+\epsilon;Q,\Psi)$. Thus, by entering sub-market $S(\theta'+\epsilon)$ instead of $S(\theta'-\epsilon)$ firms get a marginal gain in productivity but a discrete loss in wages, making the off-path sub-market $S(\theta'-\epsilon)$ attractive. To prevent such downward deviations, the worker match probability cannot jump at the cutoff. Hence, there is some interval (θ',θ'') over which firms are rationed and workers are on the long side. This however complicates the market composition which can no longer be solved in closed form without additional assumptions on F. Nonetheless, I show that an equilibrium exists and derive a general characterization for it. Furthermore, I show that if firm bargaining power is high enough, or if certain sufficient conditions on F are satisfied, the firms are less picky and enjoy a higher equilibrium payoff when search is random.

[15] $Supp(q)$ is the support of the density q, i.e., $\{\theta : q(\theta)>0\}$. It captures the sub-markets in which firms enter.

with Q_d, and (ii) the market composition Ψ_d is consistent with a steady state outcome given by (1)-(3).

Proposition 5

Under Assumption 1, there exists a direct search SME (Q_d^{**}, Ψ_d^{**}) *and a cutoff* $\theta_d^{**} \in \Theta$ *such that*

(i) $\Psi_d^{**} = \langle \boldsymbol{G}\big(\cdot; \theta_d^{**}\big), \boldsymbol{M}^a\big(\theta_d^{**}\big), \boldsymbol{M}^o\big(\theta_d^{**}\big)\rangle$,

(ii) $$Q_d^{**}(\theta) = \begin{cases} 0 & \textit{if} \quad \theta \le \theta_d^{**} \\ \int_{\theta_d^{**}}^{\theta} \frac{k_o\gamma(\omega - \rho(1-\gamma)\theta_d^{**})}{\left(k_a - k_o(1-\gamma)(1-F(\theta_d^{**}))\right)\theta_d^{**}(1-\rho(1-\gamma))} dF(\omega) & \textit{for} \quad \theta \ge \theta_d^{**} \end{cases},$$

(iii) $\mathcal{U}^d(\theta; Q_d^{**}, \Psi_d^{**}) = \rho\theta_d^{**}, \forall \theta \ge \theta_d^{**}$

The cutoff $\theta_d^{**} \in \Theta$ *is characterized by*

$$\theta_d^{**} = \frac{k_o\gamma}{k_a(1-\rho(1-\gamma)) - k_o(1-\gamma)(1-\rho)(1-F(\theta_d^{**}))} \int_{\theta_d^{**}}^{1} \theta dF(\theta). \tag{13}$$

Proof. If the pair (Q_d, Ψ_d) constitute a directed search SME, then by Lemma 2, there exists a cutoff $\theta_d \in \Theta$ such that firms only enter sub-markets $S(\theta)$ with $\theta > \theta_d$. I will look for equilibrium where $\beta(\theta; Q_d, \Psi_d) = 1$ for all $\theta > \theta_d$. Hence, in an on-path sub-market $S(\theta)$, the firm's payoff is given by

$$\mathcal{U}^d(\theta; Q, \Psi) = \frac{\rho\alpha(\theta; Q, \Psi)\theta}{\gamma + (1-\gamma)\Big(\rho\alpha(\theta; Q, \Psi) + 1 - \rho\Big)} \le \rho\theta,$$

and, the wage is given by

$$w^d(\theta; Q, \Psi) = \frac{\theta(1-\rho)}{\gamma + (1-\gamma)(\rho\alpha(\theta; Q_d, \Psi_d) + 1 - \rho)}.$$

Off-path, a firm's payoff by entering a sub-market $S(\theta)$ with $\theta < \theta_d$ is given by $\mathcal{U}^d(\theta; Q_d, \Psi_d) = \theta - w^d(\theta; Q_d, \Psi_d)$ which is increasing in θ given Assumption 1. Furthermore, by the continuity of wages, $\lim_{\theta \uparrow \theta_d} \mathcal{U}^d(\theta; Q_d, \Psi_d) = \theta_d - \lim_{\theta \downarrow \theta_d} w^d(\theta; Q_d, \Psi_d)$.

If $\lim_{\theta' \downarrow \theta_d} \alpha(\theta'; Q_d, \Psi_d) < 1$, then for a small $\epsilon > 0$, the firms would strictly prefer to deviate to an off-path sub-market $S(\theta)$ with $\theta \in (\theta_d - \epsilon, \theta_d)$. Hence, $\lim_{\theta' \downarrow \theta_d} \alpha(\theta'; Q_d, \Psi_d) = 1$, which implies that

$$\lim_{\theta \uparrow \theta_d} \mathcal{U}^d(\theta; Q_d, \Psi_d) = \theta_d - \lim_{\theta \downarrow \theta_d} w^d(\theta; Q_d, \Psi_d) = \rho\theta_d = \lim_{\theta \downarrow \theta_d} \mathcal{U}^d(\theta; Q_d, \Psi_d).$$

As the firms must be indifferent among all the sub-markets they enter, $\mathcal{U}^d(\theta''; Q_d, \Psi_d) = \mathcal{U}^d(\theta'; Q_d, \Psi_d)$ for all $\theta'' > \theta' > \theta_d$. Thus, $\mathcal{U}^d(\theta; Q_d, \Psi_d) = \rho\theta_d$ for all $\theta \geq \theta_d$.

The indifference condition $\mathcal{U}^d(\theta; Q_d, \Psi_d) = \rho\theta_d$ for all $\theta > \theta_d$ necessitates that

$$\alpha(\theta; Q_d, \Psi_d) = \begin{cases} 1 & \text{for} \quad \theta \leq \theta_d \\ \frac{\theta_d(1-\rho(1-\gamma))}{\theta - \rho(1-\gamma)\theta_d} & \text{for} \quad \theta \geq \theta_d \end{cases}.$$

Because $\alpha(\theta; Q_d, \Psi_d) < 1$ for all $\theta > \theta_d$, firms must be on the long-side of each sub-market above the cutoff. By definition, we also have that $\alpha(\theta; Q_d, \Psi_d) = \frac{g_d(\theta)}{q_d(\theta)}\zeta_d$ for all $\theta > \theta_d$, where $\zeta_d = \frac{M_d^o}{M_d^a}$. Equating these two expressions for the firm match probability and integrating over the interval $[\theta_d, 1]$ gives

$$\begin{aligned} \theta_d(1 - \rho(1-\gamma)) \underbrace{\int_{\theta_d}^1 q_d(\theta) d\theta}_{=1} =& \zeta_d \int_{\theta_d}^1 \theta dG_d(\theta) - \zeta_d \rho(1-\gamma)\theta_d(1 - G_d(\theta_d)) \\ \Leftrightarrow \theta_d =& \frac{\zeta_d \int_{\theta_d}^1 \theta dG_d(\theta)}{1 - \rho(1-\gamma) + \zeta_d\rho(1-\gamma)\big(1 - G_d(\theta_d)\big)}. \end{aligned}$$

The market composition Ψ_d must be consistent with $\langle \boldsymbol{G}\big(\cdot; \theta_d\big), \boldsymbol{M}^a\big(\theta_d\big), \boldsymbol{M}^o\big(\theta_d\big)\rangle$ as given by (7)-(9). Substituting in these expressions for G_d, M_d^a, and M_d^o gives

$$\theta_d = \frac{k_o\gamma}{k_a(1-\rho(1-\gamma)) - k_o(1-\gamma)(1-\rho)(1-F(\theta_d))} \int_{\theta_d}^1 \theta dF(\theta).$$

Recall the function $\mathbb{U}(\hat{\theta}) = \int_{\hat{\theta}}^{1} \theta dF(\theta)$ which (i) is strictly decreasing in $\hat{\theta}$, (ii) $\mathbb{U}(0) = \mathbb{E}_F[\theta]$, and (iii) $\mathbb{U}(1) = 0$. Also, recall the function

$$\mathbb{H}(\theta) = \frac{k_o \gamma}{k_a(1-\rho(1-\gamma)) - k_o(1-\gamma)(1-\rho)(1-F(\theta))}$$

which (i) is also strictly decreasing in θ, and (ii) $\mathbb{H}(\theta) \in (0,1]$ for all $\theta \in \Theta$. Hence, there is a unique fixed point to the function $\mathbb{H} \cdot \mathbb{U}$ given by θ_d^{**}. The market composition Ψ_d^{**} is then derived from the cutoff θ_d^{**} using (7)-(9). Similarly, the queuing CDF is pinned down from the agent match probability with

$$\alpha(\theta; Q_d^{**}, \Psi_d^{**}) = \frac{\boldsymbol{M}^o(\theta_d^{**})\boldsymbol{g}(\theta; \theta_d^{**})}{\boldsymbol{M}^a(\theta_d^{**}) q_d^{**}(\theta)} = \frac{\theta_d^{**}(1-\rho(1-\gamma))}{\theta - \rho(1-\gamma)\theta_d^{**}}$$

for all $\theta > \theta_d^{**}$. ■

Remark 1 *Proposition 5 characterizes an equilibrium outcome when all on-path sub-markets are congested (more firms than workers). It may be possible that there is another equilibrium in which some of the intermediate productivity sub-markets are in fact thin (more workers than firms). I explore such equilibrium outcomes when Assumption 1 is relaxed (in the online appendix Mekonnen (2018)).*

A firm faces a contemporaneous trade off; she can either settle for a worker she can match to without waiting or she can queue in a congested sub-market for the possibility of matching with a more productive worker. The indifference condition (13) captures the resolution of this contemporaneous trade off. The cutoff θ_d^{**} is the highest productivity worker firms can get without waiting in a congested sub-market.

For a given $(\rho, \gamma) \in (0,1)^2$, let $\theta_d^{**}(\rho, \gamma)$ be the equilibrium cutoff satisfying the indifference condition (13). Let $\boldsymbol{\mathcal{U}}_{\rho,\gamma}^d = \rho\theta_d^{**}(\rho, \gamma)$ be the firm's equilibrium payoff of directed search.

As a firm's bargaining power increases, she extracts more of the surplus generated by a successful match. Thus, as ρ increases, the firms become more picky. In the limit when the firms have all the bargaining power, the equilibrium outcomes under transferable and non-transferable utility coincide.

Corollary 5 *For any $\gamma > 0$, $\theta_d^{**}(\rho, \gamma)$ and $\boldsymbol{\mathcal{U}}_{\rho,\gamma}^d$ are increasing in ρ. Furthermore, $\lim_{\rho \to 1} \theta_d^{**}(\rho, \gamma) = \theta_d^*(\gamma)$ and $\lim_{\rho \to 1} \boldsymbol{\mathcal{U}}_{\rho,\gamma}^d = \boldsymbol{U}_\gamma^d$.*

Unlike the case of non-transferable utility, the comparative statics with respect to γ is no longer constant when $\rho < 1$. As in the case with non-transferable utility, the mass of productive workers with $\theta > \theta_d^{**}$ is unaffected by changes to γ as all these workers exit the market only through successful matches. However, more of the unmatched firms survive into the subsequent periods, making the congestion worse. Furthermore, a decrease in γ improves the discounted continuation value of the workers. Thus, when the productive workers have some bargaining power, they demand higher wages not only because they are relatively rarer but also because they have a better disagreement point. As a result, the firms become more willing to settle for low productivity workers who accept lower wages instead of queue in an increasingly congested market where the workers demand high wages. Additionally, if the firms don't have full bargaining power, the directed search equilibrium outcome converges to a competitive market outcome in which firms hire all workers at wages $w^d(\theta) = \theta$ when the cost of searching vanishes. In contrast, if the firms have full bargaining power, we have the non-transferable utility case: the directed search equilibrium outcome is insensitive to changes in γ (Corollary 2).

Corollary 6 *For any $\rho < 1$, $\theta_d^{**}(\rho, \gamma)$ and $\boldsymbol{\mathcal{U}}^d_{\rho,\gamma}$ are increasing in γ. Furthermore, as $\gamma \to 0$, the directed search SME converges to a competitive market outcome if, and only if, $\rho < 1$.*

Unsurprisingly, the equilibrium outcome in a Proposition 5 is inefficient. The efficient outcome would be for all firms to queue in all sub-markets which is not sequentially rational for any single firm. In equilibrium, a firm instead chooses to queue at any sub-market above $\theta_d^{**}(\rho, \gamma)$ after taking into account the congestion externalities that other firms impose on the market. However, firms do not fully internalize the negative externalities of their own strategic choices. Furthermore, the Nash bargaining protocol only splits the ex-post surplus once a match forms; it does not take into account the search externalities that arise prior to the formation of a match. Hence, the directed search SME with transferable utility and ex-post Nash bargaining is inefficient. This stands in stark contrast to the vast literature on directed search with posted wages. Posted wages provide an ex-ante mechanism for one side of the market to internalize the search frictions. However, the ability to post wages must be complemented with commitment power. If the posted wages are non-binding, then one side of the market may find it profitable to renege and propose new wages, undermining the efficiency properties of directed search markets with posted prices.

Similar to the case of non-transferable utility, random and directed search markets with transferable utility and Nash bargaining lead to inefficient equilibrium outcomes. I conclude this section by showing that the inefficiencies created by directed search are worse than the inefficiencies arising from random search. The intuition is precisely the same as the one for the case with non-transferable utility.

Let $i = r, d$ be an index for random and directed respectively. To simplify, I assume that there are an equal mass of firms and workers. Specifically, $k_a = k_o$. For given parameters $(\rho, \gamma) \in (0,1)^2$, let

$$\begin{aligned} \boldsymbol{\mathcal{W}}^i_{\rho,\gamma} =& \frac{k_o}{M^o_i(\rho,\gamma)} \left(\boldsymbol{\mathcal{U}}^i_{\rho,\gamma} + \int_{\theta^{**}_i(\rho,\gamma)}^1 \underbrace{\frac{(1-\rho)(\theta - (1-\gamma)\boldsymbol{\mathcal{U}}^i_{\rho,\gamma}}{1-\rho(1-\gamma)}}_{= \text{ worker's payoff } \mathcal{V}^i(\theta)} dF(\theta) \right) \\ =& \frac{\gamma}{\gamma + (1-\gamma)F(\theta^{**}_i(\rho,\gamma))} \int_{\theta^{**}_i(\rho,\gamma)}^1 \theta dF(\theta) \end{aligned}$$

be welfare when search is $i = r, d$, the expected payoff delivered to the fraction of the population that is new.[16]

Proposition 6

For any $(\rho, \gamma) \in (0,1)$*,*

(i) $\boldsymbol{\mathcal{U}}^r_{\rho,\gamma} > \boldsymbol{\mathcal{U}}^d_{\rho,\gamma}$*, and*

(ii) $\boldsymbol{\mathcal{W}}^r_{\rho,\gamma} > \boldsymbol{\mathcal{W}}^d_{\rho,\gamma}$*.*

Furthermore, $\lim_{\gamma \to 0} \boldsymbol{\mathcal{U}}^r_{\rho,\gamma} - \boldsymbol{\mathcal{U}}^d_{\rho,\gamma} = 0$.

Proof. The proof is similar to that of Proposition 3. ■

[16]For a fixed $(\rho, \gamma) \in (0,1)^2$, $M^o_i(\rho,\gamma)$ is the endogenous worker population (and firm population because $k_o = k_a$) when search is $i = r, d$.

4 Conclusion

This paper provides a tractable and parsimonious search model to study how changes in the search protocol or the timing of information affect equilibrium outcomes and welfare. The overall theme of the paper is that when agents search for scarce resources of heterogeneous quality, giving agents more flexibility in how they search or more ex-ante information could be detrimental. Specifically, I show that when there are more agents than resources, directed search creates worse inefficiencies and leads to lower equilibrium payoffs than random search.

While the main result of the paper is framed in a search model with non-transferable utility, I also show that the same conclusions extend into a search model when utility is transferable through Nash bargaining. In the online appendix (Mekonnen, 2018), I consider several extensions and show that the main insights in this paper are unaffected if matching is frictional (only a fraction of all possible matches arrive), or if entry is endogenous (only the objects that are of desirable quality enter the search market).

Of course, there are other search procedures than just random and directed search. An interesting avenue for future work would be to adopt a market design approach to characterize the welfare-maximizing search procedure. This would require a more general treatment, including non-stationary search procedures such that the procedure implemented in each period depends on the procedures and the market compositions of the past.

Appendix

Proof of Lemma 1:

Proof. The existence and uniqueness arguments are straightforward and skipped. I refer any interested reader to Nöldeke and Tröger (2009).

From (2), for any $\theta' < \theta''$,

$$\begin{aligned}
k_o\big(F(\theta'') - F(\theta')\big) =& \boldsymbol{M}^o(\beta) \int_{\theta'}^{\theta''} \Big((1-\beta(\theta))\gamma + \beta(\theta)\Big) d\boldsymbol{G}(\theta;\beta) \\
\geq& \gamma \boldsymbol{M}^o(\beta)\Big(\boldsymbol{G}(\theta'';\beta) - \boldsymbol{G}(\theta';\beta)\Big) \\
\geq& \gamma k_o \Big(\boldsymbol{G}(\theta'';\beta) - \boldsymbol{G}(\theta';\beta)\Big),
\end{aligned}$$

where the last inequality follows because, from (1), $\boldsymbol{M}^o(\beta) \geq k_o$. Thus, $\boldsymbol{G}(\theta''; \beta) - \boldsymbol{G}(\theta'; \beta) \leq \gamma^{-1}\big(F(\theta'') - F(\theta')\big)$. As F is absolutely continuous and $\gamma > 0$, so is $\boldsymbol{G}$, and has a positive and bounded density function.

Using (3), we can express $\boldsymbol{M}^a(\beta) - \boldsymbol{M}^o(\beta)$ as

$$
\begin{aligned}
&\frac{1}{\gamma}\left[k_a - (1-\gamma)\boldsymbol{M}^o(\beta)\int_\Theta \beta(\theta) d\boldsymbol{G}(\theta;\beta) - \boldsymbol{M}^o\gamma\right] \\
=&\frac{1}{\gamma}\left[k_a - \boldsymbol{M}^o(\beta)\int_\Theta \Big(\beta(\theta) + (1-\beta(\theta))\gamma\Big) d\boldsymbol{G}(\theta;\beta)\right] \\
=&\frac{1}{\gamma}[k_a - k_o],
\end{aligned}
$$

where the last equality follows from (1). Hence, $\boldsymbol{M}^a(\beta) - \boldsymbol{M}^o(\beta) \geq (=) 0$ if, and only if, $k_a \geq (=) k_o$. ■

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