Inequality, Portfolio Choice and the Business Cycle

Heejeong Kim*
Concordia University

Abstract

I study the effect of heterogeneity, in both the level and composition of wealth, in a dynamic stochastic overlapping-generations economy where households face uninsurable unemployment, earnings, and liquidity risk. Households pay transactions costs when they adjust savings held in high expected return assets, which make such savings illiquid. They also hold liquid, lower return assets. I show that household-level disparities in liquidity are important for understanding differences in their behavior, as well as aggregate changes in consumption and investment, over the Great Recession. When I allow for a rise in both unemployment and disaster risk, reducing households' expected income and the expected return on their illiquid savings, aggregate consumption and investment fall to levels seen in the recession.

The response of aggregate consumption is sensitive to the behavior of wealth poor households with a high marginal utility of consumption. Facing a large possible fall in earnings, they build precautionary savings in liquid assets. However, in a typical incomplete markets model with a single asset, all households would respond to the fall in the expected return on savings following a rise in disaster risk. The resulting substitution effect would offset much of the negative wealth effect on aggregate consumption. In contrast, when much of wealth is illiquid, many households do not respond to a fall in its expected return, substantially dampening the substitution effect. Moreover, wealthier households, more likely to adjust their portfolios, increase their shares of liquid assets. This results in a large fall in aggregate investment.

Keywords: Business cycle, disaster risk, portfolio choice, wealth inequality

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* Department of Economics, Concordia University, CA. E-mail: heejeong.kim.macro@gmail.com. I thank Aubhik Khan and Jonathan Heathcote for their invaluable comments. I also thank Mariacristina De Nardi , Gianluca Violante, Fabrizio Perri, Andrew Glover, Pok-sang Lam, and Jesus Fernandez-Villaverde. I thank session participants at the 2016 European Winter Meeting of the Econometric Society, 2017 North American Summer Meeting, 2017 Midwest Macro Meeting and workshop participants at Ohio State University for comments and suggestions that have improved this paper.

1 Introduction

Models of the business cycle have typically assumed a single asset held by households. This includes recent heterogeneous agent models studying the effects of wealth inequality on the propagation of large recessions (Krueger et al., 2016). However, the data shows large differences in the composition of household wealth. Households not only make consumption-savings decisions but also choose how to allocate savings across assets of varying liquidity and returns. I quantify the importance of this channel for understanding business cycles and differences in their impact on rich and poor households. While these are natural questions to explore, the difficulty of solving a heterogeneous household model with multiple assets in dynamic stochastic general equilibrium has meant that there has been little work done in developing an answer.

Households' portfolios of assets vary systematically with their age and wealth.[1] In a large recession, reductions in earnings and increases in unemployment risk may lead to a reduction in the share of illiquid but productive assets. When accompanied by a rise in precautionary savings, this may help explain large declines in aggregate investment and consumption as well as a slow recovery. An evaluation of the quantitative importance of this channel is the goal of this paper. In particular, I examine how optimal portfolio choices, across liquid and illiquid assets that vary in their rates of return, affect consumption and savings of different households. Importantly, this heterogeneity in the composition of wealth, previously disregarded in business cycle models with uninsurable earnings risk, has a quantitatively important role in reproducing the large declines in aggregate quantities seen in the Great Recession.

While heterogeneity in the composition of wealth has recently received attention in quantitative macroeconomic models, it has either been studied in endowment economies (Glover et al., 2016) or in partial equilibrium (Kaplan and Violante, 2014), making it difficult to quantitatively evaluate the importance of this channel for the business cycle. Moreover, in the absence of intra-generational inequality, it is hard to study the effects of changes in household portfolios on the cross-sectional distributions of net worth, income

[1] See Glover et al., 2016 and Khan and Kim, 2015.

and consumption (Glover et al., 2016). Households of different ages hold different shares of illiquid wealth, which implies dissimilar impacts of asset price changes on their income and consumption. These differences are equally evident when examining households of different wealth. Thus, a realistic assessment of the effect of portfolio choice requires intra-generational differences that have been omitted in general equilibrium studies. To my knowledge, I am the first to explore a quantitative DSGE OLG framework where households, who face earnings, unemployment and liquidity risk, choose both their consumption and savings in low-yield liquid and high-yield illiquid assets. Liquidity risk arises through transaction costs of actively adjusting illiquid assets. It reinforces the response of households to a rise in earnings and unemployment risk.

Another contribution of this paper is to introduce disaster risk into a heterogeneous household economy and explore the role of a rise in the risk of economic disaster in the presence of multiple assets of varying liquidity. The rise in disaster risk lowers households' expected income and wealth. Studying the micro-data using a simple permanent income model, De Nardi et al. (2011) argue that negative wealth effects, which might be driven by the fall in the expected asset prices and the expected income, are crucial determinants for understanding the fall in aggregate consumption during the Great Recession. In my model, a rise in disaster risk, alongside a fall in total factor productivity, generates a strong response in precautionary savings by all but the wealthiest households. This leads to a shift into liquid assets. Important in this mechanism, which matches well household level data over the Great Recession, is the assumption that economic disasters, when they happen, involve a relatively large fall in TFP which reduces households' expected life-time earnings and the expected return to illiquid assets. This is in contrast to the existing implementation of disaster risk taken by Gourio (2012), who studies its asset pricing implications in a representative agent model. In the current work, there is a large recession when, following a fall in TFP, a rise in disaster risk makes households increase precautionary savings and reduce illiquid, productive assets.

Calibrating the model economy using household-level data, the distribution of transactions costs that gives rise to illiquidity in capital investment is quantitatively disciplined by reproducing the share of illiquid assets to net worth across households of different levels of wealth and age. In deciding their savings in liquid and illiquid assets, which vary in their usefulness for consumption smoothing, households face borrowing limits that are a common percentage of age-specific natural debt limits that arise naturally in a life-cycle model with retirement. These age-varying borrowing limits are crucial in reconciling the micro-evidence showing a high share of illiquid assets for the young and wealth poor. The model reproduces a share of liquid assets of about one-third, similar to the SCF. Additionally, allowing for heterogeneity in the return on savings across households reproduces a significant fraction of the distributions of net worth, liquid assets, and illiquid assets seen in the data.

The economy, with assets that vary in their liquidity and return, exhibits an aggregate response to shocks that varies with the cyclicality of the return to liquid assets supplied by the government. When this supply is weakly countercyclical, the model can exhibit unusually large declines in investment and consumption over a recession. The steep fall in earnings, and rise in unemployment risk, that characterizes a recession compels households to smooth consumption by monetizing their illiquid assets. This substitution of liquid for illiquid assets leads to an unusually large decline in investment when compared to a single-asset economy.

A rise in the risk of a large economic disaster, during a recession, amplifies the effect of the fall in productivity and leads to larger declines in aggregate consumption and investment. The heightened risk of a further worsening in earnings exacerbates an already large negative wealth effect, driving a rise in precautionary savings and reducing consumption. Investment also plummets as the increased disaster risk lowers the expected return on capital. This results in a large portfolio adjustments into liquid assets, especially when the supply of liquid assets expands to maintain a constant safe real interest rate. When the government finances a constant return on liquid savings, the wealth composition heterogeneity model with a rise in disaster risk predicts peak-to-trough declines of 4.1 percent in consumption and 18 percent in investment, similar to the changes observed over the Great Recession. This is the result of a rise in the risk of a disaster that interacts with multiple assets of varying liquidity.

Importantly, in a single-asset economy without illiquid assets, a rise in disaster risk has little effect as increases in precautionary savings' motives are largely offset by equilibrium changes in the return to capital. In the present work, households increase their precautionary savings and decrease their consumption in a large recession when they expect lower life-time earnings following a rise in disaster risk. (*wealth effect*) However, in a single asset economy, all households respond to a fall in the return to savings which dampens the fall in consumption. (*substitution effect*) Thus, the substitution effect offsets the negative wealth effect, mitigating the effect of disaster risk. In contrast, when households hold both liquid and illiquid assets as in my model, a large fraction of non-adjusting households do not respond to a fall in the return to capital (illiquid assets). This sharply decreases the magnitude of the substitution effect, resulting in a large fall in aggregate consumption consistent with the data.

Heterogeneity in the composition of wealth, and the associated countercyclical substitution of liquid, low-yield assets for illiquid, productive investment in capital, leads to a slow recovery following a large recession. The half-life of aggregate consumption rises 1.4 times compared to that in a single asset economy. As the economy begins to recover, households that paid transactions costs to reduce their holdings of illiquid assets and smooth consumption against a fall in income are initially reluctant to reinvest in capital. Their shares of illiquid assets, while less than desired, remain in a range consistent with optimal adjustment in response to fixed costs. These households' tolerance for portfolio imbalances slows aggregate investment in capital and economic recovery.

Finally, I evaluate the models' predictions for changes in the cross-sectional distribution of net worth, income, and consumption to those in the PSID. During ordinary times, the portfolio choice economy successfully explains growth rates of net worth, earnings, income, and consumption that fall in wealth. These growth rates fall in a recession. More importantly, with countercyclical supply of liquid assets, savings rates rise in a large recession. In a typical model recession, spending rates would rise as households smooth their consumption. In contrast, household level data shows a fall in spending rates over the recent recession. My model economy suggests an importance of the interplay, between a rise in the probability of a large disaster and multiple assets of varying liquidity, for understanding this fall in spending rates in the Great Recession. Thus, the model with heterogeneity in the liquidity, as well as the level of wealth, and uninsurable earnings, employment and liquidity risk, is consistent with an unusual characteristics of the household data over the recession.

The numerical method developed to solve the model may be of independent interest. As pointed out in Kaplan and Violante (2014), solving for a stochastic OLG economy with a bivariate cross-sectional distribution of assets is challenging using the Krusell-Smith algorithm as this approach involves a repeated long simulations to obtain an accurate parametric law of motion for the aggregate state. First, I manage to overcome this difficulty by using a two-stage approach to solve decision rules, which allows me to use the Endogenous grid method to solve for the choice of liquid assets. Next, I extend the Backwards Induction method of Reiter (2002, 2010) to solve a stochastic life-cycle model with heterogeneity in household wealth, multiple assets, borrowing limits that vary in age, and fixed transactions costs. In contrast to linearization methods, it is important to note that the Backward Induction method allows aggregate risk to matter for household decisions and aggregate dynamics.[2]

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 documents the cross-sectional distribution of net worth, earnings, income, and consumption from the PSID. Section 4 presents the model economy. Section 5 discusses the calibration. Section 6 presents quantitative results and Section 7 concludes.

2 Related Literature

This paper contributes to the literature that studies the impact of a recession on heterogeneous households. Krueger et al. (2016) is the first to study changes in the joint distribution of earnings, income, and consumption during the Great Recession in a heterogeneous households framework. They also explore the importance of cross-sectional wealth inequality for the aggregate dynamics of an incomplete-markets Krusell-Smith economy with unemployment and earnings risk as well as permanent differences in households' discount factors. They find that an economy with a more pronounced dispersion of wealth

[2]For linearization methods in continuous time, see Winberry et al., 2016 and Kaplan et al., 2016.

experiences a larger drop in consumption compared to an economy with less inequality as wealth-poor households sharply decrease their consumption in a recession. I further study changes in the joint distribution during the Great Recession by allowing differences in households not just at the level of wealth but also in the composition of their wealth. In addition, I explore how this endogenous portfolio channel alongside a rise in disaster risk amplifies the Great Recession.

Guerrieri and Lorenzoni (2015) study the effects of a credit crunch during a recession in a heterogeneous-agent incomplete markets model with variable labor supply and idiosyncratic income shocks. They find that a credit crunch forces financially constrained households to repay their debt while it increases precautionary savings by unconstrained households, resulting in a drop in real interest rates. As financially unconstrained households respond to a fall in real interest rates by increasing their consumption, the resulting aggregate output and consumption responses are modest.[3] In their extended model with durable goods, output even increases by 0.4 percent following a credit crunch due to the high substitutability between bonds and durables. In contrast, liquidity risk embedded in illiquid wealth weakens substitutability between two assets in my model, generating a larger drop in consumption in a recession.

My work is also related to a body of work that studies the effects of multiple assets on heterogeneous households' behavior. Glover et al. (2016) study the intergenerational redistribution effects of the Great Recession in a stochastic complete-markets 6-period OLG economy with i.i.d aggregate shocks. Studying financial markets characterized by two assets - a risk-free bond and a risky equity, they find that older cohorts are hurt more than younger cohorts as a result of a larger decline in asset prices compared to a decline in wages during the Great Recession. However, in the absence of cross-sectional heterogeneity in each cohort, they are unable to address effects of wealth inequality in the recession. Moreover, given fixed total stocks of capital and labor, all changes in the demand translate to security prices changes. On the contrary, wealth inequality arising from financial friction with rich heterogeneity across households, in my model, makes it

[3] In their benchmark economy, a 10 percent drop in debt-to-GDP ratio only leads to one percent drop in output.

possible to explore the impact of the Great Recession across households with different levels of wealth. Furthermore, my model economy possibly explain both price and quantity effects of the asset demand during the Great Recession by allowing the aggregate capital to change.

Kaplan et al. (2015) explore the consumption response to changes in house prices over the Great Recession. Their OLG economy has two assets, housing and liquid savings. However, by assuming an exogenous interest rate on liquid savings and exogenous labor supply, which is the only factor of production, the dynamics of consumption, investment, and GDP over the business cycle cannot be addressed by their framework.

My model economy is closely related to the framework of Kaplan and Violante (2014) which studies the consumption responses to fiscal stimulus in an incomplete-markets life-cycle model with an optimal portfolio choice between a low-return liquid asset and a high-return illiquid asset involving a deterministic transaction cost to adjust a high-return illiquid asset. In contrast to a partial equilibrium analysis in their work, I solve the model in a general equilibrium to study implications of multiple assets that vary by liquidity and risk over the business cycle. In addition, I explore the effects of a disaster risk affecting households' belief during the Great Recession.

3 Impact of the Great Recession on heterogeneous households

Understanding changes in the joint distribution of net worth, consumption, earnings, and income may be important for studying the Great Recession. In this section, using the 2005-2011 PSID data, I document changes in the cross-sectional distributions of these variables before and during the Great Recession.[4][5] Disaggregated wealth data is contained in the PSID since 2003, consisting of transaction accounts, stocks, bonds, IRAs, business and farm equity, and debt. Consistent with the definition of net worth in the SCF, I define

[4]Krueger et al. (2016) also examined changes in the joint distribution of income, consumption, and net worth during the recent recession.

[5]The PSID consists of the SRC and SEO samples. Though SEO sample is designed to oversample the poor, I only used SRC sample to be consistent with sample selection for earnings estimation.

net worth as the sum of total assets minus total debt.[6] Table 1 compares the distribution of wealth in the PSID to that in the SCF. Although the PSID constructed measures of wealth are less dispersed than those in the SCF data, it still explains around two-thirds of total wealth held by the top 1 percent of household as well as the wealth Gini of 0.72 in 2007. Recent waves of the PSID also provide detailed spending data. I aggregate this data to measure total expenditure, defined as the sum of total spending on nondurable goods and services.[7]

Table 1. The distribution of wealth

Year	Gini	top 1%	5%	10%	50%	90%	≤ 0
2007 SCF	0.78	29.1	52.3	64.3	96.8	100	10.3
2007 PSID	0.72	19.1	40.5	56.3	95.2	101	9.0
2009 SCF	0.79	29.8	53.2	65.5	98.2	101	14.8
2009 PSID	0.73	15.4	39.6	55.9	96.6	102	14.3

Notes: Table 1 shows the wealth Gini coefficient, the share of wealth held by the top 1, 5, 10, 50 and 90 wealthiest households, the share of households with zero or negative asset holdings in the U.S. economy. For the PSID, drop three samples with wealth less than negative 99 million dollars.

Table 2 summarizes annualized growth rates of the average level of net worth, earnings, income, and consumption between 2005 and 2007 over 2005 wealth quintiles. It also documents percentage point differences in expenditure rates which are defined as total spending as a fraction of disposable income, over the sample period.[8] Earnings are the sum of after-tax wages and salaries, bonuses, overtime, tips, commissions, and transfers. Disposable income is defined as the sum of labor income, unemployment benefits and income from assets minus federal and state income taxes calculated using the NBER TAXSIM calculator.[9]

[6] See Appendix A for details on sample selection and wealth categories.

[7] Items on expenditure surveyed in the PSID are listed in Appendix A. I also compare the PSID constructed expenditure data to BEA data in Appendix A.

[8] To study impact of the Great Recession for households with different levels of wealth, I keep households in each quintile fixed and calculate the annualized percentage change of the averages between two years following Krueger et al. (2016).

[9] Note that, in the PSID, income and earnings data are retrospective while reporting unit for consumption varies by items.

Table 2. Growth rates of variables across wealth quintiles before the Great Recession 2005-2007

NW Quantile	Net worth	Earnings	Income	Expenditure	Expen. rate
Q1(*poor*)	n/a	3.3	3.3	11	3.6
Q2	25	1.7	1.9	7.1	2.3
Q3	22	1.8	2.0	3.0	0.5
Q4	11	1.3	2.9	2.9	0.0
Q5(*wealthy*)	2.1	2.6	1.2	4.3	1.5
all	6.8	2.2	2.2	5.1	1.4

Notes : Table 2 shows annualized growth rates of the average net worth, earnings, and disposable income and percentage points change in expenditure rates from the PSID.

Towards understanding the growth rates of wealth, income and consumption prior to the Great Recession, Table 2 reports annualized growth rates in these series between 2005 and 2007. The last row shows that aggregate net worth increases by 7 percent on average per year between 2005 and 2007 while aggregate earnings and income rise by 2.2 percent. Aggregate consumption rises by more than 5 percent annually and the average consumption rate rises by 1.4 percent over the sample period. Table 2 also reveals a few facts regarding the dynamics of each variable over wealth. First, it shows declining growth rates of net worth across wealth quintiles. This may be driven by mean-reversion which implies a higher probability of the rise in earnings or income for those who are currently poor compared to thr rich. Upward bias in growth rates associated with initial small levels of wealth may also contribute. Second, my PSID constructed spending data shows that growth rates of consumption generally decline in wealth (column 5). For instance, consumption grows the most for the bottom 20 percent of households and the least for households in the fourth quintile of the wealth distribution.[10] Finally, the poorest 20 percent of households in 2005 experienced the largest increase in their spending rates and most households increased their spending rates between 2005 and 2007.

Table 3 summarizes annualized growth rates of net worth, earnings, income, and consumption between 2007 and 2011 to study how wealthy and poor households were affected during the Great Recession. It reports the growth of the average of each category for the 2007 wealth quintiles. As seen in the last row of Table 3, in the aggregate, net worth fell

[10] Krueger et al. (2016) also find consumption growth rates falling in wealth over the same sample period.

approximately by 2 percent while earnings and income exhibited little growth. Next, in the aggregate, consumption changes little and the expenditure rate fell by 0.1 percentage point.[11] Thus, overall growth rates of wealth, income and consumption fell over this period compared to the two years preceding.

Table 3. Growth rates of variables across wealth quintiles during the Great Recession 2007-2011

NW Quantile	Net worth	Earnings	Income	Expenditure	Expen. rate(pp)
Q1(*poor*)	n/a	2.1	2.5	0.2	-2.3
Q2	11	1.1	0.8	1.9	1.0
Q3	-5.1	-0.3	-0.1	-0.2	-0.1
Q4	-0.5	-0.7	-0.1	-1.6	-1.7
Q5(*wealthy*)	-3.5	0.0	0.3	-2.5	-2.8
all	-1.7	0.0	0.1	-0.1	-0.1

Notes : Table 3 shows annualized growth rates of the average net worth, earnings, and disposable income and percentage points change in expenditure rates from the PSID.

Across households of different wealth levels, Table 3 shows net worth growth rates decreasing in wealth and becoming negative for the richest 60 percent of households. Households in the third and fifth quintiles experience approximately 5 and 3.5 percent declines in their wealth while households in the second quintile saw more than 10 percent rise in their wealth. I also observe rich households having lower growth rates of earnings and income relative to wealth poor households (third and fourth columns of Table 3). Consumption also falls more for rich households during the Great Recession, declining by 2.5 percent for the top 20 percent of households. Lastly, I find a fall in expenditure rates for most households but the second quintile, which implies that most households increase their savings rates over the recession.[12]

To study the severity of the Great Recession across the wealth distribution, Table 4 presents differences in the growth rates of variables between normal times (2005-2007) and recession (2007-2011). For net worth, households with less wealth tend to experience

[11] In comparison, Krueger et al. (2016) report a 1.6 percentage points drop in the expenditure rate.

[12] Over 2007-2011, Krueger et al. (2016) also find net worth growth rates falling in wealth as well as negative consumption growth rates for the top distribution of households. Moreover, they also report rise in saving rates across all wealth quintiles.

a larger drop in their growth rates of wealth. For example, the poorest 20 percent of households experience 14 percent decline in their wealth while the richest 20 percent of households experience 6 percent of decline. Turning to earnings and income, while Table 3 shows a decline only for the third and fourth quintiles over the recession, earnings and income growth slow down for all households, with the largest fall for the top 20 percent. Consumption growth rates decline across all wealth quintiles and, most importantly, all households decrease their expenditure rates with the largest fall for the bottom 20 percent of households. [13]

Table 4. Changes in growth rates of variables between prior- and during the Great Recession

NW Quantile	Net worth	Earnings	Income	Expenditure	Expen. rate
Q1(*poor*)	n/a	-1.2	-0.8	-11	-5.9
Q2	-14	-0.6	-1.1	-5.2	-1.3
Q3	-27	-2.1	-2.1	-3.2	-0.6
Q4	-12	-2.0	-3.0	-4.5	-1.7
Q5(*wealthy*)	-6	-2.6	-0.9	-6.8	-4.3
all	-5.1	-2.2	-2.1	-5.2	-1.5

Notes : Table 4 shows changes in annualized growth rates of the average net worth, earnings, disposable income, and percentage points change in expenditure rates from the PSID.

The fall in expenditure rates across wealth quintiles during the Great Recession compared to normal times is hard to explain in a standard model. Consumption smoothing motives lead households to raise their spending rates in a recession and the fall in consumption tends to be less than that in income. However, PSID data shows a rise in savings rates for most households, which likely reflects a strong precautionary savings motive. Below, I introduce disaster risk in my model and calibrate it to explain this change in saving rates.

[13] Likewise, Krueger et al. (2016) find the largest drop in expenditure rate for households in the bottom quintile.

4 Model

In my model economy, there are four agents: households, an investment firm, a production firm, and a government. Households differ by age, productivity, employment status, and wealth. Household wealth is itself the sum of low-yield liquid assets and high-yield illiquid assets. Each period, a household chooses its consumption, total saving, and asset portfolio. It has to pay an idiosyncratic transaction cost if it chooses to cash in its illiquid wealth and re-balance its portfolio. Illiquid wealth is capital and is held in an investment firm, which rents it to the production firm. Government supplies the net quantity of the liquid asset. Interest payments on which are financed by tax revenue.

In this section, I first describe households' decision problems. Second, I discuss the production economy and government. Lastly, I present the definition of recursive equilibrium.

4.1 Households

Households live for a finite number of periods. They enter the labor market at age $j = 1$, retire at age $J_r = 35$, and their last possible age is $J = 60$. While working, households face a stochastic idiosyncratic unemployment risk $e(z) \in \{0, p_e(z), 1\}$ where z is the exogenous aggregate state which consists of a total factor productivity shock η and disaster state d. This unemployment risk, which moves as a function of the exogenous aggregate state, determines households' working time in each period.[14] Households can be partially ($e = p_e$) or completely ($e = 0$) unemployed with probability $\pi_u(z)$ while they are full-time workers ($e = 1$) with probability $1 - \pi_u(z)$. A fraction $\pi_p(z)$ of those not-fully-employed are working for a fraction $p_e(z)$ of a period and unemployed for the remaining time. In contrast, not fully employed households are laid off for the full period with probability $1 - \pi_p(z)$. Summarizing, the probability for each idiosyncratic employment status is as follows:

$$\text{a full-time worker} : 1 - \pi_u(z)$$

[14] Following Khan et al. (2016), partial unemployment is introduced to match the mean and median duration of unemployment less than a model period of one year.

$$\text{a part-time worker} : \pi_u(z)\pi_p(z)$$

$$\text{unemployed} : \pi_u(z)(1 - \pi_p(z))$$

Partially and fully unemployed workers receive unemployment benefits from the government proportional to their possible earnings and unemployment duration in the period; the replacement rate is θ_u. Households draw idiosyncratic productivity shocks ε each period, which follow a Markov chain $\varepsilon \in \{\varepsilon_1, ..., \varepsilon_{n_\varepsilon}\}$, where $Pr(\varepsilon' = \varepsilon_k|\varepsilon = \varepsilon_l) = \pi_{lk} \geq 0$ and $\sum_{k=1}^{n_\varepsilon} \pi_{lk} = 1$.[15] These idiosyncratic productivity shocks alongside a labor market experience premium, $l(j)$, determine the total efficiency units of labor for workers. After retirement, households receive social security benefits proportional to their last earnings shock $s(\epsilon^{J_r - 1})$.

As mentioned already, the economy has two assets - a high-yield illiquid asset, a, and a low-yield liquid asset, b. Each unit of illiquid wealth pays a dividend $d(z)$ and has an ex-dividend price $p(z)$. Adjusting illiquid assets to a value other than a non-adjusted post-dividend balance involves an idiosyncratic fixed adjustment cost, ξ, denominated in units of output, and drawn from a time-invariant distribution, $H(\xi)$. Households paying these costs adjust illiquid wealth to a desired value; otherwise, the after-tax dividend payments are re-invested in illiquid wealth.

Borrowing is only allowed in the liquid asset, which is supplied by the government at a price q. Households face borrowing limits that are a fixed fraction of their age-varying natural debt limits, $\phi \underline{b}_j$.[16] Note that ϕ is common across households of different age. Following Kim (2016), I derive age-varying natural debt limits $\underline{b}_j$ in an overlapping-generations economy.[17] Given a natural debt limit for the next age $\underline{b}_{j+1}$ and a lowest possible future earnings x_{j+1}, a natural debt limit for age j is defined as:

$$\underline{b}_j \geq q\underline{b}_{j+1} - x_{j+1}$$

[15] I combined persistent and transitory productivity shocks into one shock process for ease of notation.

[16] In a life-cycle model, the natural debt limit falls with age as households borrow against their future income.

[17] Given that borrowing is not allowed at the last possible age, I can solve natural debt limits backward by age.

This implies that the maximum debt a household can borrow is the discounted value of the maximum debt it can borrow tomorrow and the lowest possible future earnings. Age-specific natural debt limits are crucial for explaining more indebted younger generations as they allow households to borrow against future income. In the following, $\phi < 1$

Each period, a household, identified by its age $j \in \mathbf{J} = \{1, ..., J\}$, illiquid asset, $a \in \mathbf{A} \subset \mathbf{R}_+$, liquid asset, $b \in \mathbf{B} \subset \mathbf{R}$, productivity, $\varepsilon \in \mathbf{E}$, and working status $e(z) \in \{0, p_e(z), 1\}$, chooses consumption and savings. Savings is in two assets, and adjusting the stock of illiquid wealth requires payment of a transactions cost. A household adjusts its illiquid asset from the otherwise non-adjusted post-dividend balance if it pays the current idiosyncratic cost, $\xi \in \mathbf{\Xi}$. If a household does not pay its fixed cost, it can only choose consumption and liquid savings in the current period. The distribution of households, μ, is over $(j, a, b, e, \varepsilon)$ and evolves following the mapping $\mu' = \Gamma(z, \mu)$ where $z = (\eta, d)$ is a vector summarizing the exogenous aggregate state.

I now present the problem solved by households. For the ease of notation, I define transition probabilities for aggregate state vector z by $Pr(z' = (\eta, d)_g | z = (\eta, d)_f) = \pi^z_{fg} \geq 0$. Each period, a working household has three idiosyncratic shocks; unemployment shock e, productivity shock ε, and portfolio adjustment cost ξ. If a household pays its current fixed cost ξ, it actively adjusts its illiquid wealth account a'. After any adjustment decision, a household at age j with illiquid assets a, liquid assets b, unemployment shock e, productivity shock ε_l, and adjustment cost ξ realizes the value $v_j(a, b, e, \varepsilon_l, \xi; z_f, \mu)$ given the aggregate state (z_f, μ).

$$v_j(a, b, e, \varepsilon_l, \xi; z_f, \mu) = \max \left\{ v^a_j(a, b, e, \varepsilon_l, \xi; z_f, \mu), v^n_j(a, b, e, \varepsilon_l; z_f, \mu) \right\} \tag{1}$$

where v^a_j represents the value of a household adjusting its risky illiquid assets and v^n_j is the value of a non-adjusting household. Let $v^e_j(a, b, e, \varepsilon_l; z, \mu) = \int_0^{\bar{\xi}} v_j(a, b, e, \varepsilon_l, \xi; z, \mu) H(d\xi)$ be the expected value of a household before the idiosyncratic adjustment cost ξ is realized.

A worker receives labor income and an unemployed worker receives unemployment benefits. A retiree receives a proportional social security benefits, $s(\epsilon^{J_r - 1})$. An actively adjusting household, paying a resource adjustment cost, cashes in the total stock of wealth in both assets and re-balance its portfolio. Assuming Epstein-Zin preferences, I describe the optimization problem of a household who pays its adjustment cost to actively adjust its asset portfolios between high-yield illiquid and low-yield liquid assets.

$$v_j^a(a,b,e,\varepsilon_l,\xi;z_f,\mu) =$$

$$\max_{c,\,a',\,b'} \left[(1-\beta)c^{1-\sigma} + \beta \left\{ \sum_{k=1}^{n_\varepsilon} \pi_{lk} \sum_{g=1}^{n_z} \pi_{fg}^z \sum_{e=0}^{1} \pi_e(z_g) v_{j+1}^e(a',b',e',\varepsilon_k;z_g,\mu')^{1-\gamma} \right\}^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}}$$

subject to

$$c + q(z_f,\mu)b' + p(z_f,\mu)a' \leq b + (p(z_f,\mu) + (1-\tau_a)d(z_f,\mu))a + x - \xi \tag{2}$$

$$x = \begin{cases} (1-\tau_n)w(z_f,\mu)l(j)\varepsilon_l(e+(1-e)\theta_u) & \text{if } j < J_r \\ (1-\tau_n)s(\epsilon^{J_r-1}) & \text{otherwise} \end{cases}$$

$$b' \geq \phi \underline{b}_j,\ a' \geq 0,\ c \geq 0$$

$$\mu' = \Gamma(z_f,\mu)$$

where $\underline{b}_j$ is the age-varying natural debt limit.

If a household does not pay its fixed cost, it can only choose its stock of safe liquid wealth for next period, b'. Illiquid assets pay after-tax dividends which are re-invested.

$$v_j^n(a,b,e,\varepsilon_l;z_f,\mu) =$$

$$\max_{c,\,b'} \left[(1-\beta)c^{1-\sigma} + \beta \left\{ \sum_{k=1}^{n_\varepsilon} \pi_{lk} \sum_{g=1}^{n_z} \pi_{fg} \sum_{e=0}^{1} \pi_e(z_g) v_{j+1}^e(a',b',e',\varepsilon_k;z_g,\mu')^{1-\gamma} \right\}^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}}$$

subject to

$$c + q(z_f,\mu)b' \leq b + x \tag{3}$$

$$x = \begin{cases} (1-\tau_n)w(z_f,\mu)l(j)\varepsilon_l(e+(1-e)\theta_u) & \text{if } j < J_r \\ (1-\tau_n)s(\epsilon^{J_r-1}) & \text{otherwise} \end{cases}$$

$$a' = (1+(1-\tau_a))d(z_f,\mu)a$$

$$b' \geq \phi \underline{b}_j,\ c \geq 0$$

$$\mu' = \Gamma(z_f,\mu)$$

4.2 Production and government

The exogenous aggregate state z is summarized by a total factor productivity shock η and disaster state d. Exogenous total factor productivity, η, follows a Markov chain $\eta \in \{\eta_1, ..., \eta_{n_\eta}\}$ with $Pr(\eta'|\eta) = \pi_{\eta,\eta'} \geq 0$. The economy can be either in a normal ($d = 1$), high risk of disaster ($d = 2$), or disaster state ($d = 3$). A high risk of disaster involves a higher probability of the disaster state next period. The disaster state has an additional drop in TFP, determined by the parameter λ, conditional on the economy already experiencing a fall in TFP. [18] TFP is $(1-\lambda(d,\eta))\eta$, where $\lambda(3,\eta=\eta_1) = \lambda < 1$ and $\lambda(3,\eta) = 0$ otherwise. I assume that d also follows a Markov chain $d \in \{1,2,3\}$ with

$$\pi^d = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}$$

where p_{ij} is the probability of transiting from i state to j state.

A competitive investment firm sells shares of total capital at an ex-dividend price $p(z,\mu)$ and pays dividends $d(z,\mu)$ to households where μ is the distribution of households. The investment firm owns the technology that creates capital and rents this capital to a production firm at a rental rate $r^k(z,\mu)$. Specifically, investment firm sells shares to future capital, $p(z,\mu)k'$, to households to transform k' units of current output into capital for the

[18] I will assume that the disaster state affects TFP when the latter falls by more than one standard deviation from its mean. This will be the case only when $\eta = \eta_1$.

next period. Moreover, the investment firm faces convex capital adjustment costs.

$$\Phi(k', k) = \left(\frac{k' - k}{k}\right)^2 k$$

This adjustment cost introduces deviation in the price of capital from that of consumption as it makes consumption and capital less than perfectly substitutable. I describe the optimization problem of the investment firm.

$$J(k, z_f, \mu) = \max_{k'} \Bigg((r^k(z_f, \mu) + 1 - \delta)k - (p(z_f, \mu) + d(z_f, \mu))k + p(z_f, \mu)k' - k' - \Phi(k', k) + \sum_{g=1}^{n_z} \pi_{fg} r(z_g, z_f, \mu) J(k', z_g, \mu') \Bigg)$$

where investment firm discounts future earnings by $r(z_g, z_f, \mu)$.[19]

The production firm employs capital k and hires labor n to produce output through a CRS production function $y = (1 - \lambda(d, \eta))\eta k^{\alpha} n^{1-\alpha}$, where $0 < \alpha < 1$ and $\eta > 0$. Thus, the optimization problem of the production firm is

$$\max_{k,n} \left((1 - \lambda(d))\eta k^{\alpha} n^{1-\alpha} - r^k(z, \mu)k - w(z, \mu)n \right)$$

The government supplies a quantity of liquid assets, B, at a price $q(z, \mu)$. Labor income is taxed at τ_n and dividend income at τ_a. Social security benefits and unemployment benefits are also taxed at τ_n. Government revenues are used to finance social security benefit payments, unemployment benefits payments, interest payments on debt, and government spending, $G(z, \mu) \geq 0$.[20]

[19] In equilibrium, this discount factor should be equal to the marginal rate of substitution of households who are holding illiquid assets. In my model economy, it is hard to solve as households endogenously hold different asset portfolios. In Appendix B, I will show how the equilibrium prices and dividend payments to the shares of capital are not affected by this discount factor.

[20] Every period, spending is chosen to balance the government budget.

4.3 Recursive equilibrium

Define the product space $\mathbf{S} = \mathbf{J} \times \mathbf{A} \times \mathbf{B} \times \mathbf{E} \times \{0, p_e, 1\}$ for the distribution of households. Given the Borel algebra $\mathcal{S}$ generated by the open subsets of $\mathbf{S}$, $\mu : \mathcal{S} \to [0, 1]$ is a probability measure over households. Households start with an initial wealth of zero and an initial labor productivity drawn from $\pi^0 \sim logN(0, \sigma_\pi^2)$.

A *recursive competitive equilibrium* is a set of functions

$$(v, v^a, v^n, c^a, c^n, h^a, h^n, b^a, b^n, \chi, k, n, r, q, w)$$

such that:

(i) (v, v^a, v^n) solves (1)−(3), and (c^a, h^a, b^a) are the policy functions associated with (2) for consumption, illiquid and liquid asset savings by a household that adjusts its illiquid asset holdings. (c^n, h^n, b^n) are the policy functions associated with (3) for consumption and savings in illiquid and liquid assets by a non-adjusting household. χ is the decision rule associated with (1), and $\chi = 1$ when the fixed cost to adjust illiquid assets is paid.

(ii) The government budget is balanced

$$G(z,\mu)+B_s+\sum_{e=0}^{1}\sum_{j=1}^{J}\sum_{l=1}^{n_\varepsilon}\int_{\mathbf{A}}\int_{\mathbf{B}}(1-\tau_n)(s(\varepsilon_l\mathbf{1}_{j\geq J_r})+(1-e)\theta_u wl(j)\varepsilon\mathbf{1}_{j<J_r})\mu(j,da,db,e,\varepsilon_l)$$
$$= \tau_a d(z,\mu)k + \tau_n w(z,\mu)n + q(z,\mu)B_s^{'}$$

(iii) Markets clear

$$n(z,\mu) = \sum_{e=0}^{1}\sum_{j=1}^{J}\sum_{l=1}^{n_\varepsilon}\int_{\mathbf{A}}\int_{\mathbf{B}} l(j)\varepsilon_l e\mu(j,da,db,e,\varepsilon_l)$$

$$k(z,\mu) = \sum_{e=0}^{1}\sum_{j=1}^{J}\sum_{l=1}^{n_\varepsilon}\int_{\mathbf{A}}\int_{\mathbf{B}} a\mu(j,da,db,e,\varepsilon_l)$$

$$B(z,\mu) = \sum_{e=0}^{1}\sum_{j=1}^{J}\sum_{l=1}^{n_\varepsilon}\int_{\mathbf{A}}\int_{\mathbf{B}} b\mu(j,da,db,e,\varepsilon_l)$$

(iv) Prices are competitively determined

$$
\begin{aligned}
w(z,\mu) &= (1-\alpha)(1-\lambda(d))\eta k^{\alpha} n^{-\alpha} \\
r^k(z,\mu) &= \alpha(1-\lambda(d))\eta k^{\alpha-1} n^{1-\alpha} \\
p(z,\mu) &= 1+\Phi_1(G_k(z,\mu),k) \\
d(z,\mu) &= \alpha(1-\lambda(d))\eta k^{\alpha-1} n^{1-\alpha} - \delta - \Phi_1(G_k(z,\mu),k) - \Phi_2(G_k(z,\mu),k)
\end{aligned}
$$

where $G_k(z,\mu)$ is the aggregate law of motion for aggregate capital. Φ_1 and Φ_2 are the derivatives of Φ with respect to G_k and k, respectively.[21][22]

(v)

$$
\mu'(j+1, A_0, B_0, e', \varepsilon_k) = \pi_{e'}(z)\sum_{l=1}^{n_\varepsilon} \pi_{lk} \left(\int_{\Delta_1} \mu(j,da,db,e,\varepsilon_l) H(d\xi) + \int_{\Delta_2} \mu(j,da,db,e,\varepsilon_l) H(d\xi) \right) \quad \forall\, j
$$

where $\Delta_1 = \{(a,b,e,\varepsilon_l,\xi) | h^a(j,a,b,e,\varepsilon_l,\xi;z,\mu) \in A_0,\ b^a(j,a,b,e,\varepsilon_l,\xi;z,\mu) \in B_0 \text{ and } \chi(j,a,b,e,\varepsilon_l,\xi;z,\mu) = 1\}$ and $\Delta_2 = \{(a,b,e,\varepsilon_l,\xi) | h^n(j,a,b,e,\varepsilon_l;z,\mu) \in A_0,\ b^n(j,a,b,e,\varepsilon_l;z,\mu) \in B_0 \text{ and } \chi(j,a,b,e,\varepsilon_l,\xi;z,\mu) = 0\}$, $(j,a,b,e,\varepsilon_l) \in S$ and $\xi \in \boldsymbol{\Xi}$.

5 Calibration

In order to bring the model to data, I calibrate the model economy using household data. Given its detailed wealth information, I use the 2007-2009 SCF panel data to calibrate parameters directly affecting asset holdings including the distribution of adjustment cost. Parameters governing unemployment are calibrated to match the mean and median unemployment duration in the CPS and the average unemployment rate in the BLS. Lastly,

[21] Note that these equilibrium price function, using $\Phi(k',k) = \left(\frac{k'-k}{k}\right)^2 k$ imply $p = 1$ and $d = \alpha\eta k^{\alpha-1} n^{1-\alpha} - \delta$ in steady state.

[22] Given the definition of recursive competitive equilibrium, I show the prices, $p(z,\mu)$ and $d(z,\mu)$ are consistent with equilibrium in Appendix C.

I estimate the earnings shock process using the PSID.

In the SCF, I define illiquid wealth as stocks, business equity, net residential property, net equity in non-residential real estate and net consumer durables.[23] The remaining assets and debt are considered liquid following Kaplan et al. (2016). I calibrate β to match the capital (productive illiquid asset) to output ratio of 2.66. Following Kaplan et al. (2016), I consider business equity, stocks and net equity in non-residential real estate as productive illiquid assets as well as 40 percent of net housing and consumer durables.[24] Given that a sampling unit in the SCF is a household, the total value of productive illiquid assets is divided by the average family size in 2007 to make it comparable to GDP per capita.[25] The calibrated economy implies a 6.4 percent return on illiquid wealth in the steady state when a zero return on liquid wealth is targeted, resulting in a liquidity premium of 6.4 percent. The capital share of output is $\alpha = 0.36$. Despite the share of liquid wealth to GDP not being a target, the calibrated economy explains a ratio of 29 percent, which is close to the 35 percent in the 2007 SCF.

I assume a model period of one-year. Households enter the labor market at age 25, retire at age 60, and live until age 84 with certainty. I assume Epstein-Zin preferences, allowing the elasticity of intertemporal substitution (EIS) to deviate from the inverse of the coefficient of relative risk aversion. I set the coefficient of relative risk aversion to 2 and the EIS is 1.5.[26] Social security payments are paid based on the average of the highest 35 years of earning by Social Security Administration (SSA). In the model, calculating average

[23] Following Glover et al. (2016), money market mutual funds and quasi-liquid retirement accounts are included in stocks. In the absence of a housing market and collateralized borrowing, I include residential property net of all debt secured by residential property (mortgages, home equity loans, and HELOCs) following Kaplan and Violante (2014). By defining housing wealth as a net value, I abstract from the issue of home equity loans which provide liquidity through housing asset.

[24] Kaplan et al. (2016) argue that a fraction of housing and consumer durables could be rented to business or used in production. I follow their approach.

[25] As GDP per capita is expressed in chain 2009 dollars, I adjust the value of productive illiquid wealth in 2009 dollars using CPI-IPUMS.

[26] Gourio (2012) shows that it is important to have the EIS greater than 1 in a model with a disaster risk to reproduce a countercyclical risk premia as seen in the data. He also shows that, in an economy with standard expected utility, investment is much less volatile than the data. As noted by Kaplan and Violante (2014), when regressing consumption growth on real interest rates, an estimate of the EIS is downward-biased because of measurement error and endogeneity issues. By contrast, the estimates derived using GMM are generally greater than 1. Similarly, Van Binsbergen et al. (2012) estimated a value of the EIS above 1 in a DSGE production economy in which households have Epstein-Zin preferences using maximum likelihood.

earnings requires one more state variable, making computation more challenging. Given the high persistent earnings process, I proxy the history of earnings over a worker's life-cycle using the level of earnings in the last working period, $s(\epsilon^{J_r-1}) = \theta_s w \frac{\sum_{j=1}^{J_r-1} l(j)}{J_r-1} \varepsilon^{J_r-1}$. Following Hosseini (2015), θ_s is chosen to match the replacement rate of 45 percent of average pre-tax earnings in the steady state. Labor income is taxed at 27 percent (Domeij and Heathcote (2004)). I chose τ_a to imply a 25 percent capital income tax rate in a steady state.[27]

I select parameters p, π_u, π_p to match mean and median unemployment durations as well as the unemployment rate as in Khan et al. (2016). The working period for a partially employed household p is chosen to match the median unemployment duration of 12 weeks between 1981 and 2016 in the CPS.[28] The probability of a partial employment π_p varies with a TFP shock, $\pi_p \in [\overline{\pi}_p - \varepsilon_p, \overline{\pi}_p + \varepsilon_p]$. A steady state level of the probability of a partial employment $\overline{\pi}_p$ is chosen to match the mean unemployment duration of 24 weeks between 1981 and 2016.[29] The mean duration of unemployment rises to 36 weeks after 2008 which is around 60 percent of a model period. I select ε_p to match this rise in duration when the economy is in recession.[30] The probability of unemployment π_u also varies with a TFP shock, $\pi_u \in [\overline{\pi}_u - \varepsilon_u^l, \overline{\pi}_u + \varepsilon_u^h]$. $\overline{\pi}_u$ is chosen to match an unemployment rate of 5 percent in the steady state. ε_u^l is chosen to explain a 5 percent rise in the unemployment rate over the Great Recession while ε_u^h results in a 3 percent unemployment rate when the economy is in a boom.[31] Lastly, I choose 43.5 percent for a replacement rate of unemployment benefits θ_u following Nakajima (2012).

The idiosyncratic earnings shock ε consists of both a persistent and a transitory component. The persistent shock follows an AR(1) process;

$$\varepsilon = \varphi + \varepsilon_v,$$

[27]Landvoigt et al. (2016) estimated a capital income tax rate of 20 percent from government corporate tax revenue as a share of GDP using BEA data. In Jermann and Quadrini (2012), the average tax rate is 35 percent. I chose a value in this range.

[28] Partial unemployment is introduced to handle the median unemployment duration less than a model period. As one year has 52 weeks, the median unemployment duration corresponds to 23 percent of a model period.

[29]Note that the mean unemployment duration is calculated as $\frac{24}{52} = \pi_p(1-p) + (1-\pi_p) * 1$.

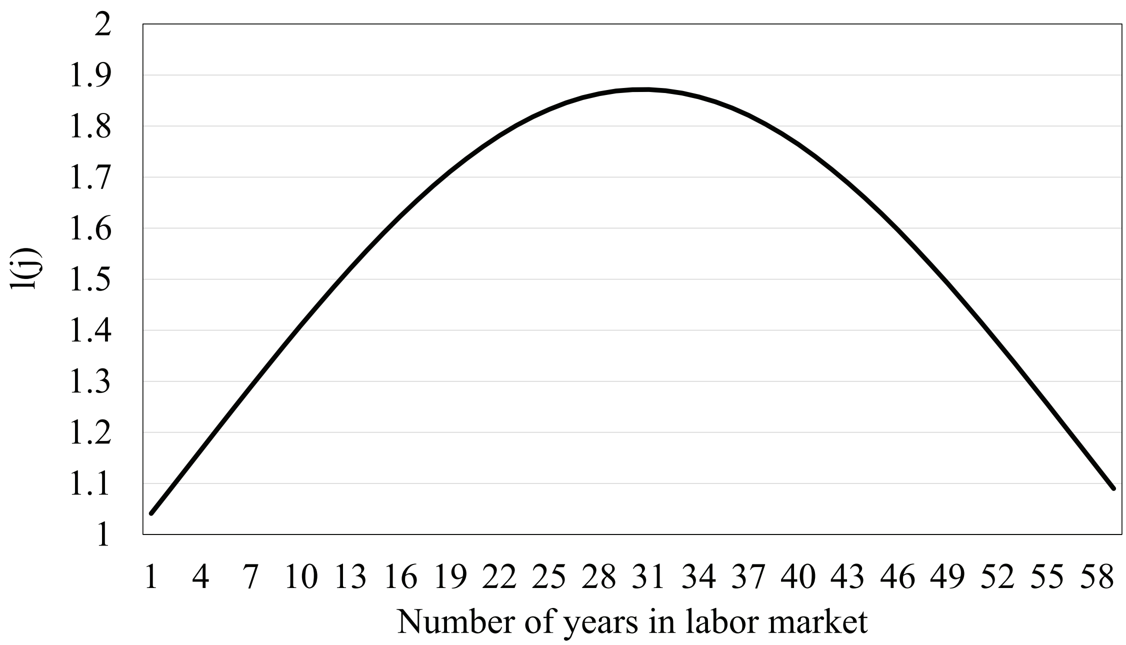
[30]Given p, this implies that $\frac{36}{52} = (\overline{\pi}_p - \varepsilon_p)(1-p) + (1 - (\overline{\pi}_p - \varepsilon_p)) * 1$

[31]Khan and Thomas (2013) measured a TFP drop by 2.18 percent in the recent recession. Thus, ε_u is chosen to imply 10 percent of unemployment rate when a TFP falls by 2.18 percent.

$$\varphi' = \rho_\varepsilon \varphi + \varepsilon_s,$$

where $\varepsilon_s \sim N(0, \sigma_s^2)$ and $\varepsilon_v \sim N(0, \sigma_v^2)$. The values of ρ_ε, σ_s^2, σ_v^2 are the 2007 estimates in Khan and Kim (2015). They also estimated a labor market experience premium $l(j)$ by running an OLS regression of the log of hourly wages on time dummies; an interaction term with education and time dummies; labor market experience approximated by age minus years of schooling minus 5; and experience-squared. Figure 1 illustrates the resulting labor market experience premium.

Figure 1. Labor market experience premium



The distribution of idiosyncratic fixed costs $\zeta \in [0, \overline{\zeta}]$ is assumed to be drawn from an uniform distribution. Households can borrow up to a fixed fraction ϕ of their age-varying natural debt limits, $\underline{b}_j$. I choose these parameters $(\overline{\zeta}, \phi)$ to match the share of households holding zero or negative net worth and the share of households holding positive illiquid assets but little liquid assets as seen in the 2007 SCF.These liquidity constrained households are comparable to poor and wealthy hand-to-mouth households respectively as defined in Kaplan and Violante (2014) and Kaplan et al. (2016).[32] Kaplan and Violante (2014) point out that these households have a high response in consumption following a transitory income change.[33] Thus, these parsimonious targets have the virtue of allowing for the mechanism that Kaplan and Violante (2014) emphasize: a potentially important role for liquidity constrained households in driving a large aggregate consumption response.

Borrowing limits are important in determining the number of liquidity constrained households as these households have little liquid assets. Figure 2 shows the resulting age-specific borrowing limits which, in contrast to the standard age-invariant limit, allows more borrowing by younger cohorts. Thus, I calibrate parameters $(\overline{\zeta}, \phi)$ to match liquidity constrained households in the 2007 SCF. As seen in the next section, given this calibration strategy, the calibrated economy explains a significant fraction of the distributions of the net worth as well as illiquid and liquid wealth. Furthermore, the benchmark economy gives rise to the share of illiquid assets to net worth over wealth deciles and age cohorts without being a calibration target.

Aggregate TFP is assumed to follow an AR(1) process in logs, $\log \eta^{'} = \rho_\eta \log \eta + \varepsilon_\eta$ where $\varepsilon_\eta \sim N(0, \sigma_\eta^2)$. Khan and Thomas (2013) estimated this process using Solow residuals calculated from data on real U.S. GDP, private capital, and total employment hours. I use their estimates and discretize this process into $n_\eta = 3$.[34]

For the transition probabilities for disaster risk, I first assume that the probability of staying in normal times is equal to the persistence of ordinary TFP shocks. Next, in order to make disaster a rare event, I set the probability of going to the disaster state from a normal state to a negligible value, $p_{13} = 0$. Next, I assume that the disaster state is

[32] Kaplan and Violante (2014) defined wealthy hand-to-mouth (htm) households as those with liquid wealth less than half of their earnings but holding positive balances of illiquid wealth. They estimated a total fraction of hand-to-mouth households between 17.5 percent and 35 percent of households in the 2004 SCF, including both wealthy and poor households. Among htm households, they find 40 to 80 percent are wealthy. I instead define wealthy htm households as those with no liquid wealth but positive illiquid wealth.

[33] Kaplan and Violante (2014) showed that the average MPC for HTM households is around 40 percent while that for non-HTM households is only 7 percent.

[34] Khan and Thomas (2013) estimated an AR(1) TFP shock process in a an yearly model with the same capital depreciation rate and the capital share in production function as my model economy. Though they assumed a DRS technology instead of a CRS technology, this difference only slightly affects the standard deviation of a TFP shock process.

preceded by a rise in the risk of disaster, which itself has a higher probability of transiting into a disaster state. Imposing the following symmetry conditions, $p_{11} = p_{33}$, $p_{12} = p_{32}$, $p_{13} = p_{31}$, $p_{21} = p_{23}$, I only need to set one more parameter: the probability of staying in a high-risk state p_{22}. Given that it is hard to pin down the transition probabilities in a high disaster risk state, I assume the following transition probability matrix:

$$\pi^d = \begin{pmatrix} 0.909 & 0.091 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.091 & 0.909 \end{pmatrix}$$

Disaster risk is usually defined as wars or economic depressions which result in more than a 15 percent decline in real GDP per capita (Barro, 2006) and modelled as a shock to TFP or capital destruction. (Gourio, 2012). λ is chosen to match a drop in the expenditure rate for the bottom 20 percent of households in the Great Recession.[35]

5.1 Numerical Overview

I develop a two-stage approach to solve savings decisions with two assets. In the first stage, given a current fixed cost, a household chooses whether or not to adjust its portfolio. If it adjusts, it chooses its savings in illiquid wealth, a'. In the second stage, given a', I solve for the optimal choice of liquid wealth, b', using the endogenous grid method (Carroll, 2006). To solve the model with aggregate uncertainty, I extend the Backward Induction Method of Reiter (2002, 2010). This involves generalizing the method to solve an OLG economy with bivariate cross-sectional distribution of continuous endogenous state variables. The Backward induction method of Reiter allows the distribution of households to vary in potentially rich ways as a function of an approximate aggregate state as it does not impose a parametric aggregate law of motion. Solving individual decision rules and the consistent aggregate law of motion simultaneously, it does not require repeated simulation, reducing computation time compared to Krusell and Smith (1998). Please see Appendix D for more details.

[35]Results are not very sensitive to the transition probabilities parameters for disaster state. Details are available in next revision.

Figure 2. Borrowing limits

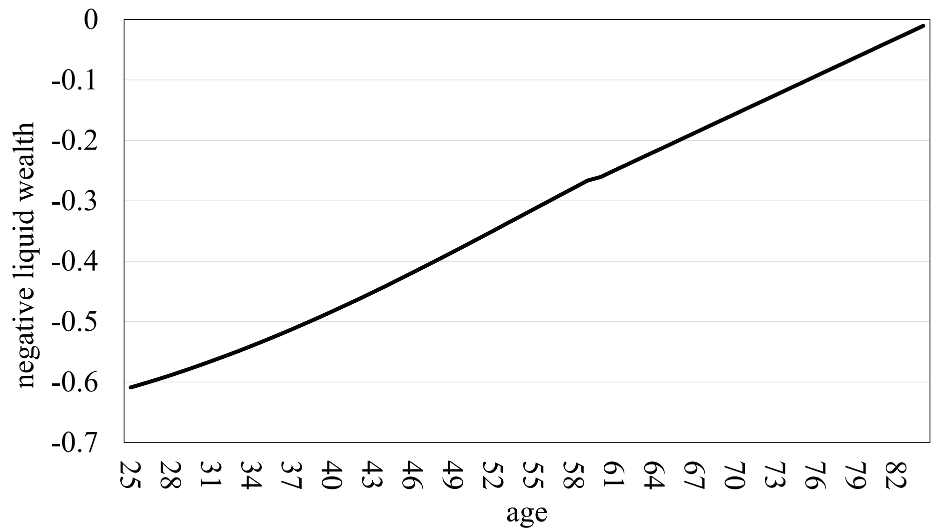


Table 5: Summary of parametrization

Parameters set externally				Value
α		capital share of output(NIPA)		0.36
δ		depreciation rate(NIPA)		0.069
τ_n, τ_a		labor and capital income taxes		0.27, 0.25
(ρ_η, σ_η)		TFP shock process (Khan and Thomas, 2013)		(0.909, 0.014)
σ		coefficient of relative risk aversion		2.0
γ		inverse of EIS		$\frac{1}{1.5}$
$\lambda(d=3)$		additional drop in tfp a in disaster state		0.1
θ_s		replacement rate of avg pre-tax earnings for social security		0.45
θ_u		replacement rate of avg pre-tax earnings for unemployment benefit		0.435
$(\sigma_s, \sigma_v, \rho_\varepsilon)$		earnings shock process (Khan and Kim, 2016)		(0.2449, 0.3937, 0.9825)
$l(j)$		male hourly wage life-cycle profile		see text
Parameters calibrated	Value	moments to match	data	model
β	0.937	capital to output ratio	2.66	2.69
p_e	0.2287	median unemployment duration as a fraction of a model period	0.2287	0.2287
π^d	see text	transition probability matrix for disaster state		
$\pi_e(z)$	see text	unemployment rate		
$\bar{\zeta}$	1.4	share of hhs holding positive illiquid asset but little liquid asset	0.32	0.32
ϕ	0.4	share of hhs holding zero or negative net worth	0.103	0.103

6 Results

I begin by discussing the steady state of the model economy. Next, I discuss the implications of wealth composition heterogeneity for business cycles driven by shocks to TFP and changes in disaster risk. Lastly, I evaluate predictions for changes in the joint household distribution of net worth, earnings, income and consumption.

6.1 Steady state

Table 6 summarizes distributions of net worth, illiquid asset, and liquid assets in the 2007 SCF and the steady state of the economy with heterogeneity in both the level and composition of net worth. Though it only targets the share of liquidity constrained households, the portfolio choice economy successfully reproduces much of the dispersion in wealth. As seen in the top panel, the quintile distributions of net worth in the benchmark (wealth composition heterogeneity) economy are broadly consistent with those in the 2007 SCF, explaining more than 70 percent of the total wealth held by the wealthiest 20 percent of households. The wealth Gini is within 9 percentage points of its empirical counterpart.[36] As pointed out in Krueger et al. (2016), reproducing enough dispersion in wealth in a model is important for explaining aggregate dynamics.[37] A realistic skewness in the distribution of wealth is also crucial for studying changes in the cross-sectional distributions of consumption and income as the recent recession had disparate effects on households with different levels of total wealth as shown in section 3.[38]

What explains differences between rich and poor households in the steady state of the benchmark economy? First, households with a favorable earnings shock have a propensity

[36]This is not driven by a high fraction of households near the borrowing limit as in Huggett (1996) but by the right tail of the distribution. The wealthiest 10 percent of households hold 57 percent of wealth while they hold 64 percent in the 2007 SCF.

[37]Krueger et al. (2016) show that the drop in aggregate consumption is 0.5 percentage points larger in an economy with more wealth inequality than in a representative agent economy.

[38]While interesting, explaining the very right tail of the distribution is not of primary importance as the analysis largely focuses on the quintile distributions of wealth where the model matches the data fairly well.

Table 6. Distributions of net worth, illiquid and liquid assets

Net worth	Q1	Q2	Q3	Q4	Q5	top 1%	5%	10%	≤ 0	Gini
2007 SCF	-0.3	1.4	5.7	14.1	79.1	29.1	52.3	64.3	10.3	0.78
Benchmark	0.2	3.8	8.9	16.1	71.1	11.6	38.2	57.1	10.3	0.69
Single asset	1.1	4.9	9.7	16.7	67.6	9.9	34.5	52.3	2.16	0.64
Illiquid wealth	Q1	Q2	Q3	Q4	Q5	top 1%	5%	10%		Gini
2007 SCF	0.14	1.6	5.9	14.4	78.0	28.2	51.2	63.2		0.76
Benchmark	0.0	3.0	8.1	15.3	72.0	11.9	40.3	59.1		0.71
Liquid wealth	Q1	Q2	Q3	Q4	Q5	top 1%	5%	10%		Gini
2007 SCF	-11.7	-0.53	0.92	7.8	103	47.1	76.2	90.5		0.92
Benchmark	-8.3	-1.4	4.89	22.9	82.0	11.3	36.7	54.9		0.90

Notes : Table 6 shows the share of net worth, illiquid asset, and liquid asset across the wealth quintiles. It also reports the share held by the top 1, 5, and 10 households for net worth, illiquid asset, and liquid asset. Lastly, it reports the share of households with zero or negative net worth in the 2007 SCF and model and the Gini coefficient for net worth, illiquid asset, and liquid asset.

to smooth consumption by accumulating a higher level of wealth compared to others. Second, a household with a high level of earnings can afford portfolio adjustment costs and invest in illiquid assets to receive a higher return on savings which accelerates its wealth accumulation. Furthermore, fixed transaction costs realized to adjust illiquid asset holdings discourage households from investing in illiquid assets until they are sufficiently wealthy. This endogeneity in the composition of wealth is crucial for explaining asset positions before the recession and the subsequent heterogeneous responses during the recession as households with different portfolios are dissimilarly affected by changes in asset prices.

Table 6 also summarizes the distributions of high-yield illiquid assets and low-yield liquid assets in the 2007 SCF and in the portfolio choice economy. The distributions of illiquid assets are comparable to those in the data, reproducing a Gini of 0.71 and 72 percent of the illiquid assets held by the top 20 percent of households compared to 78 percent in the data. However, SCF shows more dispersion in liquid assets. The liquid wealth Gini is around 0.91 and the top 20 percent of households hold all of net liquid assets in the economy. Borrowing by the bottom 20 percent of households represents approximately 12 percent of the total stock in the 2007 SCF. Though the distribution of low-yield assets falls short of the data for the top 10 percent of households, the benchmark economy qualitatively matches the more skewed distribution of liquid relative to illiquid assets, explaining the liquid wealth

Gini of 0.90.

Importantly, age-varying borrowing limits that are a common percentage of age-consistent natural debt limits, alongside fixed costs, give rise to empirically consistent average shares of illiquid assets across households of different levels of wealth and age. As seen in Figure 3, the benchmark economy predicts more than 80 percent of the total wealth held in illiquid assets for the top 80 percent of households and the high fraction of the wealth held as illiquid assets for the bottom 11-20 percent of households (2nd decile). The high ratio of illiquid assets to net worth, for the poorest 11-20 percent of households, is the result of wealth-poor households borrowing in liquid asset markets. This drives a small level of wealth which biases up their share of illiquid assets to net worth.

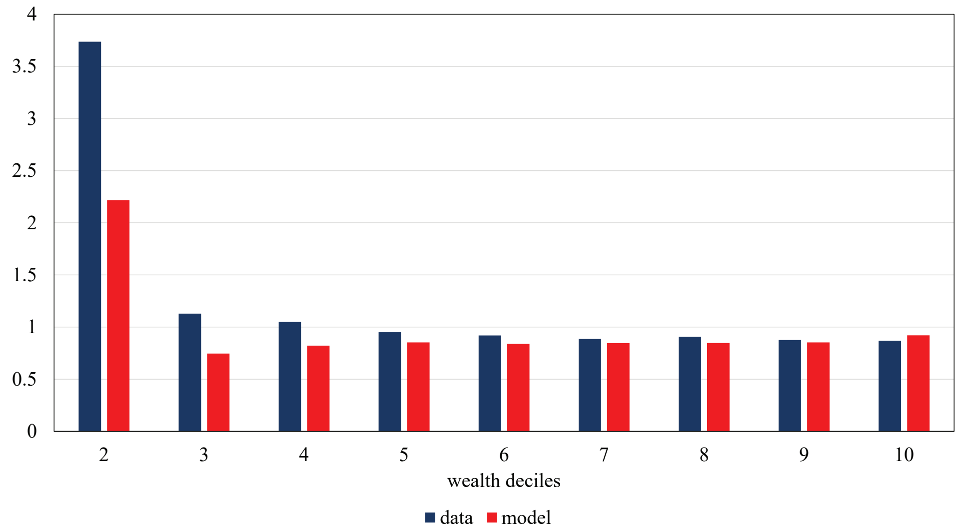
Figure 4 shows the corresponding figure over age groups. Though the benchmark economy overestimates the average share of total wealth held in illiquid assets by the youngest cohort, it captures the share's decline over age. Older cohorts tend to hold a higher fraction of safe liquid assets compared to younger cohorts as they have less time to recover from any fall in asset prices.[39] In the benchmark economy, age-varying borrowing limits are critical in explaining the high share of illiquid assets held by the young. More borrowing by younger households also biases up their share of illiquid assets to net worth embedded in the small level of wealth.

6.2 Aggregate Dynamics

In this section, I first present dynamic results for the model using TFP shocks. Next, I introduce disaster risk to show how a rise in the probability of a severe economic recession, consistent with rational expectations, amplifies the effects of a shock to TFP. Throughout, I examine the model using two different assumptions about the supply of liquid assets. The first involves a constant stock of liquid assets (B fixed) while the other assumes a constant

[39]Glover et al. (2016) also find that younger cohorts hold less safe assets than old cohorts because of their large mortgage debt which is considered safe in their classification. I still find the high share of illiquid assets to total wealth by the young when measured using the net value of housing and consumer durables.

Figure 3. Share of illiquid asset as a fraction of net worth over wealth deciles



Notes : The average share of illiquid asset as a fraction of net worth over wealth deciles in the 2007 SCF (blue bars) and the benchmark economy (red bars).

return (r_f fixed). Thus, in the second case, the supply of liquid assets is perfectly elastic while, in the first, government policy holds the supply of such assets fixed.[40] Note that, while the return on liquid savings is held constant in equilibrium in a fixed return model, this is consistent with general equilibrium characterized by the government policy which adjusts the supply of liquid assets to maintain the constant return.

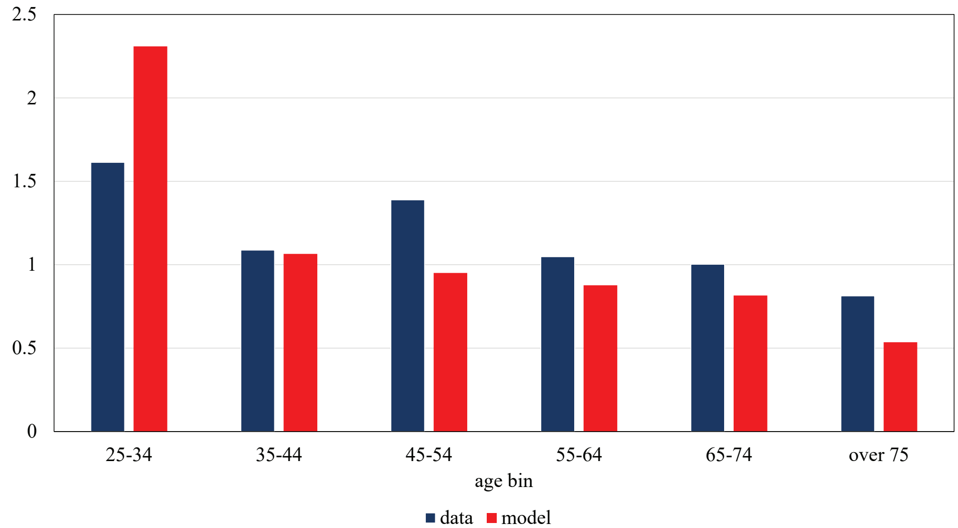
6.2.1 Heterogeneity in the composition of wealth

In Table 7 and 8, I summarize business cycles moments from simulating two portfolio choice models.[41] In both tables, we see that models with multiple assets of varying liquidity produce consumption approximately 40 percent as volatile as output and investment twice as volatile. Furthermore, the portfolio choice economy has a procyclical return on savings in liquid assets when their stock is fixed (column 9 of Table 7). In contrast, when the

[40] Each of these assumptions about government policy over the business cycle serves as a benchmark. Over the 2007 recession, real interest rates on safe, liquid assets fell markedly while, at the same time, a flight to quality led to a large rise in their supply. In a future revision, I plan to address this by modelling a countercyclical supply of liquid assets that responds to a higher probability of disaster. The supply function describing government policy can be chosen to ensure a fall in the safe real interest rate consistent with the data.

[41] I simulate the model economies with an aggregate productivity shock for 2300 periods and drop the first 300 periods.

Figure 4. Share of illiquid asset as a fraction of net worth over age groups



Notes : The average share of illiquid asset as a fraction of net worth over age group in the 2007 SCF (blue bars) and the benchmark economy (red bars).

return is rigid, the equilibrium stock becomes countercyclical (column 6 of Table 8). As a result, and somewhat surprisingly, the aggregate economy behaves similarly across the two cases where the supply of liquid assets is perfectly inelastic or elastic.

A rise in unemployment risk, during a recession, increases households' demand for liquid assets. Households attempting to smooth consumption following a decline in their earnings increase their stock of liquid savings relative to illiquid assets. Given a fixed stock of such assets, portfolio adjustments by households lower their return. When instead the government choose to hold the return on liquid savings fixed, this portfolio adjustment drives a rise in liquid assets over a recession.[42]

Table 9 shows business cycle moments for the fraction of households who adjust their illiquid assets and for the share of liquid assets to the total stock of wealth in the economy. As mentioned above, during a recession, following a reduction in earnings and a rise in un-

[42]While these forces affect the composition of household savings, their aggregate effects in ordinary business cycles is small. As a result, fluctuations in GDP, consumption, and investment resemble those in a single asset economy subject to the same TFP and unemployment risk shocks. (Table D2)

Table 7. Business cycle statistics in a portfolio choice model (B fixed)

$x=$	Y	C	I	K	B	N	$E(r)$	r_f	w
mean(x)	2.48	1.84	0.48	6.45	0.73	1.44	0.07	0.0	1.10
σ_x/σ_y	(2.75)	0.38	1.95	0.22	n/a	0.86	0.27	0.02	0.29
$corr(x,y)$	1.0	0.95	0.99	-0.04	n/a	0.96	0.94	0.79	0.59

Notes : Table 7 presents means of GDP, consumption, investment in illiquid wealth, stock of capital, supply of liquid wealth, total hours worked, expected return on illiquid wealth, return on liquid savings and wage for the model simulated data. It also lists relative standard deviation to and correlation with GDP for each variable. I smooth series using a HP-filter with a smoothing parameter of 100.

Table 8. Business cycle statistics in a portfolio choice model (r_f fixed)

$x=$	Y	C	I	K	B	N	$E(r)$	r_f	w
mean(x)	2.49	1.84	0.49	6.50	0.65	1.44	0.07	0.0	1.10
σ_x/σ_y	(2.75)	0.38	2.0	0.22	0.26	0.86	0.56	n/a	0.29
$corr(x,y)$	1.0	0.96	0.98	-0.02	-0.46	0.96	0.93	n/a	0.59

Notes : Table 8 presents means of GDP, consumption, investment in illiquid wealth, stock of capital, supply of liquid wealth, total hours worked, expected return on illiquid wealth, return on liquid savings and wage for the model simulated data. It also lists relative standard deviation to and correlation with GDP for each variable. I smooth series using a HP-filter with a smoothing parameter of 100.

employment, more households increase their holdings of liquid assets to offset the drop in their consumption, resulting in the counter-cyclical share of liquid assets across households. In the economy where the supply of such assets is allowed to vary, the share of liquid assets rises in a recession as their stock increases. This results in a more variable share than when government policy holds the supply of such assets fixed.

Now, I present impulse response functions for aggregate variables following a drop in TFP, accompanied by a rise in unemployment risk, in Figures 5 and 6 for a single asset economy as well as two heterogeneous composition of wealth economies. I assume that TFP falls for the first two periods then starts to recover at the rate implied by its persistence.[43] Following a drop in TFP, output declines by 6 percent on impact both in a model with a single asset and in models with multiple assets of varying liquidity. Given a predetermined

[43] I choose 2.18 percent drop in a TFP to be consistent with the estimated decline in measured TFP over the Great Recession reported in Khan and Thomas (2013).

Table 9. Business cycle statistics for the share of households adjusting illiquid assets and the share of liquid assets to total wealth

	B fixed			r_f fixed	
$x =$	adjusting pop	$\frac{B}{K+B}$	$x =$	adjusting pop	$\frac{B}{K+B}$
mean(x)	0.144	0.102	mean(x)	0.147	0.091
σ_x/σ_y	0.279	0.192	σ_x/σ_y	0.272	0.309
$corr(x, y)$	-0.485	0.046	$corr(x, y)$	-0.424	-0.327

Notes : Table 9 presents means of the share of households who adjust their illiquid assets and the share of liquid assets to total wealth for the model simulated data. It also lists relative standard deviation to and correlation with GDP for each variable. I smooth series using a HP-filter with a smoothing parameter of 100.

stock of capital, the fall in output is driven by the drop in TFP and aggregate labor supply. As labor supply is exogenous and all economies experience the same 5 percent rise in unemployment, the change in output on impact must be the same.

In Figure 5 (d), aggregate consumption exhibits a gradual, familiar, hump-shaped response. Moreover, aggregate consumption falls more in economies with illiquid assets compared to a single asset economy. For instance, in the absence of any change in the aggregate stock of liquid assets, aggregate consumption decreases by approximately 2.3 percent while it only falls by 2 percent in the single asset case. The larger fall in consumption is the result of illiquidity in wealth. The costs of adjusting illiquid assets deters households' ability to smooth consumption. As a result, consumption is more volatile in response to shocks to TFP and employment.

Adding liquidity risk into a model produces a slow recovery which is one of the characteristics of the Great Recession. In Figure 5 (d), the half-life of consumption in the economy with a fixed stock of liquid wealth is 1.4 times greater than that in a single asset model. This slow recovery is the result of the equilibrium fall in the real return to safe assets which depresses savings and consumption growth. Transactions costs imply a range of portfolio shares over which households are unwilling to increase their holdings of illiquid wealth. The slower growth in liquid assets for their non-investors implies fewer households reaching a sufficiently low share of illiquid wealth to invest in capital.

Aggregate capital and investment fall more in portfolio choice economies following a drop in TFP compared to a single asset economy as seen in Figures 6 (a) and 6 (b). In a single asset economy, households can raise their buffer stock of savings only through capital, hastening the recovery. However, in an economy with multiple assets that bear different risk and liquidity, households, who invested in high-yield assets at the expense of liquidity, re-balance their portfolios toward liquid assets that offer value in smoothing consumption. This results in a larger drop in investment and capital compared to a single asset economy.

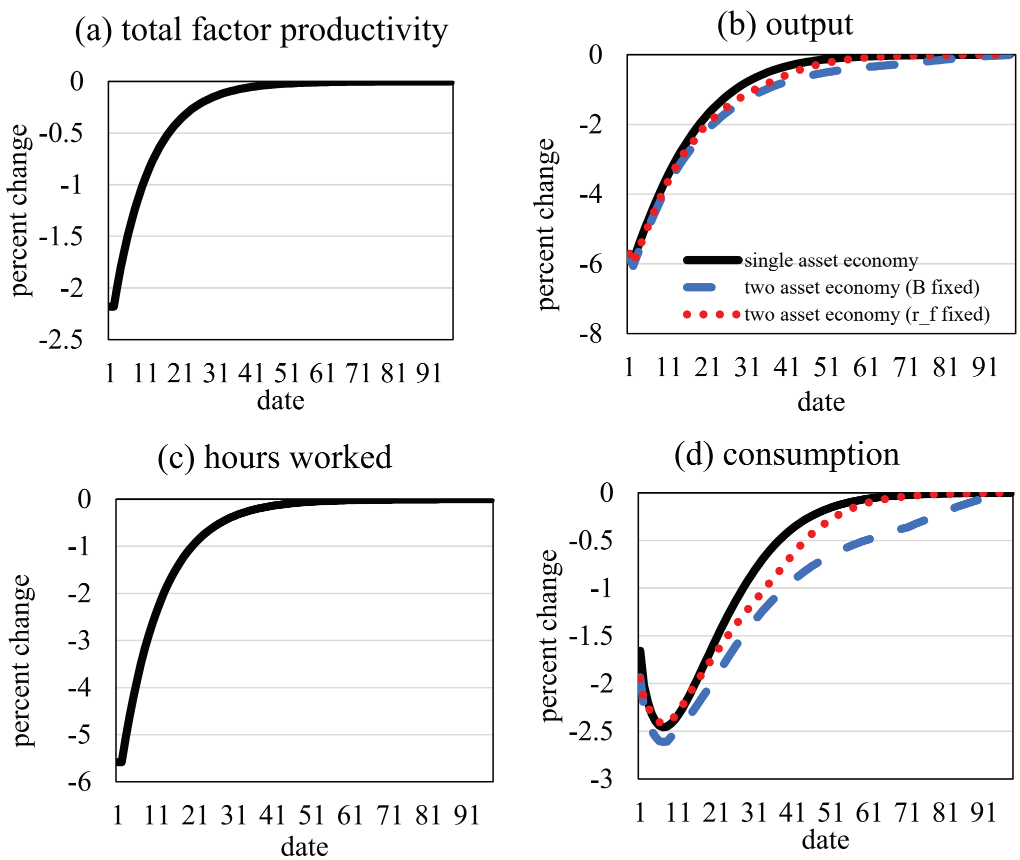
Table 10 summarizes peak-to-trough declines for each series in the Great Recession and in model economies. Following a 2.18 percent decline in TFP, an economy with one asset experiences 10 percent drop in investment and 2.4 percent drop in aggregate consumption compared to 19 percent and 4 percent, respectively, in data. As seen in the third and fourth rows of the Table 10, adding another asset that carries liquidity risk explains an additional 2 to 3 percentage points drop in aggregate investment. Although aggregate consumption falls more by 0.2 percentage points in a fixed liquid asset economy from those in a single asset economy, portfolio choice economies do not explain enough of the drop in aggregate consumption seen in data. In the following section, I introduce the disaster risk to study the amplification mechanism of a rise in risk of a disastrous recession across households.

Table 10. Peak-to-Trough declines: U.S. 2007 Recession and model

	GDP	I	N	C	TFP
data	5.59	18.98	6.03	4.08	2.18
single asset	5.92	10.00	5.58	2.45	2.18
two asset (B fixed)	6.06	12.06	5.58	2.61	2.18
two asset (r_f fixed)	5.98	12.54	5.58	2.42	2.18

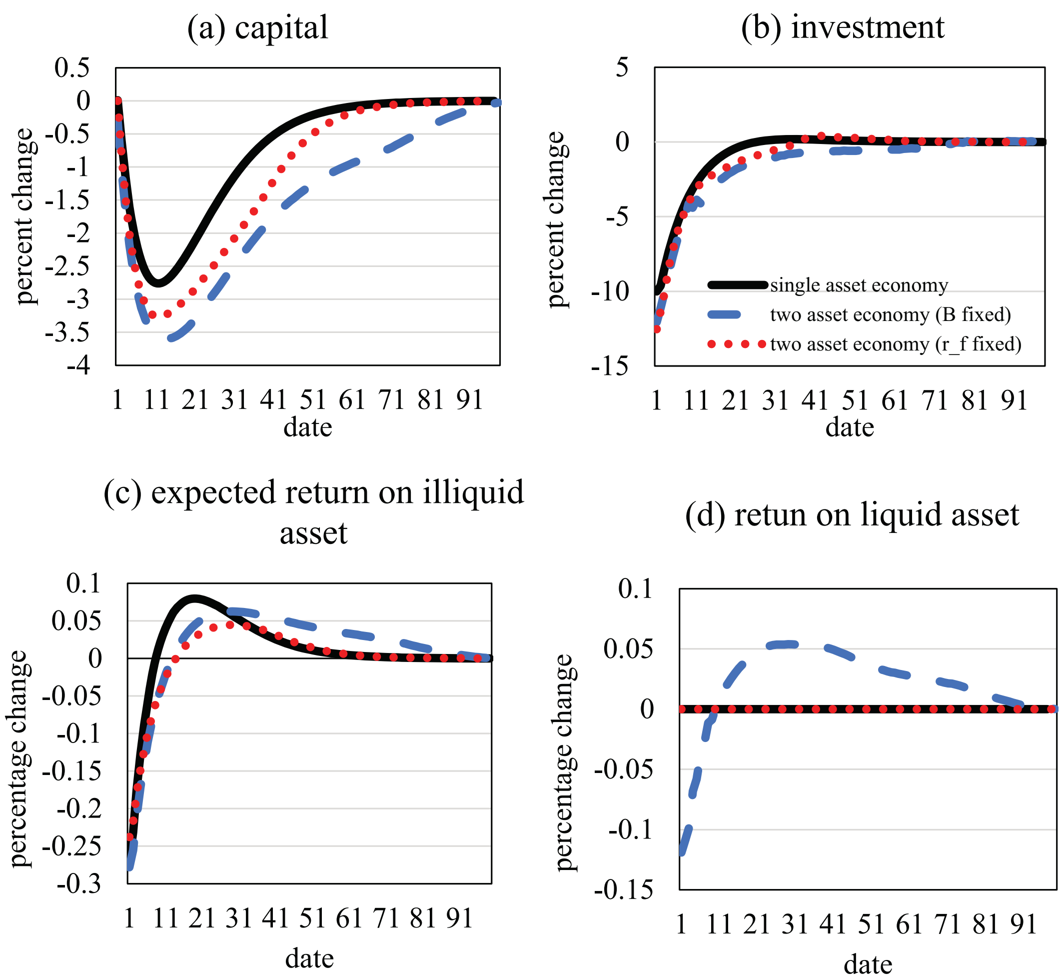
Notes : Table 10 shows peak-to-trough declines between 2008q4 and 2009q2 in the first row (Khan and Thomas, 2013). The second row is a single asset economy. The third row is a model with two assets, where the return on liquid wealth is fixed. The last row is a two asset model with a fixed stock of liquid assets.

Figure 5. Impulse response



Notes : Figure 5 shows aggregate response to a 2.18 percent drop in TFP in a single asset economy (black solid line), in a heterogeneous asset economy with a fixed quantity of liquid wealth (blue dashed line), and in a portfolio choice economy with a fixed price of liquid wealth (red dotted line). The vertical axis measures the percent change of a variable from its mean.

Figure 6. Impulse response



Notes : Figure 6 shows aggregate response to a 2.18 percent drop in TFP in a single asset economy (black solid line), in a heterogeneous asset economy with a fixed quantity of liquid wealth (blue dashed line), and in a portfolio choice economy with a fixed price of liquid wealth (red dotted line). The vertical axis measures the percent change of a variable from its mean.

6.2.2 Disaster risk

In this section, I introduce aggregate disaster risk into the wealth composition heterogeneity economy to explore how a persistent rise in the risk of economic disaster interacts with difference in liquidity of household wealth. Figure 7 presents impulse responses following a 2.18 percent drop in TFP and a rise in unemployment risk for the first two periods in a portfolio choice economy where the return on liquid savings is fixed over time while Figure 8 shows the corresponding figure in a portfolio choice model where the stock of liquid wealth is fixed. A rise in disaster risk raises households' desire for liquidity. The solid line in Figure 7 shows that, when there is no change in disaster risk, the aggregate response of the economy following a TFP shock is close to those in Figures 5 and 6 where there was no disaster. This muted effect of aggregate disaster risk in ordinary times is the result of a rare probability of further deterioration in TFP and earnings.[44]

In contrast, a rise in the risk of a sharp fall in TFP, across households, implies a different response in the aggregate economy as seen in Figure 7. First, a rise in disaster risk qualitatively changes the familiar hump-shaped response of aggregate consumption in the wealth composition heterogeneity model with a fixed rate of return on liquid wealth. This is due to a change in the average substitution effect when the future real rate of return to wealth is expected to be lower, following a rise in disaster risk, than the current average return. Over a typical TFP shock driven business cycle, the real return on assets rises over time following an initial drop. Over this period, the substitution effect leads to further reductions in consumption even as the negative wealth effect declines, generating a hump-shaped response in aggregate consumption. In contrast, when the real rate of return on liquid savings is rigid, the return to portfolios is expected to fall in the future with a rise in disaster risk. This fall in the expected real return on wealth changes the familiar hump-shaped response in consumption to a monotone one. Second, more cautious households drive a sharp initial drop in aggregate consumption. This larger drop in consumption is driven by the large negative wealth effect associated with the higher likelihood of economic disaster reducing TFP and earnings further. Lastly, aggregate investment experiences a large drop

[44] Recall that the likelihood of disaster in normal times is near zero.

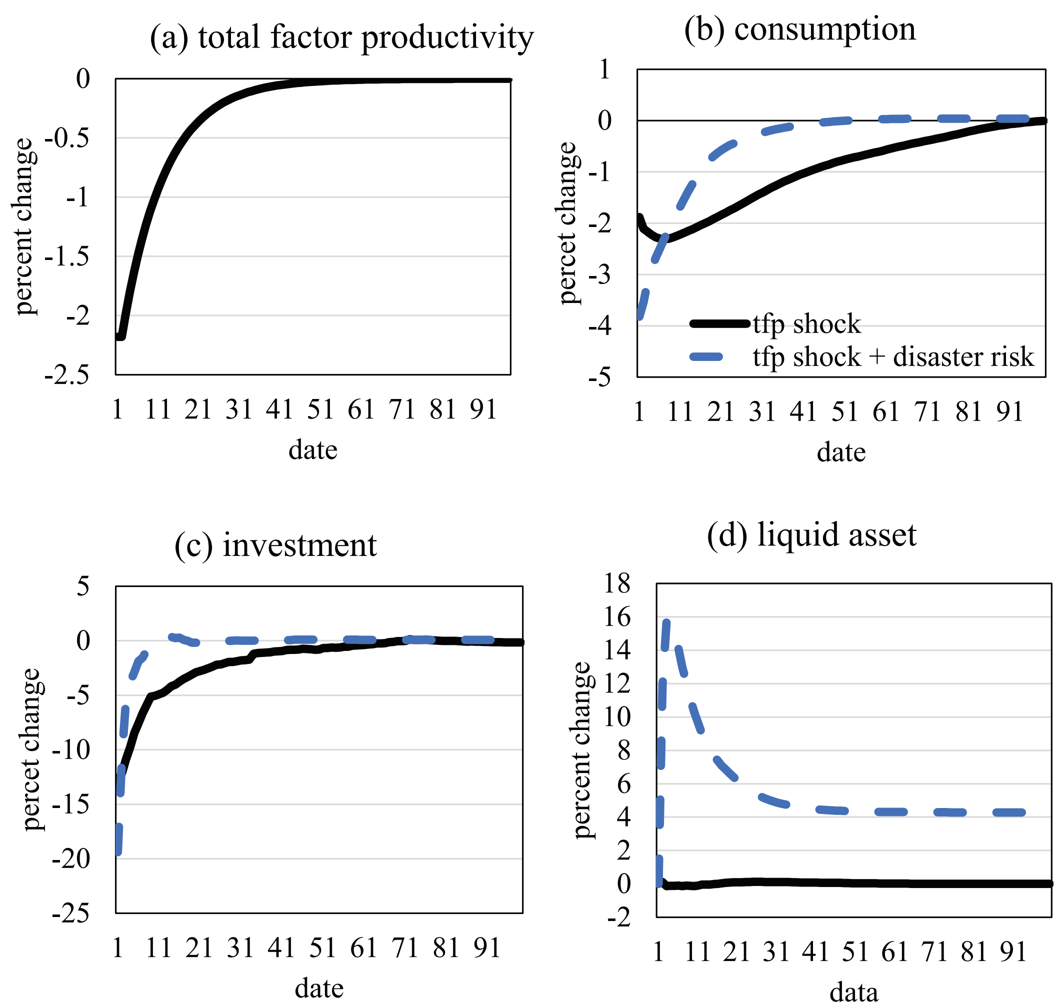
as the lower expected future return on capital causes a strong portfolio adjustment into liquid assets.[45]

As seen in Figure 8, if the stock of liquid assets is fixed, the rise in precautionary savings through liquid assets by cautious households in a recession lowers the return on liquid savings (Figure 8 (d)). This fall in the return on liquid savings dampens the effect of lower expected future average return on asset portfolios seen in Figure 7 as the current return on savings also falls more than before. While weakening the change in the substitution effect from a rise in disaster risk, the rising probability of disaster drives a far larger negative wealth effect as it sharply reduces expected earnings compared to normal times. This drives a severe recession from an empirically consistent actual fall in TFP.

A rise in the risk of a further worsening of earnings has little implication for aggregate dynamics in a single asset economy. Figure 9 shows impulse responses for aggregate consumption and investment when all households save using the same asset. While aggregate consumption falls slightly more with a rise in disaster risk, on impact, we see that the aggregate dynamics of consumption and investment are close to those with only a shock to TFP. In a single asset economy, a rise in precautionary savings by households would lower the return to savings in equilibrium, offsetting any aggregate effect.

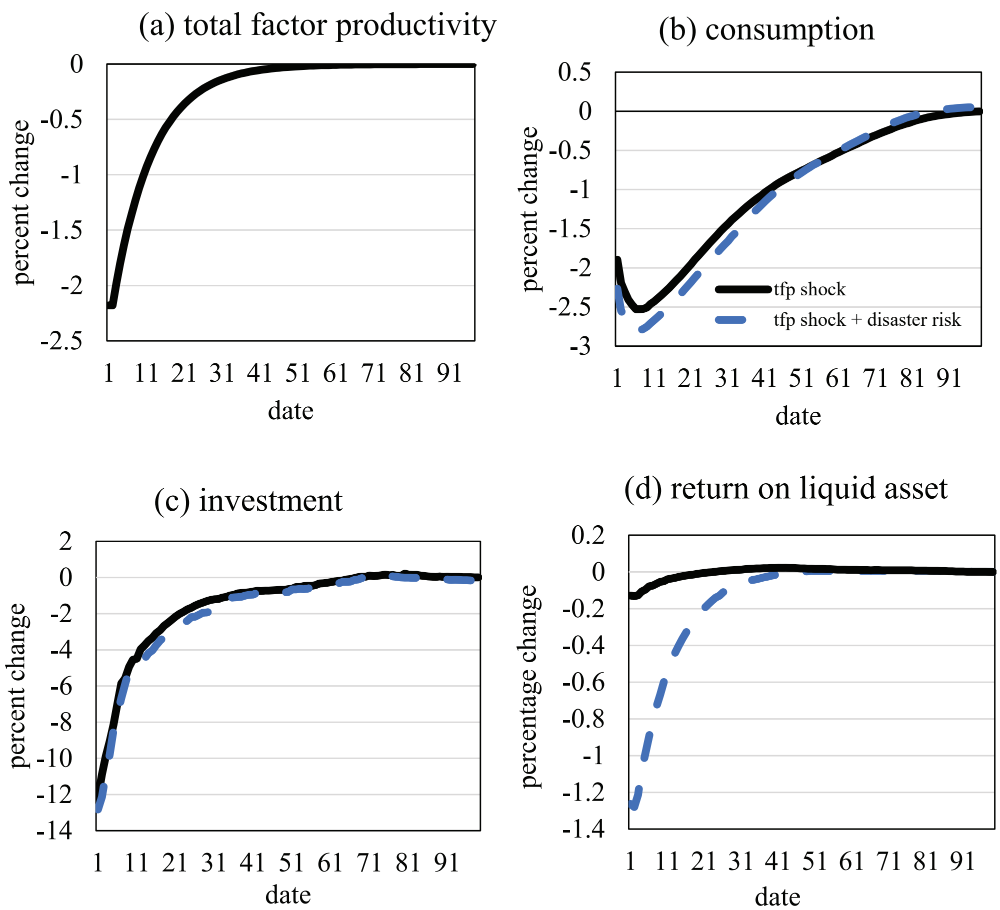
[45]Note that I model the disaster state as an additional drop in TFP that reduces earnings and the interest rate on savings. When I instead model disaster as a rise in capital depreciation which only affects the return on savings, consumption rarely falls beyond that in the response to actual TFP. It is important to generate a rising risk of a fall in earning through an economic disaster rather than a lower return on savings to explain a sharp drop in both aggregate consumption and capital investment. Thus, a rise in disaster risk does not increase precautionary savings if disaster reduces the return on assets relative to earnings.

Figure 7. Impulse response in a model with a constant return on liquid savings (r_f fixed)



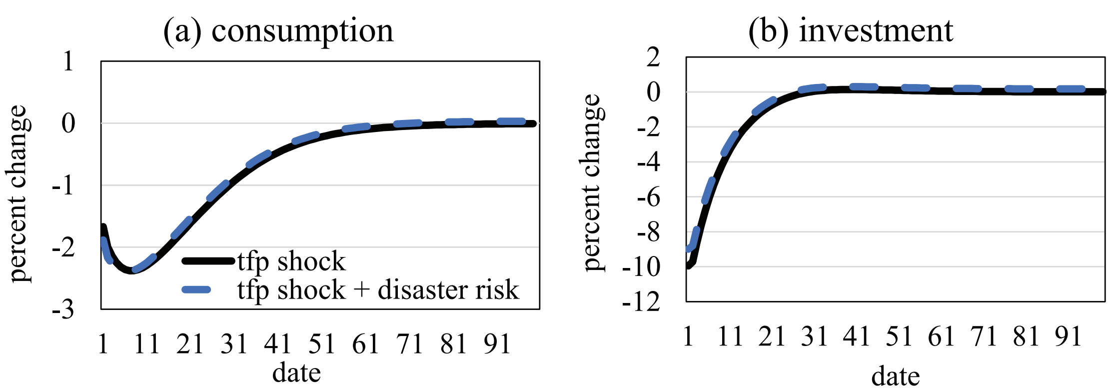
Notes : Figure 7 shows aggregate dynamics in the heterogeneous wealth composition economy following a 2.18 percent drop in TFP (black solid line) and a 2.18 percent drop in TFP alongside a rise in disaster risk (blue dashed line). The vertical axis measures percent changes of each variable from its simulation mean.

Figure 8. Impulse response in a model with a constant stock of liquid asset (B fixed)



Notes : Figure 8 shows aggregate dynamics in the heterogeneous composition of wealth economy with disaster risk in a response to a 2.18 percent drop (black solid line) and to a 2.18 percent drop alongside with a rise in disaster risk (blue dashed line).

Figure 9. Impulse response in a single asset economy



Notes : Figure 9 shows aggregate dynamics in a single economy with disaster risk in a response to a 2.18 percent drop (black solid line) and to a 2.18 percent drop alongside with a rise in disaster risk (blue dashed line).

Table 11 summarizes peak-to-trough declines in both heterogeneous wealth composition economies with a rise in disaster risk. As I have mentioned, if the real rate of return on safe assets does not drop as much as predicted by a model with a fixed stock, the rise in disaster risk drives a sharp substitution from illiquid capital to liquid assets, reproducing the drop in investment and consumption seen in the data. Indeed, a portfolio choice economy with a rigid return on liquid savings explains the 4 percent drop in consumption as well as the 20 percent drop in investment over the Great Recession.

Table 11. Peak-to-Trough declines: U.S. 2007 Recession and model

	GDP	I	N	C	TFP
data	5.59	18.98	6.03	4.08	2.18
two asset (B fixed)	5.71	12.83	5.58	2.82	2.18
two asset (r_f fixed)	5.71	19.89	5.58	3.82	2.18

Table 11 shows peak-to-trough declines between 2008q4 and 2009q2 in the first row (Khan and Thomas, 2013). The second row is a model with two assets, where the return to liquid savings is fixed. The last row is a two asset model with a fixed stock of liquid asset.

The sharp simultaneous declines in investment and consumption, following a rise in the probability of an economic disaster, is driven by dissimilar responses from households with different levels of wealth. Wealth poor households, with little liquid savings, face the risk of a sharp rise in the marginal utility of consumption following a further decline in earnings. They respond with portfolio adjustment selling illiquid assets to pay off debt and build liquid savings. The fall in investment is the result of a general reduction in savings by wealthier households who, while roughly maintaining their share of illiquid assets, smooth consumption by reducing overall savings. Table 12 shows the fraction of liquid assets to total net worth over wealth quintiles in the steady state and over the first date of this recession accompanied by a rise in disaster risk. As seen in the third row, higher risk of economic disaster boosts precautionary savings by the poorest 20 percent of households; their share of liquid savings rises from -127 percent to 18 percent when the return on liquid savings is fixed.[46]

[46] While the increase in the share of liquid savings by the poorest 20 percent of households is amplified

Table 12. Fraction of liquid assets to total net worth over the quintile distributions, $\frac{B}{(K+B)}$

B fixed	steady state	impact dates		r_f fixed	steady state	impact dates	
	t=0	t=1	t=2		t=0	t=1	t=2
all	0.096	0.100	0.101	all	0.096	0.106	0.108
Q1(poor)	-1.27	0.26	0.34	Q1(poor)	-1.27	0.18	0.33
Q2	0.28	0.27	0.25	Q2	0.28	0.26	0.25
Q3	0.17	0.17	0.17	Q3	0.17	0.16	0.17
Q4	0.14	0.14	0.14	Q4	0.15	0.14	0.14
Q5(wealthy)	0.07	0.07	0.07	Q5(wealthy)	0.07	0.07	0.08

Table 12 shows the share of liquid assets as a fraction of total wealth across wealth quintiles in the steady state and on impact dates following a TFP shock with a rise in disaster risk.

Table 13 and 14 summarize the shares of households who actively adjust their illiquid assets and the share of such adjustors, in each quintile, that actually disinvest illiquid assets, respectively. Comparing the share of adjustors in the fifth quintile to that in the first quintile, wealthy households are generally more willing to adjust their asset portfolios as they can afford frequent transaction costs. While the share of adjustors changes little over the business cycle, Table 14 shows that more adjusting households monetize their illiquid assets in a recession. For instance, in the model with a rigid real return on liquid savings, the fraction of adjustors disinvesting in illiquid assets, in the recession, increases by 4 percentage points relative to the steady state. While poor households overall increase their liquid assets, a higher fraction of wealth poor adjustors increase their illiquid assets in a recession compared to the steady state as the price of illiquid assets falls.

with a fall in the rate of return on liquid savings, a rise in disaster risk still boosts their precautionary savings compared to a recession without a rise in the probability of disaster. Indeed, without disaster risk, the start of a recession sees the share of liquid savings to net worth rise to 20 percent (fixed return model) and 13 percent (fixed stock model).

Table 13. Share of households who actively adjust their portfolios across wealth quintiles

B fixed	steady state	impact dates		r_f fixed	steady state	impact dates	
	t=0	t=1	t=2		t=0	t=1	t=2
all	0.15	0.15	0.15	all	0.14	0.16	0.15
Q1(poor)	0.06	0.07	0.08	Q1(poor)	0.06	0.07	0.08
Q2	0.12	0.11	0.11	Q2	0.12	0.11	0.11
Q3	0.14	0.14	0.13	Q3	0.13	0.14	0.13
Q4	0.15	0.15	0.15	Q4	0.15	0.16	0.15
Q5(wealthy)	0.27	0.28	0.29	Q5(wealthy)	0.26	0.29	0.28

Table 13 shows the share of households who actively adjust their illiquid assets over wealth quintiles in the steady state and during impact dates.

Table 14. Share of adjustors in each quintile that disinvest in illiquid assets

B fixed	steady state	impact dates		r_f fixed	steady state	impact dates	
	t=0	t=1	t=2		t=0	t=1	t=2
Q1(poor)	0.10	0.07	0.09	Q1(poor)	0.10	0.08	0.08
Q2	0.51	0.47	0.47	Q2	0.52	0.49	0.48
Q3	0.69	0.71	0.71	Q3	0.71	0.73	0.72
Q4	0.79	0.83	0.83	Q4	0.82	0.85	0.83
Q5(wealthy)	0.82	0.85	0.86	Q5(wealthy)	0.82	0.86	0.86

6.3 Changes in the joint distribution of net worth, income, and consumption before and during the Great Recession

In this section, I examine the heterogeneous wealth composition economies for the dynamics of net worth, income, and consumption across households, comparing to the PSID before and during the Great Recession. I first compare changes in the joint distribution in the data to those in heterogeneous wealth composition economies without disaster risk. Next, I discuss how disaster risk improves the models prediction on the dynamics of the cross-sectional distributions.

6.3.1 Recession without rising disaster risk

I simulate 60,000 households for several periods in (1) a heterogeneous composition wealth model with a constant return to liquid savings and (2) a model with a fixed stock of liquid assets. Table 15 compares annualized growth rates of the average level of net worth, earnings, income, and consumption to those in the PSID between 2005 and 2007 over the 2005 wealth quintiles. In the last column of Table 15, I also report the percentage points changes in expenditure rates defined as the fraction of total consumption to income. Note that growth rates of variables during normal times in data are comparable to those in a steady state in model economies.[47]

Table 15 shows that the model economies with multiple assets successfully explain some stylized facts seen in the PSID before the Great Recession. First, net worth growth rates fall sharply in the level of wealth. For example, it explains the more than 10 percent rise in net worth for households in the second quintile compared to the small rise in net worth for the wealthiest 20 percent of households. Second, growth rates of income and earnings in model economies are broadly consistent with those in the PSID, generating the highest growth rates for the bottom 20 percent of households. Falling growth rates of net worth, earnings, and income, in the level of wealth, is mainly driven by high mean-reversion in earnings shocks. This mean-reversion in shocks also leads to a higher growth in consumption for

[47] In Table 12, the minor difference in the distribution of households drives slightly different measures between the two model economies.

the bottom compared to that for the top. However, as consumption rises less than income, expenditure rates fall for all households but the top 20 percent.

Table 15. Growth rates of variables across wealth quintiles before the Great Recession 2005-2007

	Net worth			Earning			Income			Expend.			Exp. Rate (pp)		
Quantile	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)
Q1(*poor*)	n/a	n/a	n/a	3.3	9.8	9.3	3.3	9.8	9.9	11	7.0	6.9	3.6	-1.4	-1.5
Q2	25	10.9	11.2	1.7	5.8	5.4	1.9	5.8	5.8	7.1	3.0	3.1	2.3	-1.7	-1.6
Q3	22	2.9	3.0	1.8	2.5	2.4	2.0	2.5	2.5	3.0	0.8	0.9	0.5	-1.1	-1.0
Q4	11	1.3	1.4	1.3	-0.1	-0.5	2.9	-0.1	-0.1	2.9	-2.1	-2.0	0.0	-1.4	-1.3
Q5(*wealthy*)	2.1	0.2	0.2	2.6	-2.1	-2.6	1.2	-2.1	-2.1	4.3	0.2	-0.2	1.5	1.6	1.6

Notes : Table 15 shows annualized growth rates of the average net worth, earnings, and income and percentage points change in expenditure rates from the PSID and in portfolio choice economies without disaster risk. (1) a model with fixed return on liquid assets. (2) a model with fixed stock of liquid assets.

Table 16 compares the corresponding growth rates between 2007 and 2011 over the 2007 wealth quintiles in the PSID to those in the model economies following a continuous drop in TFP, accumulating to an overall 2.18 percent across two periods. The model economies successfully reproduce a concurrent slowdown in the growth of net worth, earnings, and income compared to normal times (Table 15). Moreover, it explains a fall in the level of net worth, earnings and income for those in the highest quintile of the wealth distribution. The growth rates of the average consumption across the quintiles of the wealth distribution decline in a recession, reproducing an actual drop in the consumption for the wealthiest 60 percent of households. The model economies also explain a rise in savings rates for households in the first four quintiles as well as a rise in consumption rate for the top 20 percent of households. A rise in consumption rate for the top is due to the fact that wealthy households are likely to be the old, who have less desire to save as they have a shorter remaining lifetime compared to the young.

To further evaluate the model's predictions for the severity of the Great Recession across the wealth distribution, Table 17 documents the first difference in growth rates

Table 16. Growth rates of variables across wealth quintiles during the Great Recession 2007-2011

	Net worth			Earning			Income			Expend.			Exp. Rate (pp)		
Quantile	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)
Q1(*poor*)	n/a	n/a	n/a	2.1	6.0	6.0	2.5	7.9	7.9	0.2	4.9	5.4	-2.3	-1.6	-1.3
Q2	11	9.9	10.1	1.1	2.7	2.9	0.8	4.1	4.2	1.9	1.6	1.8	1.0	-1.6	-1.4
Q3	-5.1	1.8	2.0	-0.3	0.1	0.2	-0.1	1.1	1.0	-0.2	-0.6	-0.2	-0.1	-1.1	-0.8
Q4	-0.5	0.3	0.4	-0.7	-2.3	-2.4	-0.1	-1.4	-1.6	-1.6	-3.3	-3.0	-1.7	-1.4	-1.0
Q5(*wealthy*)	-3.5	-0.8	-0.8	0.0	-4.3	-4.2	0.3	-3.4	-3.5	-2.5	-0.8	-0.7	-2.8	1.8	2.1

Notes : Table 16 shows annualized growth rates of the average net worth, earnings, and income and percentage points change in expenditure rates from the PSID and in portfolio choice economies without disaster risk. (1) a model with fixed return on liquid assets. (2) a model with fixed stock of liquid assets.

between normal times and recession in model to the data. While not targeted, the two model economies explain a slowdown in the growth of net worth, earnings, income, and consumption in a recession compared to a normal time. Although the model economies explain fall in consumption rates across the wealth distribution in a recession (Table 16), it fails to explain changes in the growth of expenditure rates between normal times and recession as seen in the last column of Table 17. For example, the PSID data shows that a fall in consumption rates for households across wealth quintiles is pronounced in the Great Recession compared to normal times while the model economies predicts a little drop or rise in consumption rate in a recession.

Table 17. Changes in growth rates of variables between prior- and during the Great Recession

	Net Worth			Earning			Income			Expend.			Exp. Rate (pp)		
Quantile	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)
Q1(*poor*)	n/a	n/a	n/a	-1.2	-3.8	-3.3	-0.8	-1.9	-2.0	-11	-2.1	-1.5	-5.9	-0.2	0.2
Q2	-14	-1.0	-1.1	-0.6	-3.1	-2.5	-1.1	-1.7	-1.6	-5.2	-1.4	-1.3	-1.3	0.1	0.2
Q3	-27	-1.1	-1.0	-2.1	-2.4	-2.2	-2.1	-1.5	-1.5	-3.2	-1.4	-1.1	-0.6	0.0	0.2
Q4	-12	-1.0	-1.0	-2.0	-2.2	-1.9	-3.0	-1.3	-1.5	-4.5	-1.2	-1.0	-1.7	0.0	0.3
Q5(*wealthy*)	-6	-1.0	-1.0	-2.6	-2.2	-1.6	-0.9	-1.3	-1.4	-6.8	-1.0	-0.9	-4.3	0.2	0.5

Notes : Table 17 shows changes in annualized growth rates of the average net worth, earnings, and income and percentage points change in expenditure rates from the PSID and in portfolio choice economies without disaster risk. (1) a model with fixed return on liquid assets. (2) a model with fixed stock of liquid assets.

6.3.2 With disaster risk

Table 18 reports the annualized growth rates of net worth, earnings, income and consumption in the 2007-2010 PSID as well as from the simulated heterogeneous wealth composition economies with disaster risk.[48] While growth rates in net worth, earnings, and income are broadly similar to those in Table 16, the slowdown in consumption growth in a recession is exacerbated by a rise in disaster risk, especially when the return to liquid assets is fixed. For example, in the model with fixed returns, consumption for the bottom 20 percent of households rises by 4.1 percent in a recession with a rise in disaster risk compared to 4.9 percent without disaster risk (Table 16). As explained earlier, this is driven by a higher expected return on savings in the future compared to the current average return alongside with a strong negative wealth effect following a rise in disaster risk.

Table 18. Growth rates of variables across wealth quintiles during the Great Recession with rising disaster risk

	Net worth			Earning			Income			Expend.			Exp. Rate (pp)		
Quantile	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)
Q1(*poor*)	n/a	n/a	n/a	2.1	7.3	7.4	2.5	7.8	7.8	0.2	4.1	5.4	-2.3	-2.0	-1.2
Q2	11	9.7	10.6	1.1	3.7	3.9	0.8	4.3	4.0	1.9	0.9	1.9	1.0	-2.1	-1.3
Q3	-5.1	1.8	2.2	-0.3	1.0	1.0	-0.1	1.2	0.7	-0.2	-1.0	-0.1	-0.1	-1.5	-0.5
Q4	-0.5	-0.1	0.5	-0.7	-1.7	-1.6	-0.1	-1.1	-1.8	-1.6	-3.8	-2.9	-1.7	-1.9	-0.8
Q5(*wealthy*)	-3.5	-1.1	-0.8	0.0	-3.9	-3.8	0.3	-3.0	-3.7	-2.5	-1.1	-0.7	-2.8	1.4	2.2

Notes : Table 18 shows annualized growth rates of the average net worth, earnings, and income and percentage points change in expenditure rates from the PSID and in portfolio choice economies with disaster risk. (1) a model with fixed return on liquid assets. (2) a model with fixed stock of liquid assets.

Table 19 shows the annualized changes in the joint distribution between normal times and a recession. Involving a rise in disaster risk when the return on liquid savings is constant, this model economy predicts a more marked fall in consumption rates. This implies that a rise in disaster risk not only explains the aggregate response of the economy seen in the data but also the micro-prediction of the economy. In a typical recession with a single asset, households' consumption smoothing desire increases consumption rates. However,

[48] Note that the growth rates in normal times are same as those reported in Table 13.

in the recent recession, consumption rates sharply fall across households, which can be explained with a rise in the probability of a further worsening of earnings with a rigid return on liquid savings. This suggests the importance of disaster risk that amplifies the effects of a shock to TFP with multiple assets of varying liquidity.

Table 19. Changes in growth rates of variables between prior- and during the Great Recession with rising disaster risk

	Net Worth			earning			Income			Expend.			Exp. Rate (pp)		
Quantile	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)
Q1(*poor*)	n/a	n/a	n/a	-1.2	-2.5	-1.9	-0.8	-2.0	-2.1	-11	-2.9	-1.5	-5.9	-0.6	0.3
Q2	-14	-1.2	-0.6	-0.6	-2.1	-1.5	-1.1	-1.5	-1.8	-5.2	-2.1	-1.2	-1.3	-0.4	0.3
Q3	-27	-1.1	-0.8	-2.1	-1.5	-1.4	-2.1	-1.3	-1.8	-3.2	-1.8	-1.0	-0.6	-0.4	0.5
Q4	-12	-1.4	-0.9	-2.0	-1.6	-1.1	-3.0	-1.0	-1.7	-4.5	-1.7	-0.9	-1.7	-0.5	0.5
Q5(*wealthy*)	-6	-1.3	-1.0	-2.6	-1.8	-1.2	-0.9	-0.9	-1.6	-6.8	-1.3	-0.5	-4.3	-0.2	0.6

Notes : Table 19 shows changes in annualized growth rates of the average net worth, earnings, and income and percentage points change in expenditure rates from the PSID and in portfolio choice economies with disaster risk. (1) a model with fixed return on liquid assets. (2) a model with fixed stock of liquid assets.

7 Concluding Remarks

Solving an overlapping generations economy with uninsurable earnings, unemployment and liquidity risk in dynamic stochastic general equilibrium, I have examined the implications of household-level differences in the level and liquidity of wealth for aggregate dynamics and changes in the distribution of consumption, income and wealth across households. In particular, I quantify the role of household heterogeneity in the composition of wealth for understanding differences in their responses during the Great Recession. This channel is amplified when households increase precautionary savings, and substitute liquid for illiquid assets, following a rise in disaster risk.

Liquidity varies across assets as higher returns involve investments associated with random transactions cost. Calibrating the distribution of liquidity costs to the real return on liquid and illiquid assets in the data, the model economy reproduces the empirical share of illiquid assets to net worth of households of different levels of wealth and age. Moreover, the model explains much of the distribution in net worth, illiquid wealth, and liquid wealth seen in the data.

The implications of adding realistic heterogeneity not only in the level of wealth, but also in its liquidity, are important for an understanding of both aggregate and household-level changes over large recessions. In such recessions, a fall in earnings and a rise in unemployment risk force high-yield asset investors to monetize their wealth to smooth consumption, resulting in a sharp drop in investment. During the recovery, as fixed transaction costs lead households to tolerate lower than desired shares of illiquid assets in their portfolios, the economy with wealth composition heterogeneity recovers slowly. Most importantly, precautionary savings motives, following an increase in the probability of a large disaster, yield powerful changes in household behavior. These changes in households' consumption and expenditure rates are consistent with stylized facts seen in the joint distribution of consumption, income and net worth across households over the Great Recession.

In the next version of this paper, I will use the empirical cyclicality of liquidity premia to determine the cyclicality of the supply of liquid assets over the business cycle. This will provide further information on the role of household portfolio substitutions, associated with an increase in precautionary savings, in large recessions.

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A Data appendix

A.1 2007-2009 SCF panel data

The SCF is household triennial data survey conducted by the Federal Reserve Board. The SCF provides detailed information on assets and debts of households. The SCF also employs a list sample design based on the IRS data to provide disproportionate sampling of wealthy households. Historically, the SCF had panel data from 1983 to 1989 and recently released two-year panel data for 2007 and 2009.

Net worth is defined as total assets minus total debt. Total assets include both financial and non-financial assets. Total financial assets include transaction accounts, certificates of deposit, directly held pooled investment funds, savings bonds, directly held stocks and bonds, cash value of life insurance, quasi-liquid retirement accounts, and other managed and miscellaneous financial assets. Total non-financial assets include vehicles, residential property, net equity in non-residential real estate, business equity, and other miscellaneous non-financial assets. Total debt consists of debt secured by residential property, credit card balances since last payment, installment loans, and other debts. I select households with a head who is between 25 and 85 years old and not self-employed. All the variables are deflated and expressed in 2013 dollars. Full sample weights are used.

A.2 PSID data

The PSID is a longitudinal survey of a sample of US individuals and families conducted annually from 1968 to 1977, and biennially since 1997. The original 1968 PSID sample combines the Survey Research Center (SRC) and the Survey of Economic Opportunities (SEO) samples. I use the U.S population representative SRC sample.

For greater consistency with the SCF, I select households with a head who is between 25 and 85 years old and drop the self-employed. I drop a sample observation if income is positive but annual hours worked is zero. If income is top-coded, I multiply it by a factor of 1.5 of the top-coded threshold following Katz et al. (1999). All variables are deflated and expressed in 2013 dollars using IPUMS-CPI.

The PSID provides disaggregated data on wealth since 2003, consisting of business and farm equity, transaction accounts, equity in real estate and vehicles, stocks, bonds, IRA, and debt. As shown in Table 1, the distributions of wealth in the PSID are comparable to those in the SCF. Table A1 also summarizes averages in the PSID and SCF.

Table A1. Summary statistics of data

Data	SCF		PSID	
year	2007	2009	2007	2009
Sample size	13,085	13,085	2,524	2,524
Age of head	49	51	42	44
Total wealth	395,750	325,060	344,161	283,959
Head income	75,336	71,949	72,365	72,690

Notes : All variables are expressed in 2013 dollars. For the PSID, drop three samples with wealth less than negative 99 million dollars. (Source: 2007-2009 PSID and SCF)

The PSID provides data on households' consumption. In addition to food and housing, the PSID included items on transportation, health care, education, utilities, and child care since 1999. In 2005, additional items such as household furnishing and equipment, clothing and apparel, trips and vacations, and recreation and entertainment were added.

I construct total expenditure in the PSID as the sum of nondurable goods and services.[49] The PSID measures total spending on each item for the family. Since each item has a different reporting time unit, I adjust it to an annual measure. Reporting time unit varies by samples for food delivered, food eaten out, and food at home. Approximately half of respondents reported it weekly while the rest report it monthly or biweekly. I measure annual spending on these items based on individual reported time units. Following Krueger et al. (2016), I imputed the amount of rental services from home owners by multiplying the value of the main residence by 4 percent. Imputed rent and property taxes are included in expenditure on housing to be consistent with the BEA measure. Given that reported income is earned in the preceding year, there may exist time inconsistency

[49]Durable goods include motor vehicles and parts, furnishings and equipment, recreational goods and vehicles, and other durable goods.

between consumption and income.

In Table A2 and A3, I compare composition of total expenditure in the PSID to that in the BEA to comprehend the representativeness of the former micro data for macro aggregates.[50] I used NIPA Table 2.3.6 Real Personal Consumption Expenditures in chain 2009 dollars (seasonally adjusted).[51] Since the BEA measures current year consumption while the PSID reporting time unit varies by item, I take the average of the current and preceding years of the expenditure in the BEA to make the time unit closer to the expenditure measured in the PSID. Moreover, I divide total expenditure in PSID by total family size and multiply the consumption per capita by the population size to make household consumption in PSID comparable to Personal Consumption Expenditure in BEA, Table A2 shows that the PSID aggregate consumption accounts for more than 50 percent of total expenditure in the BEA. In Table A3, spending on each item as a share of total expenditure in the PSID is broadly comparable to that in the BEA. This suggests that, while micro data captures less expenditure than aggregate data in total, the dynamics in consumption at individual level may be explained by micro data. For instance, Krueger et al. (2016) shows that the growth rates of the PSID aggregate closely follow BEA total spending, providing further support for using micro consumption data to measure individual growth rates.

[50] Total spending in the recent PSID is comparable to that in the CEX. Indeed, Charles et al. (2007) find that the 2003 PSID covers 72 percent of total expenditure measured in CEX.

[51] Given the issue raised by chain-weighted dollars, I express PSID variables in 2009 dollars for Table A2 and A3.

Table A2. Composition of total expenditure in the BEA and PSID

	2005		2007		2009	
billions of 2009 dollars	BEA	PSID	BEA	PSID	BEA	PSID
Nondurable goods	2,098	1,397	2,208	1,497	2,243	1,350
food	744	900	786	950	776	916
clothing	299	220	321	207	314	181
gasoline	299	278	297	340	284	253
other	753	n/a	814	n/a	821	n/a
Services	6,255	3,039	6,592	3,555	6,679	3,404
housing and utilities	1,753	1,371	1,832	1,843	1,871	1,689
health care	1,466	300	1,544	345	1,613	366
transportation	333	280	337	296	306	285
recreation	360	122	385	123	383	126
food services	595	n/a	629	n/a	613	n/a
financial services and insurance	686	265	731	238	728	199
other services	856	701	895	711	891	739
total	8,353	4,436	8,812	5,052	8,873	4,754

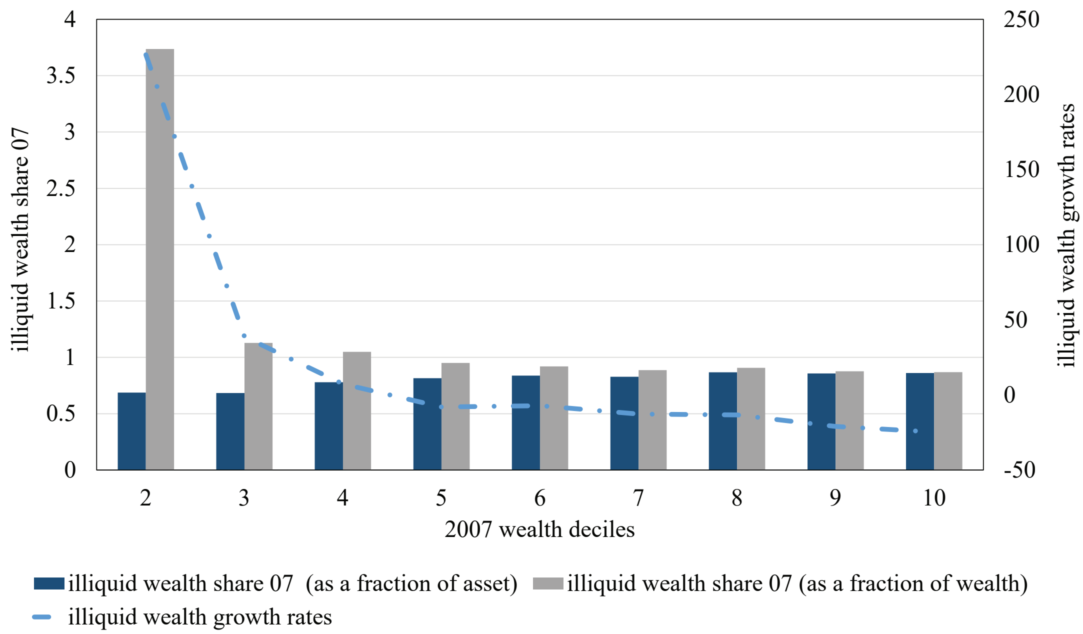
Notes : Variables in the BEA are expressed in chain-weighted 2009 dollars. Variables in the PSID are expressed in 2009 dollars. Other services include chicle care, education, communication and vehicle services.

Table A3. Spending as a fraction of total expenditure in the BEA and PSID

	2005		2007		2009	
% of total expenditure	BEA	PSID	BEA	PSID	BEA	PSID
Nondurable goods	25.1	31.5	25.1	29.6	25.3	28.4
food	8.9	20.3	8.9	18.8	8.8	19.3
clothing	3.6	5.0	3.6	4.1	3.5	3.8
gasoline	3.6	6.3	3.4	6.7	3.2	5.3
other	9.0	n/a	9.2	n/a	9.3	n/a
Services	74.9	68.5	74.8	70.4	75.3	71.6
housing and utilities	21.0	30.9	20.8	36.5	21.1	35.5
health care	17.6	6.8	17.5	6.8	18.2	7.7
transportation	4.0	6.3	3.8	5.9	3.5	6.0
recreation	4.3	2.8	4.4	2.4	4.3	2.7
food services	7.1	n/a	7.1	n/a	6.9	n/a
financial services and insurance	8.2	6.0	8.3	4.7	8.2	4.2
other services	10.3	15.8	10.2	14.1	10.0	15.5
total	100	100	100	100	100	100

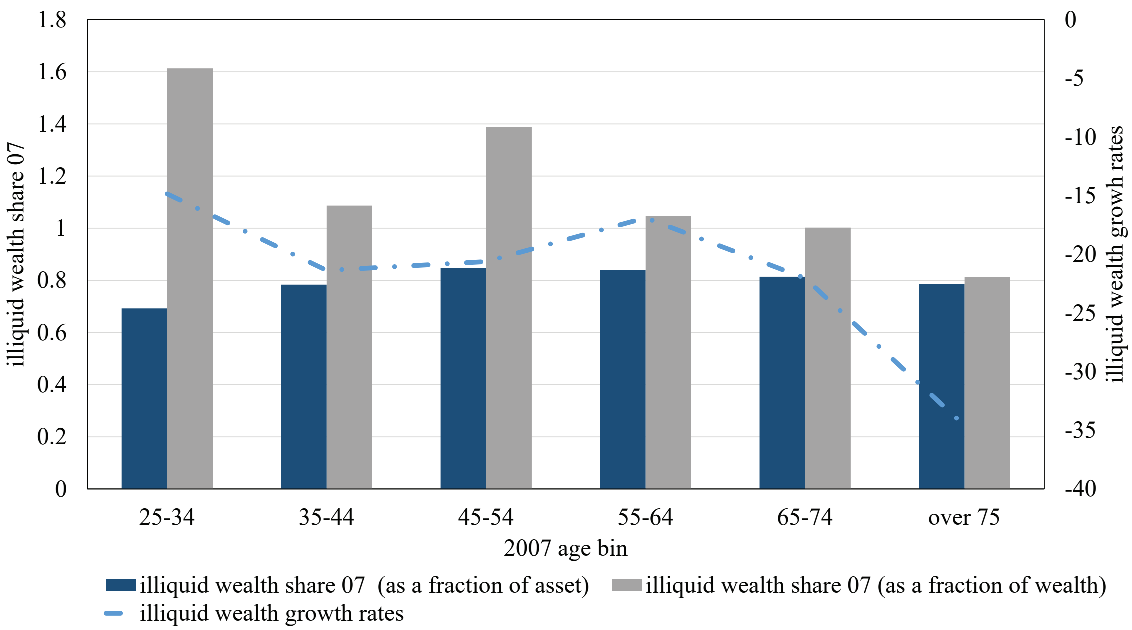
Notes : Variables in the BEA are expressed in chain-weighted 2009 dollars. Variables in the PSID are expressed in 2009 dollars. Other services include chicle care, education, communication and vehicle services.

Figure A1. Illiquid wealth share and risky wealth growth rate over 2007 wealth deciles



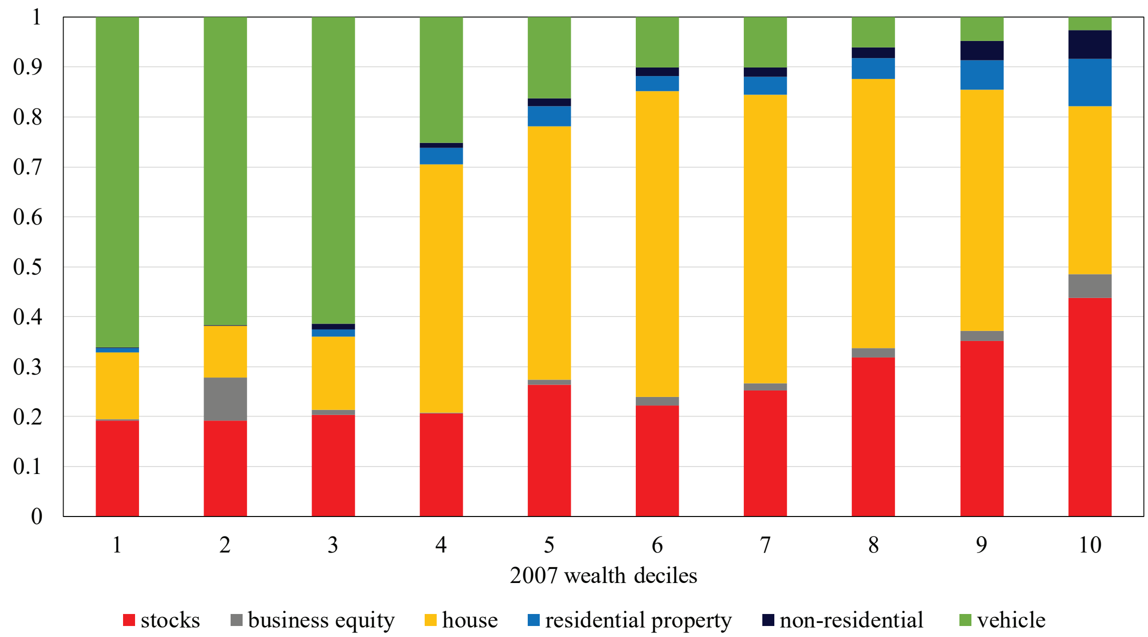
Notes : The average illiquid asset share as a fraction of total asset (blue bars) and as a fraction of net worth (grey bars) in 2007. This measure only considers samples who invest in illiquid wealth. Exclude samples with negative net worth (asset) in 2007 for net worth (asset) share measures or with net worth less than 0.1 percent of the average level of wealth. The growth rates of illiquid wealth between 2007 and 2009 (dashed line). Exclude samples with negative values in any sample period for growth rate measures. (Source: 2007-2009 SCF panel)

Figure A2. Illiquid wealth share and risky wealth growth rate over 2007 age groups



Notes : The average illiquid asset share as a fraction of total asset (blue bars) and as a fraction of net worth (grey bars) in 2007. This measure only considers samples who invest in illiquid wealth. Exclude samples with negative net worth (asset) in 2007 for net worth (asset) share measures or with net worth less than 0.1 percent of the average level of wealth. The growth rates of illiquid wealth between 2007 and 2009 (dashed line). Exclude samples with negative values in any sample period for growth rate measures. (Source: 2007-2009 SCF panel)

Figure A3. The composition of illiquid wealth in 2007



Notes : The average share of each illiquid wealth as a fraction of total illiquid wealth. Exclude samples with negative or zero risky wealth. (Source: 2007-2009 SCF panel)

B Competitive Investment Firm

A competitive investment firm has the technology that creates capital. It holds k units of capital created last period after selling $p(z,\mu)k$ in shares to households. An investment firm rents this capital to a production firm at the rental rate $r^k(z_f,\mu)$ and returns the current dividend value of shares to households. It also faces convex capital adjustment cost $\Phi(k',k)$. A competitive investment firm chooses k' to maximize its profit

$$J(k,z_f,\mu) = \max_{k'}\Bigg((r^k(z_f,\mu)+1-\delta)k - (p(z_f,\mu)+d(z_f,\mu))k \\ + p(z_f,\mu)k' - k' - \Phi(k',k) + \sum_{g=1}^{n_z}\pi_{fg}r(z_g,z_f,\mu)J(k',z_g,\mu')\Bigg) \quad (4)$$

where investment firm discounts future earnings by the marginal rate of substitution of households, $r(z_g,z_f,\mu)$.

The investment firm's optimal choices satisfy the following first-order condition

$$p(z_f,\mu) - 1 - \Phi_1(k',k) + \sum_{g=1}^{n_z}\pi_{fg}r(z_g,z_f,\mu)D_2J(k',z_g,\mu') = 0 \quad (5)$$

and the Benvensite-Scheinkman condition provides

$$D_2J(z_f,k,\mu) = r^k(z_f,\mu) + 1 - \delta - (p(z_f,\mu)+d(z_f,\mu)) - \Phi_2(k',k). \quad (6)$$

Assuming perfect competition for an investment firm, zero profit condition, $D_2J(k',z_g,\mu') = 0$, determines the equilibrium price of capital and dividends as follows.

$$p(z_f,\mu) = 1 + \Phi_1(k',k) \quad (7)$$

$$d(z_f,\mu) = r^k(z_f,\mu) - \delta - \Phi_1(k',k) - \Phi_2(k',k) \quad (8)$$

C Equilibrium Ex-dividend Price and Dividends

In this section, I show that definitions of ex-dividend price and dividend are consistent with equilibrium and imply the aggregate resource constraint. Here, I simplify the model by abstracting from government and liquid asset market.

Aggregate budget constraint for all households who are adjusting illiquid wealth is as follows:

$$c_a + p(z,\mu)k'_a \leq w_a(z,\mu) + (p(z,\mu) + d(z,\mu))k_a - \zeta_a$$

where x_a is the sum of a variable x for all households who are adjusting and $w_a(z,\mu)$ is the labor income for all active adjustors.

Using definitions of ex-dividend price and dividend, above aggregate budget constraint can be re-written as:

$$c_a + (1 + \Phi_1(k',k))k'_a \leq w_a(z,\mu) + (\alpha\eta k^{\alpha-1}n^{1-\alpha} - \delta - \Phi_2(k',k))k_a - \zeta_a \tag{9}$$

where k is the aggregate stock of capital.

Likewise, aggregate budget constraint for all households who are non-adjusting illiquid wealth is:

$$c_n \leq w_n(z,\mu) \tag{10}$$

$$k'_n = (1 + \alpha\eta k^{\alpha-1}n^{1-\alpha} - \delta - \Phi_1(k',k) - \Phi_2(k',k))k_n \tag{11}$$

Imposing $x_a + x_b = x$ and $\Phi_1(k',k)k' + \Phi_2(k',k)k = \Phi(k',k)$, equations (4)-(6) imply the aggregate resource constraint

$$c + k' + \Phi(k',k) \leq y + (1-\delta)k - \zeta_a$$

D Numerical Method

D.1 Steady state

Stationary equilibria involve finite horizon dynamic programming problems with two endogenous state variables - illiquid wealth, a, and liquid wealth, b. The state space is six-dimensional: age, illiquid wealth, liquid wealth, unemployment shock, persistent and transitory earnings shocks. Solving the model with two endogenous state variables involves a significant amount of memory (RAM) to store decision rules as well as a lot of computation time. In solving the model, I combine unemployment shock, persistent and transitory shock processes into a single shock process ε, decreasing two dimension. More importantly, I develop a two-stage approach to solve savings decisions with two assets. In the first stage, given a current fixed cost, a household chooses whether or not to adjust its portfolio. If it adjusts, it chooses its savings in illiquid wealth, a'. In the second stage, given a', I solve for the optimal choice of liquid wealth, b', using endogenous grid method (Carroll, 2006).[52]

As the aggregate supply of liquid wealth is calibrated to match a rate of return of zero percent on liquid wealth, solving for stationary equilibria involves three prices (w, p, d). Note that, in the absence of endogenous labor supply, wages are determined by the aggregate stock of capital. Moreover, p is fixed to one given equilibrium price function as there is no aggregate capital adjustment cost in a steady state. Given the initial guess of prices, I compute decision rules and the distribution of households. The latter is determined using a large grid over age, idiosyncratic shock, illiquid and liquid wealth. I use bilinear interpolation to place decision rules onto this grid. I update prices by bisecting for the aggregate capital, iterating through the above steps, until prices converge.

D.2 Decision rules

I solve the household's problem in two stages. Here, I abstract from aggregate states for the ease of notation. In the first stage, it has decided its illiquid wealth for the next period

[52] The prime denotes variables in the next period.

, a'. This may be $(1+(1-\tau_a)d)a$ if a household chooses not to adjust, or it is the result of an active portfolio adjustment choice for a' after paying the fixed cost ζ. Below, I describe the household's problem at age j with illiquid asset a, liquid wealth b, and productivity (working status) ε. Define v_j^0 as the intermediate value defined over cash-on-hand, m, the future stock of illiquid wealth a', and current productivity.

The illiquid wealth problem

$$v_j\left(a, b, \varepsilon_i, \zeta\right) = \max\left\{\max_{0\leq a'\leq m} v_j^0\left(m - pa', a', \varepsilon_i\right), v_j^0\left(x_i + b, \left(1 + (1-\tau_a)d\right)a, \varepsilon_i\right)\right\} \tag{12}$$

subject to

$$m = x_i(j, \varepsilon) + \left(p + (1-\tau_a)d\right)a + b - \zeta$$

Note that if a household chooses to adjust illiquid wealth to a', the remaining cash-on-hand for consumption and liquid wealth in the second stage is $m - pa'$. However, it is able to cash in its current stock of illiquid wealth, a. If a household does not pay its fixed cost, it can not adjust the current stock of illiquid wealth and the cash available for liquid wealth and consumption is the sum of labor income if working or pension benefit if retired and the current stock of liquid wealth.

In the second stage, a household has already decided its illiquid wealth for the next period, a'. Given a' and remaining cash-on-hand m, a household solves the problem below.

The consumption and liquid wealth problem

$$v_j^0\left(m, a', \varepsilon_i\right) = \max_{b'}\Big(u\left(c\right) + \beta v_j^e\left(a', b', \varepsilon_i\right)\Big) \tag{13}$$

subject to

$$c + qb' \leq m$$

where v_j^e represents the expected value of a household at the beginning of the next period before fixed cost is drawn.

D.3 Aggregate Dynamics

To solve the model with aggregate uncertainty, I extend the Backward induction method of Reiter (2002, 2010) to solve a stochastic overlapping-generations economy. This involves generalizing the method to handle bivariate cross-sectional distributions of endogenous state variables. The Backward induction method of Reiter allows the distribution of households to vary in potentially rich ways as a function of an approximate aggregate state as it does not impose a parametric aggregate law of motion. Moreover, this method does not involve repeated simulation, reducing computation time compared to Krusell and Smith (1998).[53][54]

Backward induction method of Reiter (2002, 2010) selects a *proxy distribution* across a grid of approximate aggregate state based on *distribution selection function* (DSF) which maps approximate aggregate states to cross-sectional distributions. A DSF selects the proxy distribution that minimizes the distance to the reference distribution subject to moment consistency conditions.[55] Solving for the DSF entails solving a large system of linear equations. With proxy distributions solved, backward induction simultaneously solves for households' decision rules and an end-of-period distribution implying a future approximate aggregate state consistent with households' expectations. This enforces consistency between individual behavior and the aggregate law of motion. Lastly, I simulate the model economy and weight simulated distributions using an inverse quadratic to update the reference distributions, and thus the DSF.

This paper contributes to the backward induction method of Reiter (2002, 2010) in two ways. First, I solve a stochastic OLG economy involving distributions defined over

[53] Both Krusell and Smith (1998) and Reiter (2002, 2010) solve households' decisions over approximate aggregate states which summarizes infinite-dimensional cross-sectional distributions using a finite vector of moments.

[54] Krusell and Smith (1998) assume a parametric function to forecast an approximate aggregate state. Though Krusell and Smith (1998) update forecast rules based on realistic simulation-generated distributions, it is critical to have a long simulation to avoid sampling errors. These repeated long period simulations make it costly to solve a model with rich distribution of households using the Krusell and Smith's method.

[55] I initially use the steady state distribution as a reference distribution.

60 different cohorts. The distribution of households over age and idiosyncratic shocks as well as two assets increases the dimension of the system of equations solved for proxy distributions making it intractable. To mitigate this problem, I aggregate full reference distributions over age and idiosyncratic shocks into a small subset of age and idiosyncratic type groups and calculate the weights mapping full distributions to aggregated ones before solving for proxy distributions. These weights keep the shape of proxy distributions over age and idiosyncratic shocks conditional on the level of wealth close to that of the original full distributions. Second, I solve the model with two endogenous state variables - illiquid and liquid wealth. To make the solution feasible, I solve the model with aggregate uncertainty over asset grids with a lower number of grid points than used for the steady state, both for decision rules and distributions. Having finished solving this model, I simulate the model economy over finer grids to have more accurate solution for households' decisions.

I summarize the outline of the algorithm as follows:

(1) Approximate the cross-sectional distribution in the aggregate state, $z = \{z_1, ..., z_{n_z}\}$, with a finite vector of statistics (moments) $m = \{m_1, ..., m_{n_m}\}$. Here, I assume m_i is i^{th}-moment.

(2) Determine asset grids for decision rules and distributions with a lower number of grid points, $A = \{a_1, .., a_{n_a}\}$ and $B = \{b_1, .., b_{n_b}\}$, keeping bounds the same as those in the steady state. These grids are used until the backward induction is solved.

(3) Aggregate the full reference distribution, $r^{\mu}(j, a, b, \varepsilon; z, m)$, across all age groups and a small subset of idiosyncratic types $n_{\tilde{\varepsilon}} \leq n_{\varepsilon}$, resulting in the reduced distribution, $r_0^{\mu}(a, b, \tilde{\varepsilon}; z, m)$ where $\tilde{\varepsilon} \in \{\tilde{\varepsilon}_1, ..., \tilde{\varepsilon}_{n_{\tilde{\varepsilon}}}\}$, and calculate weights of this mapping.

$$\omega_0(j, a, b, \varepsilon, \tilde{\varepsilon}; z, m) : r_0^{\mu}(a, b, \tilde{\varepsilon}; z, m) \rightarrow r^{\mu}(j, a, b, \varepsilon; z, m)$$

(4) Choose a DSF which gives the proxy distribution, $p_0^{\mu}(a, b, \tilde{\varepsilon}; z, m)$. I solve for $p_0^{\mu}(a, b, \tilde{\varepsilon}; z, m)$ as the solution to a problem that minimizes the distance to the reduced distribution $r_0^{\mu}(a, b, \tilde{\varepsilon}; z, m)$ while imposing that each $\tilde{\varepsilon}$ sums to its reference value and moment consistency constraints. For each approximate aggregate state (z, m), a DSF solves :

$$\min_{\{p_0^\mu(a_i,b_k,\tilde{\varepsilon}_l)\}_{i=1,k=1,l=1}^{n_a,\,n_b,\,n_{\tilde{\varepsilon}}}} \sum_{i=1}^{n_a}\sum_{k=1}^{n_b}\sum_{l=1}^{n_{\tilde{\varepsilon}}} \left(p_0^\mu(a_i,b_k,\tilde{\varepsilon}_l) - r_0^\mu(a_i,b_k,\tilde{\varepsilon}_l)\right)^2$$

subject to

$$\sum_{i=1}^{n_a}\sum_{k=1}^{n_b} p_0^\mu(a_i,b_k,\tilde{\varepsilon}_l) = \sum_{i=1}^{n_a}\sum_{k=1}^{n_b} r_0^\mu(a_i,b_k,\tilde{\varepsilon}_l),\ \ l = 1,\dots,n_{\tilde{\varepsilon}} \tag{14}$$

$$\sum_{i=1}^{n_a}\sum_{k=1}^{n_b}\sum_{l=1}^{n_{\tilde{\varepsilon}}} p_0^\mu(a_i,b_k,\tilde{\varepsilon}_l)a_i^{i_m} = m_{i_m}^a,\ \ i_m = 1,\dots,n_m \tag{15}$$

$$\sum_{i=1}^{n_a}\sum_{k=1}^{n_b}\sum_{l=1}^{n_{\tilde{\varepsilon}}} p_0^\mu(a_i,b_k,\tilde{\varepsilon}_l)b_k^{i_m} = m_{i_m}^b,\ \ i_m = 1,\dots,n_m \tag{16}$$

$$p_0^\mu(a_i,b_k,\tilde{\varepsilon}_l) \geq 0,\ \forall\ i,k,l$$

where equation (14) represents type consistency conditions. Equations (15) and (16) are moment consistency constraints for both assets. Lastly, probabilities should be positive.

Ignoring non-negativity constraints for probabilities, the first-order condition for $p_0^\mu(a_i,b_k,\tilde{\varepsilon}_l)$, with λ_l, $\lambda_{i_m}^a$, and $\lambda_{i_m}^b$ as Lagrange multipliers for (14), (15), and (16) respectively, is

$$2(p_0^\mu(a_i,b_k,\tilde{\varepsilon}_l) - r_0^\mu(a_i,b_k,\tilde{\varepsilon}_l)) - \lambda_l - \sum_{i_m=1}^{n_m} \lambda_{i_m}^a a_i^{i_m} - \sum_{i_m=1}^{n_m} \lambda_{i_m}^b b_k^{i_m} = 0 \tag{17}$$

Finally, I solve a system of $n_{\tilde{\varepsilon}}n_a n_b + n_{\tilde{\varepsilon}} + 2n_m$ linear equations in $(\{p_0^\mu(a_i,b_k,\tilde{\varepsilon}_l)\}_{i=1,k=1,l=1}^{n_a,\,n_b,\,n_{\tilde{\varepsilon}}}$, $\{\lambda_l\}_{l=1}^{n_{\tilde{\varepsilon}}}, \{\lambda_{i_m}^a\}_{i_m=1}^{n_m}, \{\lambda_{i_m}^b\}_{i_m=1}^{n_m})$.

As I ignored non-negativity constraints, the resulting solution to the system of equations may have negative elements. If any of elements of solution are negative, I set those elements equal to zero and reduce the system to the remaining elements. I solve the reduced system iteratively until the solution has no negative elements.

(5) Using weights in (3), restore the full proxy distribution over age and idiosyncratic shocks, $p^{\mu}(j, a, b, \varepsilon; z, m)$.

(6) Simultaneously solve for households' decision rules and an intrameporally consistent future approximate aggregate m'. Guess the aggregate law of motion $G_k(z, m)$ for approximate aggregate states. Given $v(J+1, a, b, \varepsilon; z, m) = 0$, solve for decision rules and value functions backwards by age over aggregate states. Compute the full proxy distribution consistent end-of-period aggregate state m' and update $G_k(z, m)$. Iterate until $G_k(z, m)$ converges. Note that this solves for an aggregate law of motion alongside households' value functions.

(7) Given the value function solved by backward induction, simulate the model economy for T periods, then drop the first T_0 periods to develop new reference distributions. The simulation bisects for m'. Let $\mu_t(j, a, b, \varepsilon)$ be the distribution of households over the simulation period $t = T_0 + 1, ..., T$.

I create new reference distributions as a weighted sum of μ_t, putting higher weights for distributions generating moments, m_t, closer to the vector of moments m. Define index sets that group dates for the same exogenous aggregate state, z, $I(z) = \{t | z_t = z\}$ where $z = z_1, ..., z_{n_z}$. Let $N(z)$ be the length of the vector $I(z)$. The reference distribution for each (z, m) is

$$r^{\mu}(j, a, b, \varepsilon; z, m) = \frac{1}{N(z)} \sum_{t \in I(z)} \frac{\delta_1(m, m_t)}{\delta(z, m)} \mu_t(j, a, b, \varepsilon)$$

where $\delta_1(m_0, m_1)$ is defined as the inverse of the Euclidian norm and $\delta(z, m) = \sum_{t \in I(z)} \delta_1(m, m_t)$.

(8) Iterate (3)-(7) to improve a DSF until no additional accuracy is achieved.

E Additional Tables

Table D1. Share of earnings, income, expenditure and expenditure rates over wealth quantiles during normal times

	% share of						% Expenditure rate			
	Earnings		Disp income		Expend.		Earnings		Disp income	
Quantile	data	model	data	model	data	model	data	model	data	model
Q1	13.8	7.1	13.4	6.3	14.7	5.2	57.6	63.5	56.9	83.3
Q2	16.4	11.7	16.3	11.1	14.9	10.2	49.1	72.1	47.7	90.9
Q3	19.0	13.6	18.7	12.9	17.9	12.2	51.0	79.7	49.9	96.1
Q4	21.6	15.8	21.3	16.1	22.1	16.1	55.2	87.3	53.9	99.8
Q5	29.1	51.7	30.3	53.7	30.2	56.4	56.1	93.1	51.9	102

Table D2. Single asset economy

$x=$	Y	C	I	K	B_s	N	$E(r)$	r_f	w
mean(x)	2.66	2.14	0.54	7.81	n/a	1.44	0.05	n/a	1.18
σ_x/σ_y	(2.75)	0.35	1.94	0.21	n/a	0.86	0.92	n/a	0.29
$corr(x,y)$	1.0	0.94	0.99	-0.05	n/a	0.96	0.95	n/a	0.59

Notes : Table D2 presents means of GDP, consumption, investment in illiquid wealth, stock of capital, supply of liquid wealth, total hours worked, expected return on illiquid wealth, return on liquid savings and wage for the model simulated data. It also lists relative standard deviation to and correlation with GDP for each variable. I smooth series using a HP-filter with a smoothing parameter of 100.

Table D3. portfolio choice economy with disaster risk

$x =$	Y	C	I	K	B_s	N	p	q	w
mean(x)	2.51	1.85	0.49	6.53	0.65	1.44	1.0	0.0	1.10
σ_x/σ_y	(2.56)	0.39	1.99	0.23	0.27	0.83	0.27	0.0	0.30
$corr(x, y)$	1.0	0.97	0.98	0.02	-0.48	0.96	0.96	0.02	0.63

Notes : Table 6 presents means of GDP, consumption, investment in illiquid wealth, stock of capital, supply of liquid wealth, total hours worked, price of illiquid wealth, and wage for the model simulated data. It also lists relative standard deviation to and correlation with GDP for each variable. I smooth series using a HP-filter with a smoothing parameter of 100.