Are negative nominal interest rates expansionary?*

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Abstract

In recent years, a handful of central banks have adopted negative policy rates. In this paper we address the effects of negative interest rates on the macroeconomy. Using aggregate and bank-level data, we document a collapse in pass-through to deposit and lending rates once the policy rate turns negative. Motivated by these empirical facts, we construct a New Keynesian model with financial intermediaries. The central bank controls the interest rate on reserves, which is transmitted to borrowing and saving rates through the bank sector. Once the deposit rate reaches its lower bound, the usual transmission mechanism of monetary policy breaks down. Moreover, because a negative interest rate on reserves reduces bank profits, the total effect on the economy can be contractionary.

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1 Introduction

Nominal interest rates have been declining over the past decades, resulting in record low policy rates. Several countries have interest rates close to or at zero percent, and some have gone even further. Between 2012 and 2016, a handful of central banks reduced their key policy rates below zero for the first time in history. While *real* interest rates have been negative on several occasions, the use of negative *nominal* rates prompted a new discussion on the relevance of the zero lower bound. The recent experience with negative interest rates in Japan and a number of European countries makes it clear that negative nominal interest rates should be viewed as part of the central bankers toolbox.

Understanding how negative nominal interest rates affect the economy is important in preparing for the next economic downturn. Interest rates have been declining for more than three decades, resulting in worries about secular stagnation (Eggertsson and Mehrotra 2014). In a recent working paper, Kiley and Roberts (2017) estimate that the zero lower bound on nominal interest rates will bind 30-40 percent of the time going forward. Whether or not accommodative monetary policy below zero is in fact expansionary is therefore a question of great policy interest. Proponents of negative nominal interest rates argue that there is nothing special about interest rates falling below zero. When announcing a negative interest rate, the Swedish Riksbank wrote in their monetary policy report that "*Cutting the repo rate below zero, at least if the cuts are in total not very large, is expected to have similar effects to repo-rate cuts when the repo rate is positive, as* ***all channels in the transmission mechanism can be expected to be active*** (The Riksbank 2015)". Consistent with this view, Rognlie (2015) shows that the interest rate need not be bounded by zero in a model with money storage costs, and concludes that negative interest rates can be used to stimulate aggregate demand. Others remain skeptical. For instance, Waller (2016) and Stiglitz (2016) argue that negative interest rates function like a tax on the banking system and as such will inhibit credit growth - thereby failing to stimulate the economy via the bank-lending channel.

In this paper we investigate the impact of negative central bank rates on the macroeconomy, both from an empirical and theoretical perspective[1]. Using aggregate data, across five different economies, we show how negative policy rates have had limited pass-through to deposit and lending rates. Making inferences about whether negative interest rates are expansionary based on aggregate data is challenging however. We therefore proceed by using a novel, high-frequent bank-level dataset on interest rates from Sweden, to explore the decou-

[1]Note that we do not attempt to evaluate the impact of other monetary policy measures which occurred simultaneously with negative interest rates. That is, we focus exclusively on the effect of negative interest rates, and do not attempt to address the effectiveness of asset purchase programs or programs intended to provide banks with cheap financing (such as the TLTRO program initiated by the ECB).

pling of lending rates from the policy rate. We document a striking decline in pass-through, and a substantial increase in heterogeneity across banks, once the policy rate becomes negative. We show that this increase in heterogeneity is linked to variation in the reliance on deposit financing: the higher the dependence on deposit financing, the smaller the effect on borrowing rates[2].

Motivated by these empirical results, we embed a New Keynesian DSGE model with a banking sector along the lines of Benigno, Eggertsson, and Romei (2014) with two modifications. First, we explicitly incorporate storage costs of money and second, we incorporate demand for central bank reserves as in Curdia and Woodford (2011). The central bank sets the interest rate on reserves, and can choose to implement a negative policy rate as banks are willing to pay for the transaction services provided by reserves. However, due to the possibility of using money to store value, the deposit rate faced by the commercial bank depositors is bounded at some level (possibly negative), in line with our empirical findings. The reason is simple: the bank's customers will choose to store their wealth in terms of paper currency if charged too much by banks. We derive the relevant bound assuming proportional money storage costs. Away from the lower bound on the deposit rate, the central bank can stimulate the economy by lowering the key policy rate. This reduces both the deposit rate and the rate at which people can borrow, thereby increasing demand. We show however, that once the deposit rate reaches its effective lower bound, reducing the policy rate further ceases to be expansionary. As the central bank no longer affects the deposit rate, it cannot stimulate the demand of savers via the traditional intertemporal substitution channel. Furthermore, as banks funding costs (via deposits) are no longer responsive to the policy rate, the bank lending channel of monetary policy also breaks down and there is no stimulative effect via lower borrowing rates. Hence, as long as the deposit rate is bounded, negative central bank rates fail to bring the economy out of a recession. We further show that if bank profits affect banks' intermediation costs, due to for instance unmodeled informational asymmetries between the bank and its creditors, negative interest rates can even be contractionary.

Literature Review Jackson (2015) and Bech and Malkhozov (2016) document the limited pass-through of negative policy rates to aggregate bank rates, but do not evaluate the effects on the macroeconomy. In an empirical paper on the pass through of negative policy rates

[2]Replicating the analysis in Heider, Saidi, and Schepens (2016) for Euro Area banks, we also show that Swedish banks that were relatively more reliant on deposit financing had lower growth in lending volumes relative to other banks after the Riksbank decreased its policy rate below zero. Aside from a detailed examination of Swedish data, we also review the evolution of aggregate interest rates in six economies that have implemented negative rates: Sweden, Denmark, Switzerland, Japan and the Euro Area (highlighting in particular Germany). Our overall reading of the evidence is that there appears to be a bound on the interest rate commercial banks will charge their depositors, even if central bank's policy rates can go negative.

to Euro Area banks, Heider, Saidi, and Schepens (2016) find that banks with higher deposit shares have lower lending growth in the post-zero environment. While the authors argue that this is a result of the lower bound on deposit rates, no attempt is made to formalize the mechanisms at play. Because negative nominal interest rates are a fairly recent phenomenon, few papers address the implications in a formal model[3][4]. Two exceptions are Brunnermeier and Koby (2016) and Rognlie (2015). The former paper defines the reversal rate as the interest rate at which further interest rate reductions become contractionary. While the reversal rate can in principle be either positive or negative, it is unrelated to the observed lower bound on deposit rates resulting from money storage costs, which is central to our paper. Rognlie (2015) allows for a negative interest rate due to money storage costs. However, because there is only one interest rate in the model, it is assumed that the central bank can directly control the interest rate faced by households at all times. Hence, none of these papers capture the key mechanism that is the focus of our paper: as the lower bound on the commercial bank deposit rate becomes binding, the connection between the central bank's policy rate (which can be negative) and the rest of the interest rates in the economy breaks down.

2 Negative Interest Rates In Practice

In this section we use aggregate data and bank level data to document three facts about the pass-through of negative policy rates to bank interest rates[5]. These facts will motivate our theoretical model presented in the next section. The aggregate data is retrieved from the central banks or the statistical agencies for each of the five currency areas we discuss. For Sweden, we also use two bank level datasets. First, we make use of bank level data on monthly lending volumes from Statistics Sweden. Second, we use daily bank level data on mortgage rates for thirteen Swedish banks and credit institutions, which was kindly provided by the price comparison site *compricer.se*[6].

[3]There is however a large literature on the effects of the zero lower bound. See for example Krugman (1998) and Eggertsson and Woodford (2006) for two early contributions.

[4]Our paper is also related to an empirical literature on the connection between interest rate levels and bank profits (Borio and Gambacorta (2017), Kerbl and Sigmund (2017)), as well as a theoretical literature linking credit supply to banks net worth (Holmstrom and Tirole (1997), Gertler and Kiyotaki (2010)).

[5]Although we do not focus on money market rates, we note that the pass-through to money market rates has been stronger than the pass-through to banks deposit rates and lending rates. This is illustrated in Figure (9) in the appendix.

[6]We have mortgage rates for different maturities, ranging from three months to ten years. The maturities for which all financial institutions provide interest rates are 3 months, 1 year, 2 years, 3 years and 5 years. In the text we depict the results using five year maturities, but we replicate our results using lower maturities in Figure (10) in the appendix.

Fact 1: Limited Pass-Through to Deposit Rates In Figure (1) we plot deposit rates for six economic areas in which the policy rate is negative. Starting in the upper left corner, the Swedish central bank lowered its key policy rate below zero in February 2015. Deposit rates, which are usually below the policy rate, did not follow the central bank rate into negative territory. Instead, deposit rates for both households and firms remain stuck at, or just above, zero. A similar picture emerges for Denmark, as illustrated in the upper right corner. The Danish central bank crossed the zero lower bound twice, first in July 2012 and then more aggressively in September 2014. As was the case for Sweden, the negative policy rate has not been transmitted to deposit rates.

Consider next the Swiss and Japanese case in the middle row of Figure (1). Switzerland implemented a negative policy rate in December 2014, while the central bank in Japan lowered its key policy rate below zero in early 2016. The deposit rates in both countries were already very low, and did not follow the policy rate into negative territory. As a result, the impact on the deposit rate was limited.

Finally, interest rates for the Euro Area are depicted in the bottom row of Figure (1). The ECB reduced its key policy rate below zero in June 2014. As seen from the left panel, the deposit rate does not normally follow the policy rate as closely as in the other cases we consider. One reason for this may be the underlying heterogeneity of the Euro Area, with some banks needing to pay a risk premium on their deposits. Looking at Germany only, in the bottom right of the figure, we see a similar picture emerge as in the other countries. That is, despite negative policy rates, the deposit rate appears bounded by zero. What explains the reduction in the aggregate Euro Area deposit rate after the ECB lowered its policy rate to negative levels? A key point to emphasize is that the negative interest rate policy was implemented together with a host of other credit easing measures, some of which implied direct lending from the ECB to commercial banks at a (potentially) negative interest rate[7]. That policy is better characterized as a credit subsidy rather than charging interest on reserves, which the commercial banks hold in positive supply at the central bank. One would also expect ECB credit easing towards banks in the periphery to reduce the risk premium that those banks had to pay to their customers, which would explain the decline in deposit rates in the Euro area on average. That decline, then, had less to do with negative interest rate, and more to do with the general credit intervention by the ECB - which largely could have been done without policy rates being negative. To sum up, the aggregate evidence is strongly suggestive of a lower bound on deposit rates.

[7]We discussed the ECBs targeted longer-term refinancing operations (TLTROs) briefly in section 5.

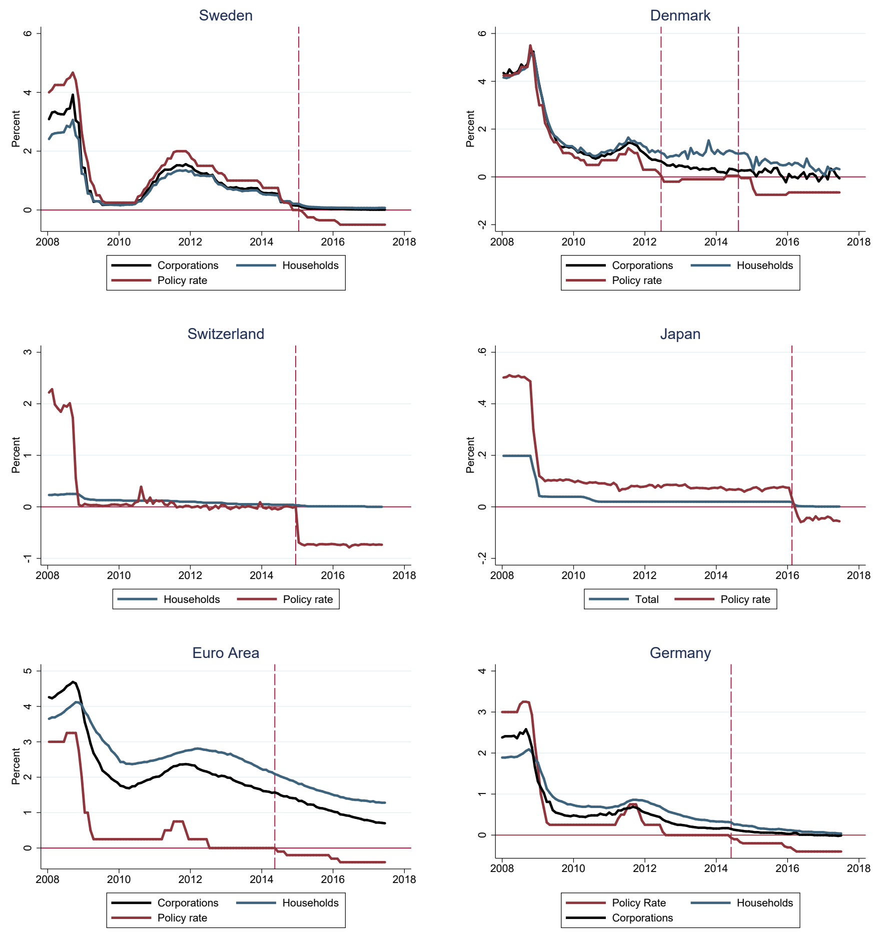

Figure 1: Aggregate Deposit Rates for Sweden, Denmark, Switzerland, Japan, the Euro Area and Germany. The policy rates are defined as the Repo Rate (Sweden), the Certificates of Deposit Rate (Denamark), SARON (Switzerland), the Uncollaterized Overnight Call Rate (Japen) and the Deposit Rate (Euro Area and Germany). The red vertical lines mark the month in which policy rates became negative. Source: The Riksbank, Statistics Sweden, the NB, the SNB, Bank of Japan, and the ECB.

Fact 2: Limited Pass-Through to Lending Rates Although deposit rates appear bounded by zero, one might still expect negative policy rates to lower *lending* rates. As lending rates are usually above the central bank policy rate, they are all well above zero. Here we show that the pass through of the policy rate to lending rates appears affected by the policy rate becoming negative, an empirical finding our model will replicate. In Figure (2) we plot bank lending rates for the six economic areas considered above. While lending rates usually follow the policy rate closely, there appears to be a disconnect once the policy rate breaks the zero lower bound, a feature which will become more stark once we consider disaggregated bank data. Looking at the aggregate data in Figure (2), lending rates in Sweden, Denmark and Switzerland seem less sensitive to the respective policy rates once they become negative. There appears to be some reduction in Japanese lending rates at the time the policy rate went negative, but because there are no further interest rate reductions in negative territory the Japanese case is less informative[8]. Again, the Euro Area is somewhat of an outlier, as lending rates appear to have decreased. This is not surprising in light of the higher-than-zero deposit rates we documented in the previous fact, which we suspect had to do with banks in the Euro Area periphery paying a risk premium on deposits. The ECBs introduction of large-scale credit subsidies simultaneously with negative interest rates could be expected to lower this risk premium, resulting in lower deposit rates and thus lowering the financing cost of banks in the periphery. Again, for the case of Germany, in which the zero lower bound on deposit rates *is* binding, lending rates appear less responsive.

[8]The initial reduction in Japanese lending rates could be caused by the positive part of the policy rate cut, i.e. going from a positive policy rate to a zero policy rate.

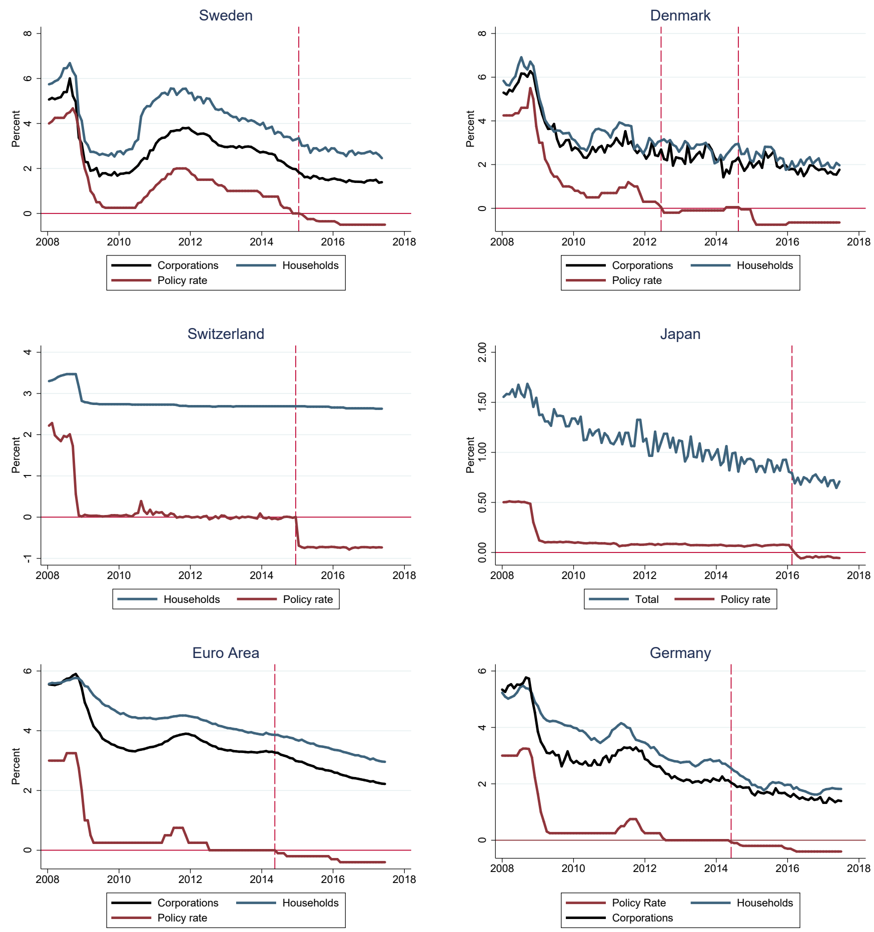

Figure 2: Aggregate Lending Rates for Sweden, Denmark, Switzerland, Japan, the Euro Area and Germany. The policy rates are defined as the Repo Rate (Sweden), the Certificates of Deposit Rate (Denamark), SARON (Switzerland), the Uncollaterized Overnight Call Rate (Japen) and the Deposit Rate (Euro Area and Germany). The red vertical lines mark the month in which policy rates became negative. Source: The Riksbank, Statistics Sweden, the NB, the SNB, Bank of Japan, and the ECB.

Perhaps the most compelling evidence on the breakdown in correlation between the policy rate and the lending rate of banks, comes from daily bank level data from Sweden. In the left panel of Figure (3) we plot bank level mortgage rates for thirteen banks or credit institutions. The vertical lines capture days on which the repo rate was lowered. The first two lines capture repo rate reductions in positive territory. On both of these occasions, there is an immediate and homogeneous decline in bank lending rates. The solid vertical line marks the day the repo rate turned negative for the first time, and the three proceeding lines capture repo rate reductions in negative territory. The response of bank lending rates to these interest rate cuts are strikingly different. While there is some initial reduction in lending rates, most of the rates increase again shortly thereafter. As a result, the total impact on lending rates is limited. There is also a substantial increase in dispersion, with several banks keeping their lending rate roughly unchanged despite repeated interest rate reduction below zero. In the right panel of Figure (3) we plot the minimum and maximum bank lending rate, along side the repo rate (the dashed black line). Again, the increase in dispersion after the repo rate went negative is striking. We also note that the minimum bank lending rate has stayed constant since the first quarter of 2015, despite three policy rate reductions.

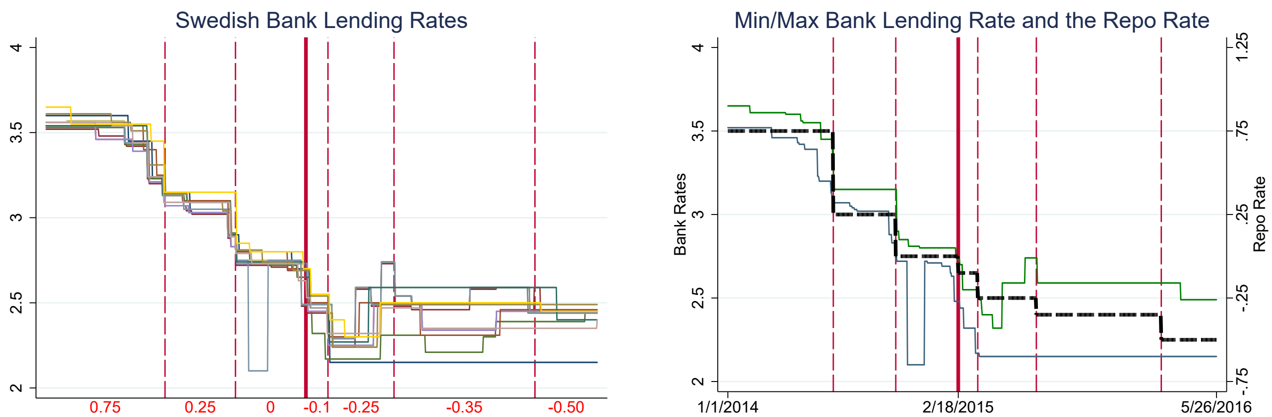

Figure 3: Bank Level Lending Rates Sweden. Interest rate on five-year mortgages. The red vertical lines mark days in which the repo rate was lowered. Left panel: lending rates by bank - the label on the x-axis shows the value of the repo rate. Right panel: the solid green (blue) line depicts the maximum (minimum) bank lending rate, while the dashed black line depicts the repo rate.

Fact 3: Increased Dispersion in Pass-Through Figure (3) showed an increase in the dispersion of bank lending rates once the policy rate fell below zero. In the left panel of Figure (4), we illustrate this explicitly by plotting the standard deviation of lending rates over time. We first note that dispersion in bank rates appear to spike around the time when changes to the repo rate are announced. Second, and more importantly for our purpose, there is a sustained increase in dispersion after the zero lower bound is breached.

In the right panel of Figure (4) we plot the bank level correlations between the lending rates and the policy rate. The correlations captured by the blue bars are calculated for the pre-zero period, and show correlations close to one for all banks in our sample. The red bars capture correlations for the post-zero period, and show a very different picture. First, the correlations are much lower, averaging only 0.02. Second, the correlations now vary substantially across banks, ranging from -0.46 to 0.62. We therefore conclude that bank responses to negative interest rates are more heterogeneous than bank responses to positive interest rates.

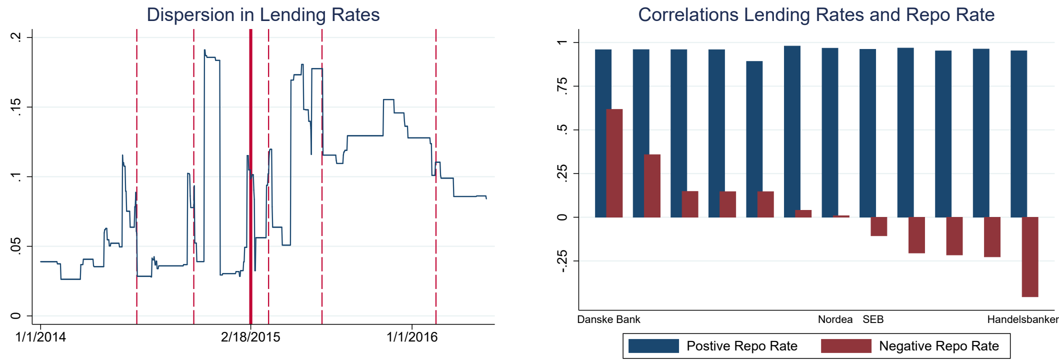

Figure 4: Cross-Sectional Standard Deviation in Lending Rates Sweden. Interest rate on five-year mortgages.

What is causing the dispersion in bank responses to negative interest rates? We first note that several banks initially cut interest rates when the policy rate became negative, only to raise them again shortly thereafter. This could indicate that banks are uncertain about how to set prices in the new environment. However, there could also be more structural reasons why bank responses are heterogeneous. Given that there are frictions in raising different forms of financing and some sources of financing are more responsive to monetary policy changes than others, cross-sectional variation in balance-sheet components can induce cross-sectional variation in how monetary policy affects banks (Kashyap and Stein (2000)). As negative interest rates have had limited pass-through to deposit financing relative to other sources of financing (see Figure (14) in the appendix), this is especially relevant for negative interest rates. To investigate this hypothesis we plot the bank level correlation between lending rates and the repo rate after the repo rate turned negative, as a function of banks deposit shares. The result is depicted in Figure (5), and shows a correlation of -0.16. The small number of observations makes it difficult to draw any firm conclusions. However, we note that banks with high deposit shares consistently have small lending rate responses, in line with our proposed explanation. This suggests that the lower bound on deposits is leading to cross-sectional differences in pass-through based on banks share of deposit financing.

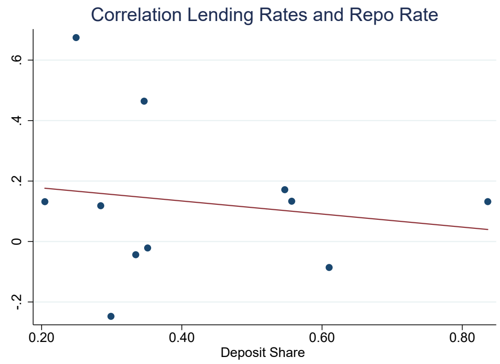

Figure 5: Correlation between lending rate and repo rate after the repo rate turned negative, as a function of the deposit share. Swedish banks.

Another way to investigate whether the degree of deposit financing affects bank behavior is to look at lending volumes, rather than interest rates. This is done in Heider, Saidi, and Schepens (2016) for Euro Area banks. Consistent with our proposed explanation, they find that banks with higher deposit shares had lower lending growth after the policy rate turned negative. Here we show that their results also hold for Swedish banks. Following Heider, Saidi, and Schepens (2016) we use the difference in difference framework specified in equation (1). I_t^{post} is an indicator variable equal to one after the policy rate became negative, while $DepositShare_i$ is the deposit share of bank i in year 2013. We include bank fixed effects δ_i and month-year fixed effects δ_t. Standard errors are clustered at the bank level. We restrict our sample to start in 2014, following Heider, Saidi, and Schepens (2016) in choosing a relatively short time period around the event date. The coefficient of interest is the interaction coefficient β. If banks with high deposit shares have lower credit growth than banks with low deposit shares after the policy rate breaches the zero lower bound, we expect to find $\hat{\beta} < 0$.

$$\Delta log(Lending_{it}) = \alpha + \beta\, I_t^{post} \times DepositShare_i + \delta_i + \delta_t + \epsilon_{it} \tag{1}$$

The regression results are reported in Table (1). The interaction coefficient is negative as expected, and significant at the ten percent level. Hence, the results for European banks from Heider, Saidi, and Schepens (2016) seems to hold also for Swedish banks[9]: credit growth in the post-zero environment is lower for banks which rely heavily on deposit financing.

[9]Our estimate of β is larger in absolute value than the one found in Heider, Saidi, and Schepens (2016), but we lack the statistical power to establish whether the coefficients are in fact statistically different.

	(1)
	$\Delta log(Lending)$
$I^{post} \times DepositShare$	-0.0297*
	(-1.72)
Constant	0.00962
	(1.46)
Clusters	31
Bank FE	yes
Month-Year FE	yes
Observations	1,046

t statistics in parentheses, Std. err. clustered at bank level
* $p < .10$, ** $p < .05$, *** $p < .01$

Table 1: Regression results from estimating equation (1).

To summarize, we conclude that the repeated reductions in central bank rates below zero have not led to negative deposit rates. In fact, deposit rates appear stuck at zero, and do not react to further interest rate reductions. Further, lending rates appear elevated as well, causing the spread between deposit rates and lending rates to remain fairly constant. Finally, the variation in bank responses to negative interest rates is consistent with the theory that the pass-through to lending rates is lower for banks which largely finance themselves with deposits, due to the apparent lower bound on deposit rates. Motivated by these empirical facts, we now move on to developing a formal framework for understanding the impact of negative central bank policy rates. As in the data, the deposit rate will be subject to a lower bound, resulting from storage costs associated with holding money. This lower bound on the deposit rate will affect banks willingness to lower lending rates. Although the policy rate in our model can be negative, what matters for the effect on the macroeconomy is to what degree negative policy rates are transmitted to other interest rates in the economy.

3 Model

In this section, we outline a New Keynesian model with a banking sector. The household and firm sectors are based on Benigno, Eggertsson, and Romei (2014), while we add central bank reserves to the bank problem along the lines of Curdia and Woodford (2011). In addition, we include storage costs of holding money, which will determine the lower bound on the deposit rate. The model features three distinct interest rates, which allows us to model the pass-through of the policy rate to bank rates. Monetary policy in our model is implemented through the banking sector. By changing the interest rate on reserves, the central bank can affect the interest rates charged by banks. These are the interest rates that feature in the households optimization problems.

3.1 Households

We consider a closed economy, populated by a unit-measure continuum of households. Households are of two types, either patient (indexed by superscript s) or impatient (indexed by superscript b). Patient households have a higher discount factor than impatient agents, i.e. $\beta^s > \beta^b$. The total mass of patient households is $1-\chi$, while the total mass of impatient households is χ. In equilibrium, impatient households will borrow from patient households via a banking system, which we specify below. We therefore refer to the impatient households as "borrowers" and the patient households as "savers".

Households consume, supply labor, borrow/save and hold real money balances. At any time t, the optimal choice of consumption, labor, borrowing/saving and money holdings for a household $j \in \{s, b\}$ maximizes the present value of the sum of utilities

$$\mathcal{U}_t^j = \mathbb{E}_t \sum_{T=t}^{\infty} \left(\beta^j\right)^{T-t} \tilde{\zeta}_T \left[U\left(C_T^j\right) + \Omega\left(\frac{M_T^j}{P_T}\right) - \ V\left(N_T^j\right) \right] \tag{2}$$

where $\tilde{\zeta}_t$ is a random variable following some stochastic process and acts as a preference shock[10]. C_t^j and N_t^j denotes consumption and labor for type j respectively.

We assume that households have exponential preferences over consumption, i.e. $U(C_t^j) = 1 - exp\{-qC_t^j\}$ for some $q > 0$. The assumption of exponential utility is made for simplicity, as it facilitates aggregation across agents. Households consume a bundle of consumption goods. Specifically, there is a continuum of goods indexed by i, and each household j has preferences over the consumption index

$$C_t^j = \left(\int_0^1 C_t\left(i\right)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} \tag{3}$$

where $\theta > 1$ measures the elasticity of substitution between goods.

Agents maximize lifetime utility (equation (2)) subject to the following flow budget constraint:

$$B_t^j + M_{t-1}^j - S\left(M_{t-1}^j\right) + W_t^j N_t^j + \Psi_t^j + \psi_t^j = B_{t-1}^j\left(1 + i_{t-1}^j\right) + P_t C_t^j + M_t^j \tag{4}$$

[10]We introduce the preference shock as a parsimonious way of engineering a recession.

B_t^j denotes one period risk-free debt of type j ($B_t^b > 0$ and $B_t^s < 0$), Ψ_t^j is type j's share of firm profits, and ψ_t^j is type j's share of bank profits. $S\left(M_{t-1}^j\right)$ is the storage cost of holding money. Let Z_t^{firm} denote firm profits, and Z_t^{bank} denote bank profits. We assume that firm profits are distributed to both household types based on their population shares, i.e. $\Psi_t^b = \chi Z_t^{firm}$ and $\Psi_t^s = (1-\chi) Z_t^{firm}$. Bank profits on the other hand are only distributed to savers, which own the deposits by which banks finance themselves[11][12]. Hence, we have that $\psi_t^b = 0$ and $\psi_t^s = Z_t^{bank}$.

The optimal consumption path for an individual of type j has to satisfy the standard Euler-equation

$$U'\left(C_t^j\right)\tilde{\zeta}_t = \beta^j\left(1+i_t^j\right)\mathbb{E}_t\left(\Pi_{t+1}^{-1}\right)U'\left(C_{t+1}^j\right)\tilde{\zeta}_{t+1} \tag{5}$$

Optimal labor supply has to satisfy the intratemporal trade-off between consumption and labor

$$\frac{V'\left(N_t^j\right)}{U'\left(C_t^j\right)} = \frac{W_t^j}{P_t} \tag{6}$$

Finally, optimal holdings of money is implicitly defined by satisfy

$$\frac{\Omega'\left(\dfrac{M_t^j}{P_t}\right)}{U'\left(C_t^j\right)} = \frac{i_t^j + S'\left(M_t^j\right)}{1+i_t^j} \tag{7}$$

The lower bound on deposits rates are typically defined as the lowest value of i_t^j satisfying equation (7). The lower bound on interest rates depends crucially on the marginal storage cost. With zero (or constant) marginal storage cost, $S'\left(M_t^j\right) = 0$ and the existence of a satiation point in real money balances the lower bound is $\underline{i}^j = 0$. With a non-constant marginal storage cost, however, this is no longer the case. If storage cost are convex, for instance, the marginal storage cost is increasing in M_t^j. In this case, there is no lower bound. Based on the data from section (2), deposit rates seems bounded close to zero. This is consistent with a proportional storage cost $S\left(M_t^j\right) = \gamma M_t^j$ with a small $\gamma > 0$. In that case, the lower bound is $\underline{i}^j = -\gamma$. In what follows, we assume that this is the case.

[11] Distributing bank profits to both household types would make negative interest rates even more contractionary. The reduction in bank profits would reduce the transfer income of borrower households, causing them to reduce consumption. We believe this effect to be of second order significance, and so we abstract from it here.

[12] We also assume that any central bank profits are distributed to savers, following the distribution of bank profits.

Because we assume that households have exponential preferences over consumption, the labor-consumption trade-off can easily be aggregated into an economy-wide labor market condition[13]

$$\frac{V'(N_t)}{U'(C_t)} = \frac{W_t}{P_t} \tag{8}$$

Aggregate demand is given by

$$Y_t = \chi C_t^b + (1-\chi) C_t^s \tag{9}$$

3.2 Firms

Each good i is produced by a firm i. Production is linear in labor, i.e.

$$Y_t(i) = N_t(i) \tag{10}$$

where $N_t(i)$ is a Cobb-Douglas composite of labor from borrowers and savers respectively, i.e. $N_t(i) = \left(N_t^b(i)\right)^{\chi} \left(N_t^s(i)\right)^{1-\chi}$. This ensures that each type of labor receives a total compensation equal to a fixed share of total labor expenses. That is,

$$\begin{aligned} W_t^b N_t^b &= \chi W_t N_t \qquad (11) \\ W_t^s N_t^s &= (1-\chi) W_t N_t \qquad (12) \end{aligned}$$

where $W_t = \left(W_t^b\right)^{\chi} \left(W_t^s\right)^{1-\chi}$ and $N_t = \int_0^1 N_t(i)\, di$.

Given preferences, firms face a downward-sloping demand function

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t \tag{13}$$

We introduce nominal rigidities by assuming Calvo-pricing. That is, in each period, a fraction α of firms are not able to reset their price. Thus, the likelihood that a price set in period t applies in period $T > t$ is α^{T-t}. Prices are assumed to be indexed to the inflation target Π. The firm problem is standard and is solved in Appendix D.

[13]To see this, just take the weighted average of equation (6) using the population shares χ and $1-\chi$ as the respective weights.

3.3 Banks

Our banking sector is based on Benigno, Eggertsson, and Romei (2014) and Curdia and Woodford (2011). It is made up of identical, perfectly competitive banks. Bank assets consist of one-period real loans l_t. In addition to loans, banks hold real reserves $R_t \geq 0$ and real money balances $m_t \geq 0$, both issued by the central bank[14]. While the central bank controls the supply of total central bank currency, the bank sector itself determines the allocation between reserves and money. Bank liabilities consist of real deposits d_t. Real reserves are remunerated at the interest rate i_t^r , which is set by the central bank. Similarly, loans earn a return i_t^b. The cost of funds, i.e. the deposit rate, is denoted i_t^s. Intermediaries take all of these interest-rates as given.

Financial intermediation takes up real resources. Therefore, in equilibrium there is a spread between the deposit rate i_t^s and the lending rate i_t^b. We assume that banks' intermediation costs are given by the function $\Gamma\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t^{bank}\right)$. In order to allow for the intermediation cost to be time-varying for a given set of bank variables, we include a stochastic cost-shifter $\bar{l}_t$. This cost-shifter may capture time variation in borrowers default probabilities, bank regulation etc. (Benigno, Eggertsson, and Romei (2014)). A higher $\bar{l}_t$ decreases the banks' intermediation costs for a given level of lending l_t.

We assume that the intermediation costs are increasing and convex in the amount of real loans provided. That is, $\Gamma_l > 0$ and $\Gamma_{ll} \geq 0$. Following Curdia and Woodford (2011), central bank currency plays a key role in reducing intermediation costs[15]. The marginal cost reductions from holding reserves and money are captured by $\Gamma_R \leq 0$ and $\Gamma_m \leq 0$ respectively. We assume that the bank becomes satiated in reserves for some level $\bar{R}$. Hence, $\Gamma_R = 0$ for $R \geq \bar{R}$. Similarly, banks become satiated in money at some level $\bar{m}$, so that $\Gamma_m = 0$ for $m \geq \bar{m}$. Banks can thus reduce their intermediation costs by holding reserves and/or cash, but the opportunity for cost reduction can be exhausted. Finally, we assume that higher profits reduce the marginal cost of lending. That is, we assume $\Gamma_{lz} < 0$. We discuss this assumption below.

Bank profits are implicitly defined by

[14] Because we treat the bank problem as static - as outlined below - we do not have to be explicit about the price level.

[15] For example, we can think about this as capturing in a reduced form way the expected cost of the liquidity risk that banks face, as in Bianchi and Bigio (2014). When banks make loans, they take on costly liquidity risk because the deposits created when the loans are made have a stochastic point of withdrawal. More reserves helps reduce this expected cost.

$$z_t^{bank} = d_t - l_t - R_t - m_t - \Gamma\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t^{bank}\right) \tag{14}$$
$$+ \mathbb{E}_t Q_{t,t+1}\left[\left(1+i_t^b\right) l_t + \left(1+i_t^r\right) R_t + m_t - S\left(m_t\right) - \left(1+i_t^s\right) d_t\right]$$

where $Q_{t,t+1}$ is a stochastic discount factor used to price the real value of next-period income flows.

Following Benigno, Eggertsson, and Romei (2014), we transform the bank's problem into a static problem by assuming that any profits from the asset holdings realized in period $t+1$ are distributed in period t. This is equivalent to assuming that $\left(1+i_t^b\right) l_t + \left(1+i_t^r\right) R_t + m_t - S\left(m_t\right) - \left(1+i_t^s\right) d_t = 0$. We can solve this for d_t and insert it into equation (14) to get:

$$z_t^{bank} = \frac{i_t^b - i_t^s}{1+i_t^s} l_t - \frac{i_t^s - i_t^r}{1+i_t^s} R_t - \frac{i_t^s}{1+i_t^s} m_t - \frac{1}{1+i_t^s} S\left(m_t\right) - \Gamma\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t^{bank}\right) \tag{15}$$

Assuming proportional storage cost $S(m) = \gamma m$, any interior l_t, R_t and m_t has to satisfy the respective first-order conditions from the bank's problem

$$l_t \quad : \quad \frac{i_t^b - i_t^s}{1+i_t^s} = \frac{1}{\bar{l}_t}\Gamma_l\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t^{bank}\right) \tag{16}$$
$$R_t \quad : \quad -\Gamma_R\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t^{bank}\right) = \frac{i_t^s - i_t^r}{1+i_t^s} \tag{17}$$
$$m_t \quad : \quad -\Gamma_m\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t^{bank}\right) = \frac{i_t^s + \gamma}{1+i_t^s} \tag{18}$$

The first order condition for lending says that the banks trade of the profits generated from lending with the increase in intermediation costs. The next two first order conditions describe banks demand for reserves and cash. In equilibrium, banks choose to hold both. Although the income from holding reserves exceeds the income from holding money as long as $i^r > -\gamma$, the bank still chooses to hold some money in order to lower its intermediation cost. We assume that reserves and money are not perfect substitutes, and so minimizing the intermediation cost implies holding both reserves and money. This seems intuitive, but is not important for our main result[16].

[16]The assumption that banks always wants to hold some reserves is however important for the effect of negative interest rates on bank profitability. If we instead assume that the sum of money holdings and

The first-order constraint for loans pins down the equilibrium credit spread

$$1+\omega_t \equiv \frac{1+i_t^b}{1+i_t^s} \tag{19}$$

Specifically, it says that

$$\omega_t = \frac{1}{\chi \bar{b}_t}\Gamma_l\left(\frac{b_t^b}{\bar{b}_t}, R_t, m_t, z_t^{bank}\right) \tag{20}$$

where we have used the market clearing condition in equation (21) to express the spread as a function of the borrowers real debt holdings b_t^b[17].

$$l_t = \chi b_t \tag{21}$$

That is, the difference between the borrowing rate and the deposit rate is a function of the aggregate debt level relative to some exogenous and potentially time-varying debt benchmark $\bar{b}_t$, the amount of liquid bank holdings, and bank profits.

Why do bank profits affect the intermediation cost? We have assumed that the marginal cost of extending loans decreases with bank profits. That is, $\Gamma_{lz} \leq 0$. This assumption captures, in a reduced form way, the established link between banks net worth and their operational costs. We do not make an attempt to microfound this assumption, which is explicitly done in among others Holmstrom and Tirole (1997) and Gertler and Kiyotaki (2010). Instead, we provide a short overview of the current literature linking bank net worth to credit supply.

In Gertler and Kiyotaki (2010) bank managers may divert funds, which means that banks must satisfy an incentive compatibility constraint in order to obtain external financing. This constraint limits the amount of outside funding the bank can obtain based on the banks net wort. Because credit supply is determined by the total amount of internal and external funding, this means that bank lending depends on bank profits. In an early contribution, Holmstrom and Tirole (1997) achieve a similar link between credit supply and bank net worth by giving banks the opportunity to engage (or not engage) in costly monitoring of it's non-financial borrowers. For recent empirical evidence on the relevance of bank net worth in explaining credit supply see for example Jiménez and Ongena (2012).

Our decision to include bank profits in the intermediation cost is consistent with a re-

reserves enters the banks cost function as one argument, the bank would hold only money once $i^r < -\gamma$. Hence, reducing the interest rate on reserves further would not affect bank profits. However, such a collapse in central bank reserves is not consistent with data, suggesting that banks want to hold some (excess) reserves.

[17] Following equation (21) we also assume that $\bar{l}_t = \chi \bar{b}_t$.

duction in bank net worth increasing the marginal cost of lending. As in Gertler and Kiyotaki (2010), lower profits increase the lending rate through higher bank costs. We abstract from details such as what sort of agency problem gives rise to an external finance premium. Rather, we simply introduce a reduced form negative correlation between bank profits and bank intermediation costs. Importantly, our main result that negative interest rates are *not expansionary* does not depend on profits affecting intermediation costs. However, the link between profits and the intermediation cost is the driving force behind negative interest rates being *contractionary.* If we turn off this mechanism, negative interest rates still reduce bank profits, but this does feed back into aggregate demand[18].

3.4 Policy

The central bank controls the overall supply of central bank liabilities, i.e. money held by households and banks along with bank reserves. In addition, we assume that the central banks sets the interest rate on reserves i_t^r. Given these assumption, the bank sector itself controls the allocation between reserves and money. The real supply of central bank currency is

$$s_t = R_t + m_t + m_t^s + m_t^b \tag{22}$$

where R_t is reserves, m_t is the banks holdings of real money balances, and m_t^s and m_t^b denote the real money balances of savers and borrowers respectively. The central bank determines s_t and i_t^r. How s_t it is split between R_t, m_t,m_t^s and m_t^b is then a market outcome determined by the first order conditions of the banks and households.

Curdia and Woodford (2011) show that optimal central bank reserve policy implies that $\Gamma_R = 0$ whenever possible. This is equivalent to ensuring that banks hold sufficient reserves to be satiated, i.e. in equilibrium $R = \overline{R}$. This minimizes the intermediation cost, and therefore the spread between the deposit and lending rate. From the first order condition for reserves (17), we see that $i^s = i^r$ if $\Gamma_R = 0$. The easiest way to interpret this is that if banks are satiated in reserves, the central bank implicitly controls i_t^s via i_t^r. A key point, however, is that $\Gamma_R = 0$ is not always feasible due to the bound on the deposit rate. If the deposit rate is bounded at $\underline{i}^s = -\gamma$, and the central bank lowers i_t^r below$-\gamma$, the banks are no long satiated in reserves as $i^s > i^r$. The first order condition then implies $\Gamma_R > 0$, and accordingly $R < \bar{R}$. Intuitively, it is not possible to keep banks satiated in reserves when they are being charged for their reserve holdings.

[18] Alternatively, we could assume that bank profits do not affect intermediation costs, but that bank profits are distributed to all households. A reduction in bank profits would then reduce aggregate demand through the borrowers budget constraint.

We assume that the interest rate on reserves follows a Taylor rule given by equation (23)[19]. Because of the reserve management policy outlined above, the deposit rate in equilibrium is either equal to the reserve rate or to the lower bound, as specified in equation (24).

$$i_t^r = r_t^n \pi_t^{\phi_\pi} y_t^{\phi_Y} \tag{23}$$

$$i_t^s = \max \{\underline{i}^s, i_t^r\} \tag{24}$$

3.5 Equilibrium Conditions: A Summary

After having introduced central bank policy in the previous section we are now ready to define an equilibrium in our model. For a given initial price dispersion Δ_0 and debt level B_0^b , and a sequence of shocks $\left\{\zeta_t, \bar{b}_t\right\}_{t=0}^{\infty}$, an equilibrium is a process for the 14 endogenous variables $\{C_t^b,\, C_t^s,\, b_t^b,\, m_t^b,\, Y_t,\, \Pi_t,\, F_t,\, K_t,\, \Delta_t,\, \lambda_t,\, l_t,\, R_t,\, m_t,\, z_t^{bank}\}_{t=0}^{\infty}$ and the 3 endogenous interest rates $\{i_t^b,\, i_t^s,\, i_t^r\}_{t=0}^{\infty}$ such that the following equilibrium conditions hold[20]. First, from the household problem we have the borrowers budget constraint and money demand given by equations (4) and (7) with $j = b$[21], and the two Euler equations given by equation (5) with $j = \{s, b\}$. Further we have the aggregate demand condition outlined in equation (9), leaving us with five equations from the demand side of the economy.

From the firm side we have the variables Π_t, F_t, K_t, Δ_t and λ_t. Π_t denotes inflation, while F_t and K_t are defined in appendix D. Δ_t and λ_t denotes price dispersion and the weighted marginal utility of consumption respectively. The five equilibrium conditions from the firm side are identical to the ones in Benigno, Eggertsson, and Romei (2014) and are listed in appendix D as equations (35), (36), (37), (38), and (39).

From the bank side we have four equilibrium conditions. These are bank profits given by equation (15), the first order condition for lending given by equation (16), the first order condition for reserves given by equation (17), and the first order condition for money given by equation (18). In addition, market clearing in the credit market requires that equation (21) is satisfied.

The final two equilibrium conditions are the policy equations from the previous section. Specifically, the interest rate on reserves follows the Taylor rule in equation (23) and the

[19]The log-linear version is the familiar looking equation $\hat{i}_t^r = \hat{r}_t^n + \phi_\pi \hat{\pi}_t + \phi_Y \hat{y}_t$, where $\hat{X} \equiv \frac{X_t - X}{X}$ and X is the steady state value of X_t.

[20]Inflation is defined as $\Pi_t = \frac{P_t}{P_{t-1}}$, and we redefine the preference shock such that $\zeta_t \equiv \frac{\tilde{\zeta}_{t+1}}{\tilde{\zeta}_t}$

[21]Technically, we also need to solve for firm profits and labor income which enter into the borrowers budget constraint. When we list the non-linear equilibrium equations in Appendix E we have inserted for firm profits, and then labor income cancels. Hence we do not need to explicitly solve for these two variables.

deposit rate is determined by equation (24). That leaves us with 17 equations to solve for the 17 endogenous variables listed above. These non-linear equilibrium conditions are summarized in appendix E.

3.6 Generalization of Standard New Keynesian Model

We take a log-linear approximation of the equilibrium conditions around the steady state. The steady state equations, as well as the log-linearized equilibrium conditions are listed in appendix E[22]. Here we reproduce the key equations.

The linearized model can be seen as a generalization of the textbook New Keynesian model. We first note that the supply side collapses to the standard case. That is, the supply side can be summarized by the generic Phillips curve

$$\hat{\pi}_t = \kappa \hat{y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} \tag{25}$$

The demand side is governed by the IS-curve in equation (26), which is derived by combining the aggregate resource constraint and the Euler equations[23] where $\sigma \equiv \frac{1}{zY}$.

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \sigma \left(\hat{i}_t^s - \mathbb{E}_t \pi_{t+1} - \hat{r}_t^n \right) \tag{26}$$

In the standard model, the natural rate of interest r^n is exogenous. In our case the natural rate of interest is endogenous, and depends on the shocks to the economy. Specifically, the natural rate of interest takes the following form

$$\hat{r}_t^n = -\hat{\zeta}_t - \chi \hat{\omega}_t \tag{27}$$

The natural rate of interest depends on the preference shock and the spread between the deposit rate and the borrowing rate, $\hat{\omega}_t \equiv \hat{i}_t^b - \hat{i}_t^s$. This spread is pinned down by the relative level of debt in the economy, as well as deviations in bank profits:

$$\hat{\omega}_t = \frac{i^b - i^s}{1 + i^b} \left((\nu - 1)\hat{b}_t^b - \nu \hat{\bar{b}}_t + \iota \hat{z}_t^{bank} \right) \tag{28}$$

More private debt increases the interest rate spread, thereby reducing the natural rate

[22]The log-linearized equilibrium conditions listed in the appendix are the IS-curve in equation (64), the expression for the real interest rate in equation (65), the borrowers Euler equation in equation (66), the borrowers budget constraint in equation (67), the borrowers money demand in equation (68), the Phillips curve in equation (69), the definition of the interest rate spread in equation (70), the value of the interest rate spread in equation (71), bank profits in equation (72), the banks money demand in equation (73), the Taylor rule in equation (74), and the lower bound on the deposit rate in equation (75).

[23]The log-linearized resource constraint is given by $\hat{y}_t = \frac{\chi c^b}{y} \hat{c}_t^b + \frac{(1-\chi)c^s}{y} \hat{c}_t^s$.

of interest. This is the same mechanism as outlined in Benigno, Eggertsson, and Romei (2014). In addition, the interest rate spread now depends on bank profits. The reason is that higher profits reduce interemediation costs, thereby lowering the banks required interest rate margin. Both the preference shock and the shock to the economy's debt capacity is exogenous. However, the total debt level and bank profits are endogenous. In order to solve for these two variables, the entire system of equations outlined in the appendix needs to be solved.

While the standard New Keynesian model only has one interest rate, our model has three distinct interest rates. As in Benigno, Eggertsson, and Romei (2014) we have an interest rate on borrowing i_t^b, as well as an interest rate on saving i_t^s. In addition, and following Curdia and Woodford (2011), we also have an interest rate on reserves i_t^r. The reserve rate is set by the central bank according to the standard Taylor rule

$$\hat{i}_t^r = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t \tag{29}$$

Because the central bank keeps the bank sector satiated in reserves whenever feasible, the deposit rate is equal to the reserve rate when the lower bound is not binding[24]

$$i_t^s = \max\left\{\underline{\mathrm{i}}^s, i_t^r\right\} \tag{30}$$

In standard economic times, away from the lower bound, our model is conceptually identical to Benigno, Eggertsson, and Romei (2014). If the central bank lowers the reserve rate, this lowers the deposit rate through equation (30). The reduction in the deposit rate stimulates the consumption of saver households. In addition, lowering the deposit rate reduces the banks financing costs. This increases their willingness to lend, putting downwards pressure on the borrowing rate. Hence, the reduction in the reserve rate leads to a reduction in the other interest rates in the economy, thereby stimulating aggregate demand.

Most macro models either implicitly or explicitly impose a lower bound on the interest rate controlled by the central bank. Given the experience with negative policy rates in recent years, this seems counterfactual. Hence, we allow the central bank to adopt a negative reserve rate. Deposit rates on the other hand, appear to be bounded at levels close to zero. In line with the data, the deposit rate in our framework is bounded below. When the lower bound on the deposit rate is binding, the standard effect of reserve rate reductions breaks down. As evident from equation (30), lowering the reserve rate below the bound and into negative territory has no effect on the deposit rate. Further, because the deposit rate stays unchanged,

[24]We express the bound in terms of i_t rather than $\hat{i}_t$. This is consistent with the impulse responses depicted in the next section. Following the literature, interest rates are plotted in percent, while other variables are reported in percent deviation from steady state.

there is no stimulative effect on bank financing costs and so no increase in their willingness to lend. As a result, there is no longer a boost to aggregate demand. Moreover, because charging a negative interest rate on reserves reduces bank profits, the interest rate spread in equation (28) increases[25]. This implies an increase in the borrowing rate, and so aggregate demand falls. Hence, when the deposit rate is stuck at the lower bound, further reductions in the reserve rate have a *contractionary* effect on the economy. Note that this result is due to the feedback effect from bank profits to the interest rate spread. If we shut down this channel by setting $\iota = 0$ in equation (28), negative interest rates are neither expansionary nor contractionary.

To summarize, our model behaves as a standard New Keynesian model in normal times. Once the lower bound on the deposit rate becomes binding however, the central bank looses its ability to stimulate the economy by reducing the interest rate on reserves. If the central bank adopts a negative reserve rate, bank profits suffer. Given the feedback effect from bank profits to aggregate demand, negative interest rates have a contractionary effect on the economy.

4 The Effects of Monetary Policy in Positive and Negative Territory

In this section, we compare our baseline model (which we refer to as the negative rates model) to two other models. The first model is the standard case, in the sense that there is an effective lower bound on both the deposit rate and the central banks policy rate. The second model is the frictionless model, in which both the deposit rate and the central bank policy rate can fall below zero.

We consider two different shocks to the economy. First, we consider a preference shock, which effectively makes agents more patient and so delays consumption. Second, we consider a positive shock to the banks intermediation cost, making it more costly for the bank to supply loans. We then evaluate to which degree the central bank can stimulate aggregate demand by lowering the reserve rate given the different model regimes.

4.1 Calibration

We pick the size of the two shocks to generate approximately a 4.5 percent drop in output on impact and a duration of the lower bound of approximately 12 quarters. We choose parameters from the existing literature whenever possible. We target a borrowing rate of 4

[25]Note that $\iota < 0$ so that a decrease in profits implies an increase in the interest rate spread.

%[26] and a deposit rate of 1.5 %, yielding a steady state credit spread of 3.5 %. The preference parameter q is set to 0.75, which generates an intertemporal elasticity of substitution of approximately 2.75, in line with Curdia and Woodford (2011). We set the proportional storage cost to 0.01, yielding an effective lower bound of -0.01%. This is consistent with the deposit rate being bounded at zero for most types of deposits, with the exception of slightly negative rates on corporate deposits. We set $\overline{R} = 0.7$, which yields steady-state money holdings in line with average excess reserves relative to total assets for commercial banks from January 2010 and until April 2017[27]. We set $\overline{m}$ to 0.01, implying that currency held by banks in steady state accounts for approximately 1.5 percent of total assets. This currency amount corresponds to the difference between total cash assets reported at US banks and total excess reserves from January 2010 until April 2017.

The parameter ν measures the sensitivity of the credit spread to private debt. We set ν so that a 1 % increase in private debt increases the credit spread with 0.12 %, in line with estimates from Benigno, Eggertsson, and Romei (2014). Given the steady-state credit spread, $\bar{l}$ pins down the steady-state level of private debt. We choose $\bar{l}$ to target a private debt-to-GDP ratio of approximately 95 percent, roughly in line with private debt in the period 2005 - 2015 (Benigno, Eggertsson, and Romei, 2014). The final parameter is ι. In our baseline scenario we set $\iota = -0.2$. While ι is not important for our main result that negative interest rates are not expansionary, it is important for determining the feedback effect from bank profits to aggregate demand. In Table 3 in the next section we show how the potentially contractionary effect of negative interest rates depend qualitatively on ι.

All parameter values are summarized in Table (2). Due to the occasionally binding constraint on i_t^s, we solve the model using OccBin (Guerrieri and Iacoviello, 2015). We consider a cashless limit for the household's problem.

4.2 Preference Shock

We start by investigating how the economy responds to a shock to agents marginal utility of consumption which - in the standard model case - generates approximately a 4.5 percent drop in output on impact. This reduction in output is chosen to roughly mimic the average reduction in real GDP in Sweden, Denmark, Switzerland and the Euro Area, as illustrated in figure (11) in the appendix[28]. The shock to the preference parameter follows an AR(1)

[26] This is consistent with the average fixed-rate mortgage rate from 2010-2017.Series MORTGAGE30US in the St.Louis Fed's FRED database.

[27] We use series EXCSRESNS for excess reserves and TLAACBW027SBOG for total assets from commercial banks, both in the St.Louis Fed's FRED database.

[28] Detrended real GDP fell sharply from 2008 to 2009, before partially recovering in 2010 and 2011. The partial recovery was sufficiently strong to induce an interest rate increase. We focus on the second period of

Parameter	Value	Source/Target
Inverse of Frisch elasticity of labor supply	$\eta = 1$	Justiniano et.al (2015)
Preference parameter	$q = 0.75$	Yields IES of 2.75(Curdia and Woodford, 2011)
Share of borrowers	$\chi = 0.61$	Justiniano et.al (2015)
Steady-state gross inflation rate	$\Pi = 1$	For simplicity.
Discount factor, saver	$\beta^s = 0.9962$	Annual savings rate of 1.5 %
Discount factor, borrower	$\beta^b = 0.99$	Annual borrowing rate of 4 %
Probability of resetting price	$\alpha = 2/3$	Gali (2008)
Taylor coefficient on inflation gap	$\phi_\Pi = 1.5$	Gali (2008)
Taylor coefficient on output gap	$\phi_Y = 0.5/4$	Gali (2008)
Elasticity of substitution among varieties of goods	$\theta = 7.88$	Rotemberg and Woodford (1997)
Proportional storage cost of cash	$\gamma = 0.01$ %	Effective lower bound $\underline{i}_t^s = -0.01\%$
Reserve satiation point	$\overline{R} = 0.07$	Target steady-state reserves/total assets ratio of 13 %
Money satiation points	$\overline{m} = 0.01$	Target steady-state cash/total assets of 1.5 %
Marginal intermediation cost parameters	$\nu = 6$	Benigno, Eggertsson, and Romei (2014)
Level of safe debt	$\bar{l} = 1.3$	Target debt/GDP ratio of 95 %
Link between profits and intermediation costs	$\iota = -0.2$	1 % increase in profits $\approx$ 0.01 % reduction in credit spread

Shock	Value	Source/Target
Preference shock	1.8 % temporary increase in ξ	Generate a 4.5 % drop in output on impact
Intermediation cost shock	50 % temporary decrease in $\overline{L}$	Generate a 4.5 % drop in output on impact
Persistence of shocks	$\rho = 0.85$	Duration of ZLB of 12 quarters

Table 2: Parameter values

process with persistence 0.85 and no further shocks.

The results of the exercise are depicted in Figure (6). We start by considering the completely frictionless case, referred to as the *No bound* case. In this scenario it is assumed that both the policy rate and the deposit rate can turn negative, as illustrated by the dashed black lines in Figure (6). The preference shock reduces aggregate demand and inflation, triggering an immediate response from the central bank. The policy rate is lowered well below zero, which leads to an identical fall in the deposit rate. As long as the deposit rate is not bounded, optimal policy is feasible and the central bank keeps banks satiated in reserves. The reduction in deposit rates reduces bank financing costs. As a result, banks supply more credit and the borrowing rate declines. In the frictionless case, the aggressive reduction in the policy rate means that the central bank is able to perfectly counteract the negative shock. Hence, all of the reaction comes through interest rates, with no reduction in aggregate demand.

falling real GDP (which occurred after 2011), as negative interest rates were not implemented until 2014-2015. Targeting a reduction in real GDP of 5 percent is especially appropriate for the Euro Area and Sweden. Real GDP fell by somewhat less in Denmark, and considerably less in Switzerland. This is consistent with the central banks in the Euro Area and Sweden implementing negative rates because of weak economic activity, and the central banks in Denmark and Switzerland implementing negative rates to stabilize their exchange rates.

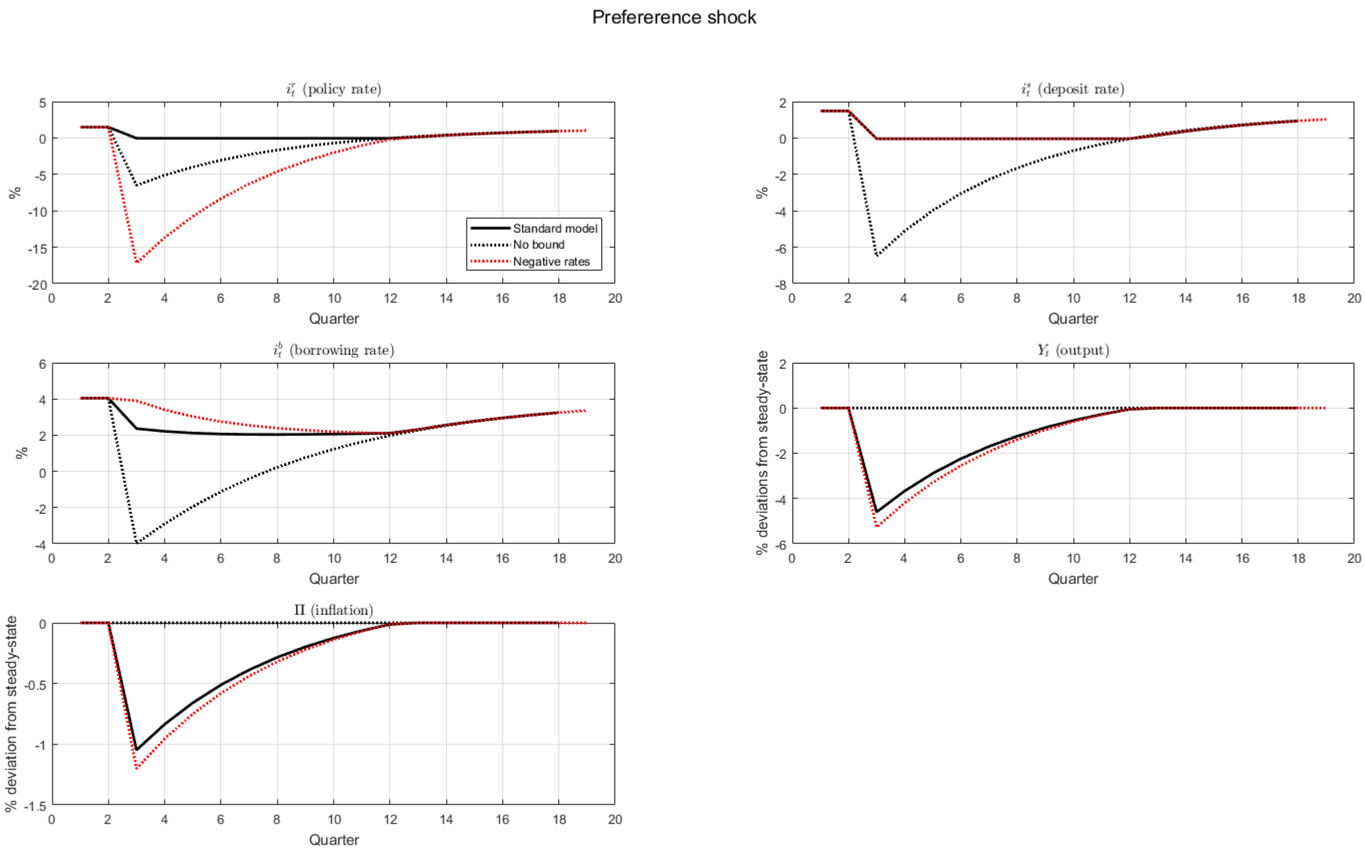

Figure 6: Impulse response functions following an exogenous increase in the marginal utility of consumption tomorrow, under three different models. *Standard model* refers to the case where there is an effective lower bound on both deposit rates and the central bank's policy rate. *No bound* refers to the case where there is no effective lower bound on any interest-rate. *Negative rates* refers to the model outlined above, where there is an effective lower bound on the deposit rate but no lower bound on the policy rate.

Contrast the frictionless case to the standard case, in which both the policy rate and the deposit rate are bounded. In this case, the central bank is not able to offset the shock, and output is below its steady state value for the full duration of the shock. This scenario is outlined by the solid black line in Figure (6). As before, the central bank reacts to the shock by lowering the policy rate. However, the policy rate soon reaches the lower bound, and cannot be lowered further. As a result, both the policy rate and the deposit rate are stuck at the lower bound for the duration of the shock. This transmits into the borrowing rate, which falls less than in the previous case, due to the limited reduction in the deposit rate. Because of the inability of interest rates to fully adjust to the shock, aggregate output and inflation remain below their steady state values for the full duration of the shock.

Finally, we consider the case deemed to be most similar to what we see in the data. While the policy rate is not bounded, there exists an effective lower bound on the deposit rate. This case is illustrated by the red dashed lines in Figure (6). The central bank reacts to the shock by aggressively reducing the policy rate. However, the deposit rate only responds until it reaches its lower bound, at which point it is stuck. As a result, the borrowing rate does not fall as much as in the frictionless case, and the central bank is once again unable to mitigate the negative effects of the shock on aggregate demand and inflation. Hence, the central bank cannot stimulate aggregate demand by lowering its policy rate below zero.

At first glance, the model with negative interest rates looks identical to the standard model. Interestingly, there is an important difference between imposing negative interest rates and not doing so - which in effect makes negative interest rates slightly contractionary. Output actually declines more if the central bank chooses to lower its policy rate below zero. The reason is the negative effect on bank profits resulting from the negative interest rate on reserves. Banks hold reserves in order to reduce their intermediation cost, but when the policy rate is negative they are being charged for doing so. At the same time, their financing costs are unresponsive due to the lower bound on the deposit rate. Hence, bank profits are lower when the policy rate is negative, as illustrated in Figure (7).

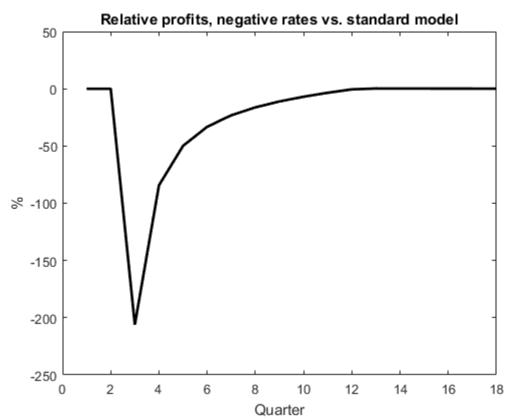

Figure 7: Profits under negative rates model relative to standard model.

This decline in bank profits feeds back into aggregate demand through the effect of bank net worth on the marginal lending cost. Lower net worth increases the costs of financial intermediation, which reduces credit supply. The importance of profits for banks intermediation costs are parametrized by ι. In Table 3 we report the effect of the same preference shock on output and the borrowing rate for different assumptions about ι. In the case in which there is no feedback from bank profits to intermediation costs, the output drop under negative rates corresponds to the output drop under the standard model. The same holds for the borrowing rate. As ι increases in absolute value, the reduction in the borrowing rate is muted due to the increase in intermediation costs. As a result, output drops by more. For ι sufficiently high, the borrowing rate actually *increase* when negative policy rates are introduced. This is consistent with the bank-level data on daily interest rates from Sweden, where some banks in fact *increased* their lending rate following the introduction of negative interest rates.

Model	Output, % deviation from SS	Reduction in borrowing rate, percentage points
Standard	4.5	1.5
$\iota = 0$	4.5	1.5
$\iota = -0.1$	4.8	1.2
$\iota = -0.15$	5	0.8
$\iota = -0.2$	5.2	0.2
$\iota = -0.25$	5.7	-1

Table 3: The effect of a preference shock on output and the borrowing rate (on impact) with negative policy rates for different values of ι.

To summarize, when $\iota \neq 0$ there is an additional negative effect on aggregate demand. Hence, we conclude that negative interest rates are not expansionary. In fact, due to the feedback from bank profits to aggregate demand, they can even be contractionary.

4.3 Intermediation Cost Shock

We next consider the impact of a shock to banks intermediation costs in Figure (8). Specifically, we consider a temporary reduction in the debt limit $\bar{l}_t$. This directly increases the interest rate spread, causing the borrowing rate to increase. Besides from this, the effects are very similar to those caused by the preference shock, and the basic takeaway is unchanged.

The shock to the intermediation cost causes an increase in the interest rate spread, which reduces lending and hence aggregate demand. In the frictionless case, the central bank can perfectly counteract this by reducing the reserve rate below zero. Given the bound on the deposit rate however, the central bank looses its ability to bring the economy out of a recession. Any attempt at doing so, by reducing the reserve rate below zero, only lowers bank profits and so lowers aggregate demand further.

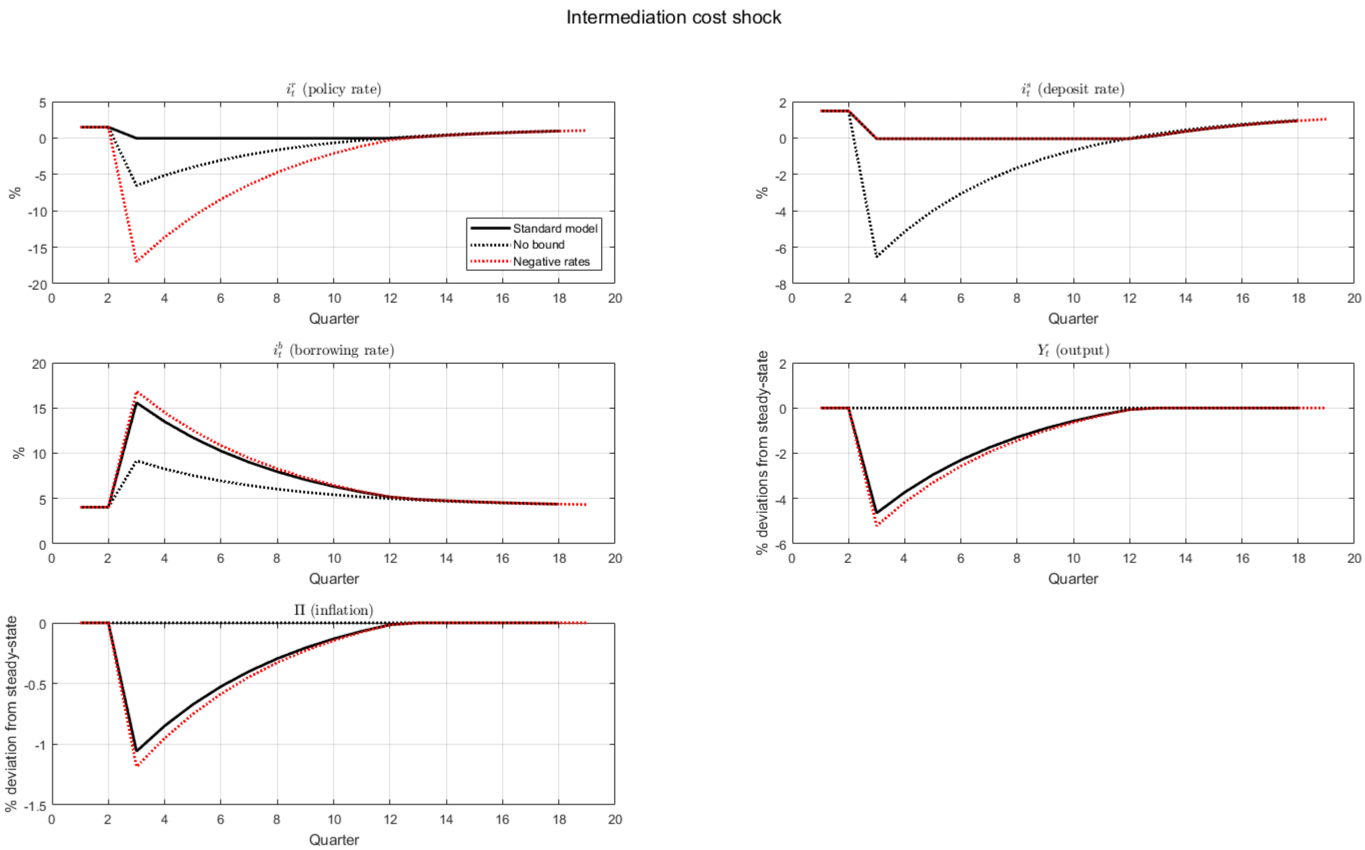

Figure 8: Impulse response functions following an exogenous increase in banks intermediation costs, under three different models. S*tandard model* refers to the case where there is an effective lower bound on both deposit rates and the central bank's policy rate. *No bound* refers to the case where there is no effective lower bound on any interest-rate. *Negative rates* refers to the model outlined above, where there is an effective lower bound on the deposit rate but no lower bound on the policy rate.

5 Discussion

Our main result is that negative central bank rates are not expansionary. This result relies crucially on deposit rates being bounded, as observed in the data. The intuition is straightforward. When deposit rates are kept from falling, banks funding costs are constant. Hence, banks are unwilling to lower lending rates, as this would reduce the spread between deposit rates and borrowing rates - thereby reducing bank profits. This link between the deposit rate and the borrowing rate means that the bound on the deposit rate transmits into a bound on the lending rate, consistent with the empirical evidence from Sweden. Note that the result that negative interest rates are non-expansionary does not rely on the effect of bank profits on the real economy. That is, as long as the deposit rate is bounded, negative interest rates are always non-expansionary. This holds also if there is no feedback from bank profits to aggregate demand.

The second result we want to highlight is the negative impact on bank profits. Charging banks to hold reserves at the central bank lowers their profits. Although banks can choose to transform their reserves into cash holdings, they still prefer to hold some positive amount of reserves. In our model this is because reserve holdings lowers the intermediation cost. The intuition being that reserves provide liquidity, which banks need to handle their day-to-day transactions. In reality, holding large amounts of cash may involve substantial fixed costs, making it costly to transform reserves into cash. Hence, negative reserve rates lower bank profits. The impact of lower bank profits on aggregate demand depends on the model specifications. There is at least two ways in which lower bank profits may reduce economic activity. First, as in our main model specification, lower bank profits could increase banks financing costs, thereby reducing credit supply. This reduces the consumption of borrower households, and thereby aggregate demand. Second, we could distribute bank profits to all household types, thereby creating a negative impact through the borrowers budget constraint. We prefer the former channel, as this is the mechanism which has been extensively discussed in the media. Under either one of these two assumptions, negative interest rates are not only non-expansionary, but are even contractionary.

Finally, it is worth mentioning that in our model all reserves earn the same interest rate. In reality, most central banks have implemented a tiered remuneration scheme, in which case the marginal and average reserve rates differ. For example, some amount of reserves may pay a zero interest rate, while reserves in excess of this level earn a negative rate. We outline the policy schemes in the different countries in appendix C. Allowing for more than one interest rate on reserves would not qualitatively alter our results[29].

We now discuss some arguments put forward by proponent of negative interest rates. Addressing all of these arguments formally would require expanding our model substantially. Instead, we elaborate informally on whether any of these changes would alter our main conclusions.

Lower Lending Rates While the lower bound on deposit rates generally seems to have been accepted as an empirical fact, the lower bound on lending rates is not as widely recognized. Some proponent of negative interest rates argue that regardless of deposit rates,

[29]To see this consider first a scenario in which the central bank offers a reserve rate of -1 percent on all reserves. Contrast this to a scenario in which half of the reserves earn a zero interest rate, and the other half earns an interest rate of -1 percent. The average reserve rate is then -0.5 compared to -1.0 in the first case. The negative impact on profits is quantitatively mitigated, but the economic intuition remains unchanged.

negative policy rates should transmit into lending rates as usual. The deputy governor of monetary policy at the Bank of England gave a speech in which he highlighted such a link between the reserve rate and the lending rate: *"Such a charge on reserves would encourage banks to substitute out of them into alternative assets, though the banking system as a whole could not get rid of the reserves – other than by converting them into cash – as the total quantity is primarily determined by the MPC's asset purchase decisions.* ***But any attempt by banks to substitute out of reserves into other assets, including loans, would lead to downward pressure on the interest rates on those assets.*** *Eventually, the whole constellation of interest rates would shift down, such that banks were content to hold the existing quantity of reserves. This is exactly the mechanism that operates when Bank Rate is reduced in normal times;* ***there is nothing special about going into negative territory****."* (Bean 2013). Similar explanations for the expansionary effects of negative interest rates have been given by other central banks (The Riksbank 2016, Jordan 2016). This argument is potentially problematic for two reasons. First, in our model, banks are reluctant to cut lending rates when the deposit rate is stuck at its lower bound. Doing so would reduce their profits, as their funding costs are not responding to the negative policy rate. This connection between the deposit rate and the lending rate seems consistent with data. Looking at aggregate data inn Figure (2) suggested that the bound on deposit rates was transmitting into a bound on lending rates as well. The bank level data from Sweden in Figure (4) further confirmed the collapse in pass-through from policy rates to lending rates once the policy rate turned negative. Another source of empirical evidence comes from the ECBs lending survey. As illustrated in Figure (15) in the appendix, 80-90 of banks in the Euro Area say that the negative policy rate has not contributed to increased lending volumes. There have also been reports of some banks *increasing* their lending rates in response to negative policy rates. This contractionary response seems particularly well documented in the Swiss case (Jordan 2016, Bech and Malkhozov 2016). Second, even if lending rates responded to negative policy rates as usual, this would imply a reduction in bank profits. The reduction in lending rates would squeeze banks profit margins, potentially reducing credit supply and thereby economic activity. This effect would only be in place when the policy rate was sufficiently low to make the lower bound on deposit rates binding, implying that negative interest rates *are* in fact special.

Additional Funding Source In our model deposits constitute the sole financing source for banks. In reality banks have access to several funding sources, with potentially different sensitivity to negative policy rates. Figure (14) in the appendix illustrate some interest rates relevant for banks financing costs in Sweden. In general, negative interest rates seem to have passed through to interbank rates. Also the interest rates on relatively short-term mortgage bonds and government bonds have gone negative. This implies that the negative policy rate may be reducing banks financing costs somewhat, even though deposit rates are bounded. In Sweden, deposits account for more than 40 percent of total liabilities (see figure (12) in the appendix), and this share is typically even higher in the Euro Area. Hence, the deposit rate is the quantitatively most important interest rate for evaluating banks financing costs. If the policy rate no longer affects the deposit rate, the central banks ability to influence banks funding costs is substantially reduced. However, one could imagine that banks respond to negative policy rates by shifting away from deposits to alternative sources of financing, in response to negative policy rates. This could increase the effectiveness of the monetary policy transmission. As illustrated in Figure (12) in the appendix, this does not seem to be the case. Swedish banks actually increased their deposit share after the central bank rate turned negative in early 2015.

Bank Fees While banks have been unwilling to lower their deposit rates below zero, there have been some discussion surrounding their ability to make up for this by increasing fees and commission income. In our model there is no fixed costs involved with opening a deposit account, but allowing for this would imply that the interest rate on deposits could exceed the effective return on deposits. If banks respond to negative policy rates by increasing their fees, this could in principle reduce the effective deposit rate, and thereby lower banks funding costs. However, as illustrated in Figure (13), the commission income of Swedish banks as a share of total assets actually fell after the policy rate turned negative. Hence, the data does not support the claim that the effective deposit rate is in fact falling. Given that depositors understand that higher fees reduce the effective return on their savings, this is perhaps not surprising.

Alternative Transmission Mechanisms Our focus is on the bank lending channel of monetary policy transmission. When the policy rate is positive, lowering it reduces the interest rates charged by banks, which increases credit supply and thereby economic activity. We have shown that this mechanism is no longer active when the policy rate is negative. However, it is still possible that negative interest rates stimulate aggregate demand through other channels. Both the Swiss and Danish central bank motivated their decision to implement negative central bank rates by a need to stabilize the exchange rate. This open economy dimension of monetary policy is absent from our analysis. The point we want to make is that the *bank lending channel* of monetary policy transmission - traditionally the most important channel - is not robust to introducing negative policy rates. The impact on the exchange rate would depend on which interest rates are most important for explaining movements in the exchange rate.

Even if lending volumes do not respond to negative policy rates, there could potentially be an affect on the composition of borrowers. It has been suggested that banks may respond to negative interest rates by increasing risk taking. This could potentially increase lending rates, resulting in upward pressure on the interest rate margin. Heider, Saidi, and Schepens (2016) find support for increased risk taking in the Euro Area, using volatility in the return-to-asset ratio as a proxy for risk taking. According to their results, banks in the Euro Area responded to the negative policy rate by increasing return volatility. This is certainly not the traditional transmission mechanism of monetary policy, and it is unclear whether such an outcome would be desirable.

Other policies Finally, it is worth emphasizing that our model exercise focuses exclusively on the impact of negative policy rates. Other monetary policy measure which occurred over the same time period are not taken into account. This is perhaps especially important to note in the case of the ECB, which implemented its targeted longer-term refinancing operations (TLTROs) simultaneously with lowering the policy rate below zero. Under the TLTRO program, banks can borrow from the ECB at attractive conditions. Both the loan amount and the interest rate are tied to the banks loan provision to households and firms. The borrowing rate can potentially be as low as the interest rate on the deposit facility, which is currently -0.40 percent[30]. Such a subsidy to bank lending is likely to affect both bank interest rates and bank profits, and could potentially explain why lending rates in the Euro Area have fallen more than in other places once the policy rate turned negative.

6 Concluding remarks

Since 2014, several countries have experimented with negative nominal central bank interest rates. In this paper, we have documented that negative central bank rates have not been transmitted to aggregate deposit rates, which remain stuck at levels close to zero. As a result of this, aggregate lending rates remain elevated as well. Using bank level data from Sweden, we documented a disconnect between the policy rate and lending rates, once the policy rate fell below zero. Moreover, there is a substantial increase in the dispersion of lending rates across banks. We showed that banks with high deposit shares generally have low pass-through. This is consistent with other interest rates relevant for banks funding

[30]In our model we only consider a negative interest rate on bank assets, as we impose $R \geq 0$. The TLTRO program implies a negative interest rate on a bank liability.

costs being somewhat more responsive to negative policy rates than deposit rates. Further, we confirmed that the results in Heider, Saidi, and Schepens (2016) also hold for Swedish banks, showing that banks with high deposit shares had lower credit growth in the aftermath of the policy rate becoming negative.

Motivated by our empirical findings, we developed a New Keynesian framework with savers and borrowers along the lines of Benigno, Eggertsson, and Romei (2014). We augmented the model by explicitly adding central bank reserves, based on Curdia and Woodford (2011). Using this setup, we considered two shocks to the economy - a preference shock and a intermediation cost shock. We showed analytically that the deposit rate is bounded at zero if there is no storage cost of money, or at some well-defined level below zero if there are proportional storage costs. This is in line with the bound observed in the data. We showed that lowering the policy rate below zero fails to stimulate aggregate demand once the deposit rate is bounded. As the deposit rate does not respond to negative policy rates, there is no expansionary effect through intertemporal substitution. Moreover, the bound on deposit rates eliminates the impact on borrowing rates, and so there is no stimulative effect through credit supply. Further, a negative interest rate on reserves reduces bank profits, which lowers aggregate demand. As a result, lowering the policy rate below zero has a contractionary effect on the economy.

Our empirical and theoretical analysis casts doubt on how potent negative nominal interest rates are in terms of stimulating real economic activity. Furthermore, if bank profits are sufficiently important in determining banks willingness to extend credit, negative rates may even have contractionary effects on the macroeconomy.

References

Bean, C. (2013): "Note on Negative Interest Rates for Treasury Committee," Discussion paper, Bank of England.

Bech, M. L., and A. Malkhozov (2016): "How have central banks implemented negative policy rates?," *Working paper*.

Benigno, P., G. B. Eggertsson, and F. Romei (2014): "Dynamic debt deleveraging and optimal monetary policy," Discussion paper, National Bureau of Economic Research.

Bianchi, J., and S. Bigio (2014): "Banks, liquidity management and monetary policy," Discussion paper, National Bureau of Economic Research.

Borio, C., and L. Gambacorta (2017): "Monetary policy and bank lending in a low interest rate environment: diminishing effectiveness?," *Journal of Macroeconomics*.

Brunnermeier, M. K., and Y. Koby (2016): "The Reversal Interest Rate: An Effective Lower Bound on Monetary Policy," *Preparation). Print*.

Curdia, V., and M. Woodford (2011): "The central-bank balance sheet as an instrument of monetary policy," *Journal of Monetary Economics*, 58(1), 54–79.

Eggertsson, G. B., and N. R. Mehrotra (2014): "A model of secular stagnation," Discussion paper, National Bureau of Economic Research.

Eggertsson, G. B., and M. Woodford (2006): "Optimal monetary and fiscal policy in a liquidity trap," in *NBER International Seminar on Macroeconomics 2004*, pp. 75–144. The MIT Press.

Gali, J. (2008): "Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework," .

Gertler, M., and N. Kiyotaki (2010): "Financial intermediation and credit policy in business cycle analysis," *Handbook of monetary economics*, 3(3), 547–599.

Guerrieri, L., and M. Iacoviello (2015): "OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily," *Journal of Monetary Economics*, 70, 22–38.

Heider, F., F. Saidi, and G. Schepens (2016): "Life Below Zero: Bank Lending Under Negative Policy Rates," *Working paper*.

Holmstrom, B., and J. Tirole (1997): "Financial intermediation, loanable funds, and the real sector," *the Quarterly Journal of economics*, 112(3), 663–691.

Jackson, H. (2015): "The international experience with negative policy rates," Discussion paper, Bank of Canada Staff Discussion Paper.

Jiménez, G., and S. Ongena (2012): "Credit supply and monetary policy: Identifying the bank balance-sheet channel with loan applications," *The American Economic Review*, 102(5), 2301–2326.

Jordan, T. (2016): "Monetary policy using negative interest rates: a status report," Discussion paper, Swiss National Bank.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2015): "Household leveraging and deleveraging," *Review of Economic Dynamics*, 18(1), 3–20.

Kashyap, A. K., and J. C. Stein (2000): "What do a million observations on banks say about the transmission of monetary policy?," *American Economic Review*, pp. 407–428.

Kerbl, S., and M. Sigmund (2017): "Negative Interest Rates: Forecasting Banks Profitability in a New Environment," *Working Paper*.

Kiley, M. T., and J. M. Roberts (2017): "Monetary policy in a low interest rate world," *Brookings Paper on Economic Activity*.

Krugman, P. R. (1998): "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, 1998(2), 137–205.

Rognlie, M. (2015): "What lower bound? Monetary policy with negative interest rates," *Unpublished manuscript, Department of Economics, Harvard University (November 23)*.

Rotemberg, J. J., and M. Woodford (1997): "An optimization-based econometric framework for the evaluation of monetary policy," *NBER macroeconomics annual*, 12, 297–346.

Stiglitz, J. (2016): "The problem with negative interest rates," Discussion paper.

The Riksbank (2015): "Monetary Policy Report February 2015," Discussion paper, Sveriges Riksbank.

——— (2016): "Perspectives on the negative repo rate," Discussion paper.

Waller, C. (2016): "Negative interest rates: a tax in sheep's clothing," Discussion paper.

Appendix A: Additional Figures

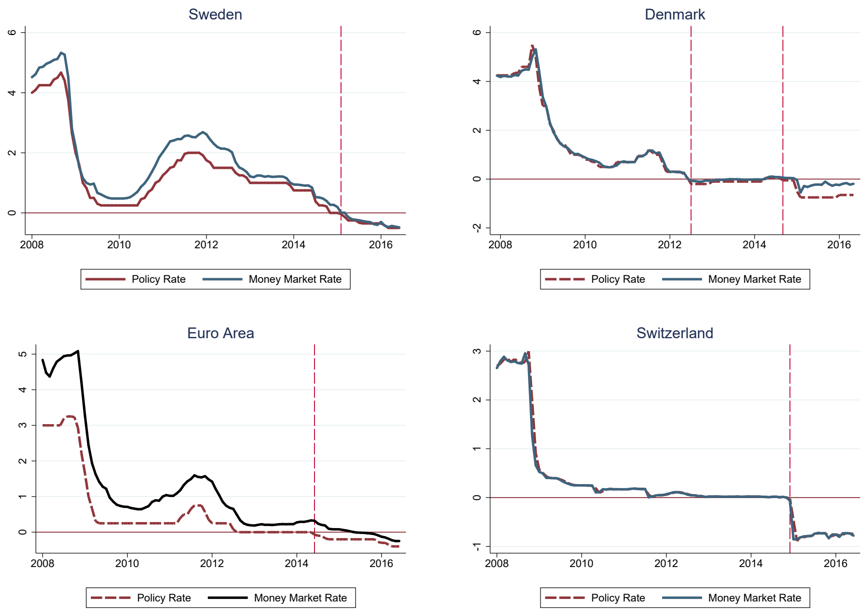

Figure 9: Money Market Rates for Sweden (3 month STIBOR), Denmark (Tomorrow/Next Rate), the Euro Area (3 month EURIBOR) and Switzerland (3 month LIBOR CHF).

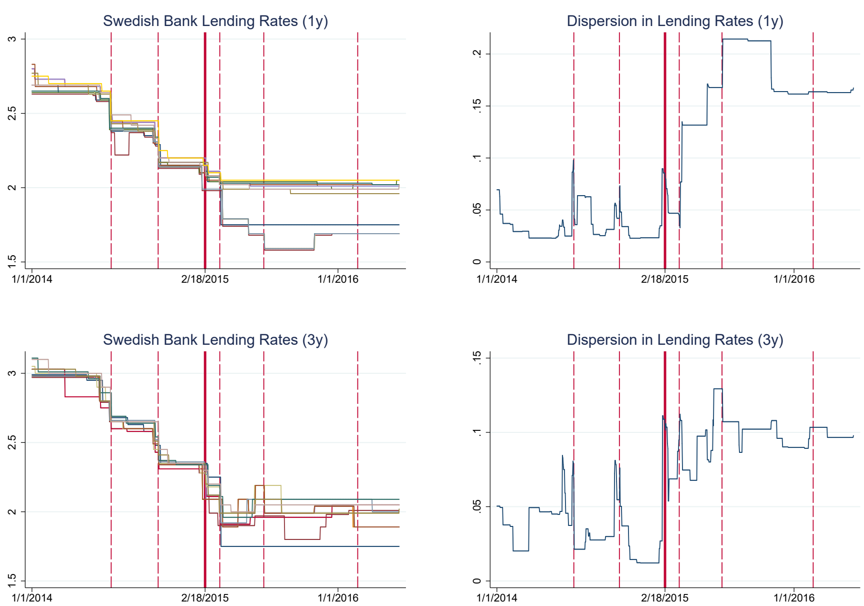

Figure 10: Bank Level Lending Rates Sweden. Interest rate on one-year and three-year mortgages.

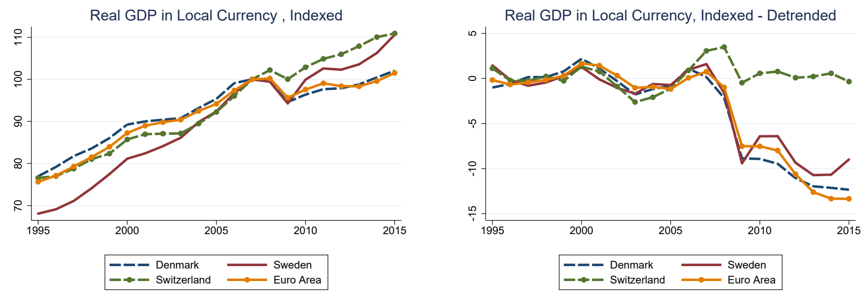

Figure 11: Gross Domestic Product in Constant Prices. Local Currency. Indexed so that GDP^{2008}=100. The right panel shows the detrended series using a linear time trend based on the 1995-2007 period.

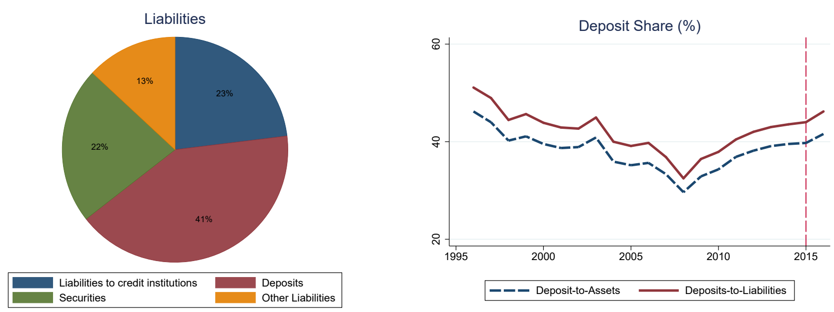

Figure 12: Decomposition of Liabilities (average 1996-2016) and Deposit Share for Swedish Banks.

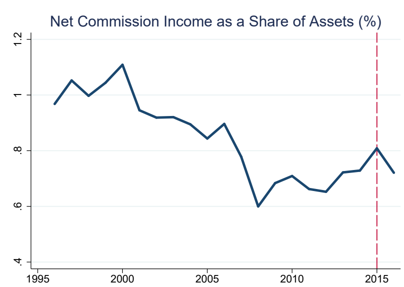

Figure 13: Net Commission Income as a Share of Total Assets for Swedish Banks. Source: Statistics Sweden.

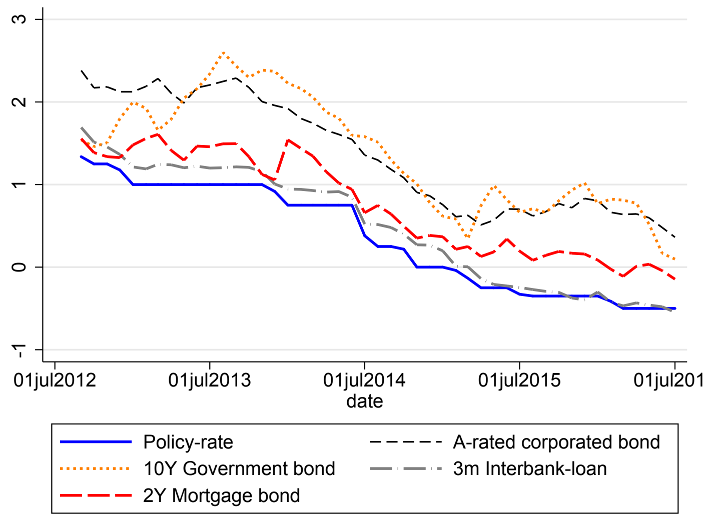

Figure 14: Other interest rates in Sweden. Source: Statistics Sweden.

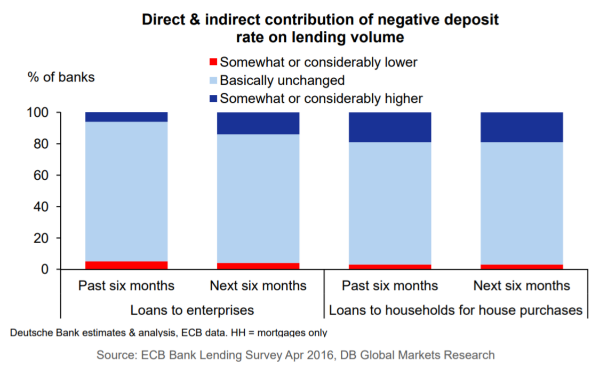

Figure 15: Share of banks answering that the negative ECB deposit rate has has a negative, neutral or positive effect on their lending volume. Source: Deutsche Bank.

Appendix C: Marginal and Average Rate on Reserves

In our model, central bank reserves earn a single interest rate i^r. In reality, central banks can adopt exemption thresholds and tiered remuneration schemes so that not all reserves earn the same interest rate. Hence, even though the key policy rate is negative, not all central bank reserves necessarily earn a negative interest rate. Here we provide a short overview of the different remuneration schedules implemented in the Euro Area, Denmark, Japan, Sweden and Switzerland. For a more detailed analysis see Bech and Malkozov (2016).

In the Euro Area, required reserves earn the main financing operations rate - currently set at 0.00 percent. Excess reserves on the other hand, earn the central bank deposit rate - currently set at -0.40 percent. Hence, only reserves in excess of the required level earn a negative interest rate. A similar remuneration scheme is in place in Denmark. Banks can deposit funds at the Danish central bank at the current account rate of 0.00 percent. However, there are (bank-specific) limits on the amount of funds that banks can deposit at the current account rate. Funds in excess of these limits earn the interest rate on one-week certificates of deposits - currently set at -0.65 percent.

The Riksbank issues one-week debt certificates, which currently earn an interest rate of -0.50 percent. While there is no reserve requirement, the Swedish central bank undertakes fine-tuning operations to drain the bank sector of remaining reserves each day. These fine-tuning operations earn an interest rate of -0.60 percent[31]. The Swiss central bank has the lowest key policy rate at -0.75 percent. However, due to high exemption thresholds the majority of reserves earn a zero interest rate. The Bank of Japan adopted a three-tiered remuneration schedule when the key policy rate turned negative. As a result, central bank reserves earn an interest rate of either 0.10, 0.00 or -0.10 percent.

Due to the tiered remuneration system, there is a gap between the average and the marginal reserve rate. Bech and Malkhozov (2016) calculate this gap as of February 2016, as illustrated in figure (16).

Central bank remuneration schedules (mid-February 2016) — Table 1

	European Central Bank	Sveriges Riksbank	Swiss National Bank	Danmarks Nationalbank
Exemption threshold	Minimum reserve requirement	.	Individual exemption	Current account limit
Aggregate amounts	Local currency, in billions			
Overnight deposits (reserves)				
Below threshold	113	–50[1]	303	29
Above threshold	650		170	.[3]
Term (one-week)	.	187	.	119
Policy rates	Basis points			
Overnight deposits (reserves)				
Below threshold	5	–60[2]	0	0
Above threshold	–30		–75	.[3]
Term (one-week)	.	–50	.	–65
Weighted average rate	–25	–52	–27	–52
Marginal minus average rate[4]	–5	–8	–48	–13

[1] Amount of fine-tuning operations. In addition, overnight deposits with central bank represent SEK 0.01 billion. [2] Rate applied to fine-tuning operations. Overnight deposits with central bank earn –125 basis points. [3] Amounts above the aggregate current account limit are converted into one-week certificates of deposit (Box 2). [4] Marginal rate is the rate on overnight deposits with central bank above exemption threshold.

Sources: Central banks; authors' calculations.

Figure 16: Reserve Rates - Source: Bech and Malkozov (2016).

[31] Any residual reserves earn the deposit rate of -1.25 percent.

Appendix D: Firm Problem

A firm that is allowed to reset their price in period t sets the price to maximize the present value of discounted profits in the event that the price remains fixed. That is, each firm i choose $P_t(i)$ to maximize

$$\mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left[\Pi^{T-t} \frac{P_t(i)}{P_T} Y_T(i) - \frac{W_T}{P_T} Y_T(i) \right] \tag{31}$$

where $\lambda_T \equiv q\left(\chi \exp\left\{-qC_T^b\right\} + (1-\chi)\exp\left\{-qC_T^s\right\}\right)$, which is the weighted marginal utility of consumption and $\beta \equiv \chi\beta^b + (1-\chi)\beta^s$[32].

Denoting the markup as $\mu \equiv \dfrac{\theta}{\theta-1}$, firms set the price as a markup over the average of expected marginal costs during the periods the price is expected to remain in place. That is, the first-order condition for the optimal price $P(i)_t^*$ for firm i is

$$\frac{P(i)_t^*}{P_t} = \mu \frac{\mathbb{E}_t\left\{\sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t}\frac{1}{\Pi^{T-t}}\right)^{\theta} \frac{W_T}{P_T} Y_T\right\}}{\mathbb{E}_t\left\{\sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t}\frac{1}{\Pi^{T-t}}\right)^{\theta-1} \frac{W_T}{P_T} Y_T\right\}} \tag{32}$$

This implies a law of motion for the aggregate price level

$$P_t^{1-\theta} = (1-\alpha) P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta} \Pi^{1-\theta} \tag{33}$$

where P_t^* is the optimal price from equation (32), taking into account that in equilibrium $P_t^*(i)$ is identical for all i. We denote this price P_t^*.

Since prices are sticky, there exists price dispersion which we denote by

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di \tag{34}$$

with the law of motion

$$\Delta_t = \alpha \left(\frac{\Pi_t}{\Pi}\right)^{\theta} \Delta_{t-1} + (1-\alpha) \left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha} \right)^{\frac{\theta}{\theta-1}} \tag{35}$$

[32] Recall that the firm is owned by both types of households according to their respective population shares.

We assume that the disutility of labor takes the form $V(N_t^j) = \frac{(N^j)^{1+\eta}}{1+\eta}$. We can then combine equations (32) - (35), together with the aggregate labor-consumption trade-off (equation (8)) to get an aggregate Phillips curve of the following form:

$$\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha} \right)^{\frac{1}{\theta-1}} = \frac{F_t}{K_t} \tag{36}$$

where

$$F_t = \lambda_t Y_t + \alpha\beta\mathbb{E}_t \left\{ F_{t+1}\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta-1} \right\} \tag{37}$$

and

$$K_t = \mu\frac{\lambda_t \Delta_t^{\eta} Y_t^{1+\eta}}{z\exp\left\{-zY_t\right\}} + \alpha\beta\mathbb{E}_t \left\{ K_{t+1}\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta} \right\} \tag{38}$$

and

$$\lambda_T \equiv z\left(\chi\exp\left\{-qC_T^b\right\} + (1-\chi)\exp\left\{-qC_T^s\right\}\right) \tag{39}$$

Since every firm faces demand $Y(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$ and $Y_t(i) = N_t(i)$, we can integrate over all firms to get that

$$N_t = \Delta_t Y_t \tag{40}$$

Appendix E: Equilibrium

Non-linear Equilibrium Conditions

Let $\zeta_t \equiv \frac{\tilde{\zeta}_{t+1}}{\tilde{\zeta}_t}$. For given initial conditions Δ_0, b_0^b and a sequence of shocks $\left\{\zeta_t, \bar{b}_t\right\}_{t=0}^{\infty}$, an equilibrium in our model is a sequence of endogenous prices $\left\{i_t^s, i_t^b, i_t^r\right\}_{t=0}^{\infty}$ and endogenous variables $\left\{C_t^b,\, C_t^s,\, b_t^b,\, m_t^b,\, Y_t,\, \Pi_t,\, F_t,\, K_t,\, \Delta_t,\, \lambda_t,\, l_t,\, R_t,\, m_t,\, z_t\right\}_{t=0}^{\infty}$ such that the 17 equations listed below are satisfied. The first two equations are the Euler equations for borrowers and savers, while the third equation is the borrowers budget constraint. Note that we have inserted for firm profits in the latter. The borrowers money demand is captured by the fourth equation. The fifth equation is the aggregate demand equation, followed by the five supply side equations. The eleventh equation is bank profits, proceeded by the three first order conditions from the bank problem. Finally, there is the Taylor rule, the lower bound on the deposit rate and the market clearing condition in the debt market.

$$\exp\left\{-qC_t^b\right\} = \beta^b\left(1+i_t^b\right)\mathbb{E}_t\left(\Pi_{t+1}^{-1}\exp\left\{-qC_{t+1}^b\right\}\right)\zeta_t \tag{41}$$

$$\exp\left\{-qC_t^s\right\} = \beta^s\left(1+i_t^s\right)\mathbb{E}_t\left(\Pi_{t+1}^{-1}\exp\left\{-qC_{t+1}^s\right\}\right)\zeta_t \tag{42}$$

$$C_t^b + \frac{1+i_{t-1}^b}{\Pi_t}b_{t-1}^b + m_t^b = \chi Y_t + b_t^b + \frac{1-\gamma}{\Pi_t}m_{t-1}^b \tag{43}$$

$$\frac{\Omega'\left(m_t^b\right)}{U'\left(C_t^b\right)} = \frac{i_t^b+\gamma}{1+i_t^b} \tag{44}$$

$$Y_t = \chi C_t^b + (1-\chi)C_t^s \tag{45}$$

$$\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{1}{\theta-1}} = \frac{F_t}{K_t} \tag{46}$$

$$F_t = \lambda_t Y_t + \alpha\beta\mathbb{E}_t\left\{F_{t+1}\left(\frac{\pi_{t+1}}{\pi}\right)^{\theta-1}\right\} \tag{47}$$

$$K_t = \mu\frac{\lambda_t\Delta_t^\eta Y_t^{1+\eta}}{q\exp\left\{-qY_t\right\}} + \alpha\beta\mathbb{E}_t\left\{K_{t+1}\left(\frac{\Pi_{t+1}}{\Pi}\right)^\theta\right\} \tag{48}$$

$$\lambda_t = q\left(\chi\exp\left\{-qC_t^b\right\} + (1-\chi)\exp\left\{-qC_t^s\right\}\right) \tag{49}$$

$$\Delta_t = \alpha\left(\frac{\pi_t}{\pi}\right)^\theta\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}} \tag{50}$$

$$z_t^{bank} = \frac{i_t^b-i_t^s}{1+i_t^s}l_t - \frac{i_t^s-i_t^r}{1+i_t^s}R_t - \frac{i_t^s+\gamma}{1+i_t^s}m_t - \Gamma\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t^{bank}\right) \tag{51}$$

$$\frac{i_t^b-i_t^s}{1+i_t^s} = \frac{1}{\bar{l}_t}\Gamma_l\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t^{bank}\right) \tag{52}$$

$$-\Gamma_R\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t^{bank}\right) = \frac{i_t^s-i_t^r}{1+i_t^s} \tag{53}$$

$$-\Gamma_m\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t^{bank}\right) = \frac{i_t^s+\gamma}{1+i_t^s} \tag{54}$$

$$i_t^r = r_t^n\pi_t^{\phi_\pi}y_t^{\phi_Y} \tag{55}$$

$$i_t^s = \max\left\{-\gamma, i_t^r\right\} \tag{56}$$

$$l_t = \chi b_t^b \tag{57}$$

Steady state

We denote the steady-state value of a variable X_t as X.

First, observe that in steady-state inflation is at the inflation target Π . As a result, there is no price dispersion ($\Delta = 1$). For simplicity, we also assume that $\Pi = 1$ in steady state.

Combining this with the Phillips curve, we have that steady-state output is pinned down by the following equation

$$\mu \frac{Y^\eta}{q \exp\{-qY\}} = 1 \tag{58}$$

From the Euler equation of a household of type j we have that

$$1 + i^j = \frac{1}{\beta^j} \tag{59}$$

Using the steady-state interest rates, we can jointly solve for all bank-variables exploiting the fact that, with $\pi = 1$, the bank's problem is independent of other household- and firm-level variables. Notice that in steady-state, banks are satiated in reserves $R = \overline{R}$ by assumption. Furthermore, if the intermediation cost function is additive between money and the other arguments (which we assume, see below), the steady-state level of money holdings for banks is independent of other bank variables. Therefore, only bank profits and bank lending have to be solved jointly.

Given total debt and interest rates, the borrowers budget constraint and money demand can be solved for steady state consumption and money holdings:

$$C^b = \chi Y - i^b b^b - \gamma m^b \tag{60}$$

$$\Omega'(m^b) = \frac{i^b + \gamma}{1 + i^b} \tag{61}$$

Then, using the aggregate resource constraint we have that

$$C^s = \frac{1 - \chi^2}{1 - \chi} Y + \frac{\chi}{1 - \chi} (i^b b^b + \gamma m^b) \tag{62}$$

Log-linearized equilibrium conditions

We log linearize the non-linear equilibrium conditions around steady state, and define $\hat{X} \equiv \frac{X_t - X}{X}$. For the intermediation cost function we assume the following functional form

$$\Gamma\left(\frac{l_t}{\bar{l}_t}, R_t, m_t, z_t^{bank}\right) = \begin{cases} \left(\frac{l_t}{\bar{l}_t}\right)^{\nu} \left(z_t^{bank}\right)^{\iota} + \frac{1}{2}\left(R_t - \overline{R}\right)^2 + \frac{1}{2}\left(m_t - \overline{m}\right)^2 & \text{if } R_t < \overline{R} \text{ and } m_t < \overline{m} \\ \left(\frac{l_t}{\bar{l}_t}\right)^{\nu} \left(z_t^{bank}\right)^{\iota} & \text{if } R_t \geq \overline{R} \text{ and } m_t \geq \overline{m} \end{cases} \tag{63}$$

By combining the two Euler equations and the aggregate demand equation we derive the IS curve in equation (64), where we define $\sigma = \frac{1}{qy}$. By combining the five supply side equations we derive the Phillips curve in equation (69). We define the real interest rate $\hat{r}_t^n$ in equation (65), and the interest rate spread in equation (70). We also use the market clearing condition to substitute $\hat{l}_t$ for $\hat{b}_t^b$. Hence, an equilibrium of the log-linearized model is a process for the 12 endogenous variables $\left\{\hat{c}_t^b,\, \hat{b}_t^b,\, \hat{m}_t^b,\, \hat{y}_t,\, \hat{r}_t^n,\, \hat{\pi}_t,\, \hat{m}_t,\, \hat{z}_t^{bank},\, \hat{\omega}_t,\, \hat{i}_t^b,\, \hat{i}_t^s,\, \hat{i}_t^r\right\}_{t=0}^{\infty}$ such that the 12 equations listed below are satisfied.

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - \sigma\left(\hat{i}_t^s - \mathbb{E}_t\pi_{t+1} - \hat{r}_t^n\right) \tag{64}$$

$$\hat{r}_t^n = -\hat{\zeta}_t - \chi\hat{\omega}_t \tag{65}$$

$$\hat{c}_t^b = \hat{c}_{t+1}^b - \frac{1}{qc^b}\left(\hat{i}_t^b - \mathbb{E}_t\hat{\pi}_{t+1} + \hat{\zeta}_t\right) \tag{66}$$

$$\begin{aligned} c^b\hat{\pi}_t + c^b\hat{c}_t^b = {} & \hat{\pi}_t\left(\chi y + b^b\right) + \chi\hat{y}_t y + \hat{b}_t^b b^b - \hat{i}_{t-1}^b b^b - \left(1 + i^b\right)\hat{b}_{t-1}^b b^b \\ & - m^b(\hat{m}_t^b + \hat{\pi}_t) + (1-\gamma)m^b\hat{m}_{t-1}^b \end{aligned} \tag{67}$$

$$\frac{\Omega''(m^b)m^b}{\Omega'(m^b)}\hat{m}_t^b = -qc^b\hat{c}_t^b - \frac{i^b + \gamma - 1}{i^b + \gamma}\hat{i}_t^b \tag{68}$$

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} \tag{69}$$

$$\hat{i}_t^b = \hat{i}_t^s + \hat{\omega}_t \tag{70}$$

$$\hat{\omega}_t = \frac{i^b - i^s}{1 + i^b}\left((\nu - 1)\hat{b}_t^b - \nu\hat{\bar{b}}_t + \iota\hat{z}_t^{bank}\right) \tag{71}$$

$$\begin{aligned} \hat{i}_t^s + \hat{z}_t^{bank} = {} & \frac{\chi b^b}{(1+i^s)\, z^{bank}}\left(\hat{i}_t^b - \hat{i}_t^{\,s} + \left(i^b - i^s\right)\hat{b}_t^b\right) - \frac{R}{(1+i^s)\, z^{bank}}\left(\hat{i}_t^s - \hat{i}_t^r\right) \\ & - \frac{m}{(1+i^s)\, z^{bank}}\left(\hat{i}_t^s + (i^s + \gamma)\,\hat{m}_t\right) - \frac{\Gamma}{(1+i^s)\, z^{bank}}\hat{i}_t^s \\ & - \frac{\Gamma_z}{\iota}\left(\nu\left(\hat{b}_t^b - \hat{\bar{b}}_t\right) + \iota\hat{z}_t^{bank}\right) - \frac{m\,(m - \overline{m})}{z^{bank}}\hat{m}_t \end{aligned} \tag{72}$$

$$\hat{m}_t = \frac{m - \bar{m}}{m}\frac{1 - i^s - \gamma}{i^s + \gamma}\hat{i}_t^s \tag{73}$$

$$\hat{i}_t^r = \hat{r}_t^e + \phi_\Pi\hat{\pi}_t + \phi_Y\hat{y}_t \tag{74}$$

$$i_t^s = \max\left\{\underline{\mathrm{i}}^s,\, i_t^r\right\} \tag{75}$$