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ABSTRACT

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Capital Reallocation and Aggregate Productivity*

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January 16, 2015

Abstract

This paper studies the productivity implications of the cyclical reallocation of capital. Frictions in the reallocation process are a source of factor misallocation. Cyclical movements in these frictions lead to variations in the degree of reallocation and thus in productivity. These frictions also impact the capital accumulation decision. The effects are quantitatively important in the presence of fluctuations in adjustment frictions and/or the cross sectional variation of profitability shocks. The cyclicality of the output loss due to costly reallocation depends on the joint distribution of capital and plant-level productivity. Instead of relying on approximative solution techniques we show analytically that a higher-order moment is needed to solve the model accurately. Even without aggregate productivity shocks, the model has quantitative properties that resemble those of a standard stochastic growth model: (i) persistent shocks to the Solow residual, (ii) positive co-movement of output, investment and consumption and (iii) consumption smoothing.

1 Motivation

Frictions in the reallocation of capital and labor are important for understanding aggregate productivity. With heterogenous plants, the assignment of capital, labor and other inputs across production sites impacts directly on aggregate productivity. Frictions in the reallocation process thus lead to the misallocation of factors of production (relative to a frictionless benchmark). This point lies at the heart of the analysis of productivity both within and across countries in Maksimovic and Phillips (2001), Hsieh and Klenow (2009), Bartelsman, Haltiwanger, and Scarpetta (2013) Restuccia and Rogerson (2008) and others.[1]

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[1] More specific differences with these and other studies are discussed below.

In this paper we consider the **cyclical dimension of reallocation** in the presence of capital reallocation costs. In important empirical contributions, Eisfeldt and Rampini (2006) and Kehrig (2011) show that capital reallocation is pro-cyclical and that the cross-sectional productivity dispersion behaves counter-cyclically.[2] This not only underlines the significance of heterogeneity in the production sector but also suggests that frictions in the reallocation of capital may produce cyclical effects on output over the business cycle. One contribution of this paper is to specify a dynamic general equilibrium model to further understand these findings about cyclical reallocation and dispersion in productivities.

Figure 1 highlights the cyclicality of capital reallocation in the US. Throughout, we define capital reallocation as sales of property, plant and equipment (PP&E). The data is described in detail in Appendix C. The correlation between the cyclical components of capital reallocation and real GDP is 0.53.[3] The data furthermore reveals counter-cyclical dispersion: the standard deviation in Tobin's Q, which serves as a proxy for the average value of capital, shows a negative correlation with US GDP of -0.14.[4]

From Olley and Pakes (1996) and other contributions, not properly taking cross-sectional heterogeneity into account will lead to a mis-measurement of total factor productivity (TFP). We are interested in the cyclical component of the output loss resulting from frictions in the reallocation process which will be reflected in mis-measured TFP. This relates to the question of how micro-frictions, like adjustment costs in the accumulation and reallocation of capital, translate into aggregate outcomes. **We find that if the only shocks in the economy are to aggregate TFP, then the productivity loss from costly reallocation has no cyclical element.**[5] If an aggregate model behaves as if there were no non-convexities at the plant-level, then the distortions in the allocation of capital across plants with different productivities will matter only for aggregate *levels*. As a result, the distribution over plants' capital stock and idiosyncratic productivity can be extremely well approximated by its first moment.

So, in addition to shocks to aggregate productivity, we also study shocks to plants' adjustment opportunities. This is similar to Eisfeldt and Rampini (2006) where plants face a time-varying adjustment cost. Furthermore, we study shocks to the distribution of idiosyncratic productivity as in Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), Gilchrist, Sim, and Zakrajsek (2013), or Bachmann and Bayer (2013).

Both of those shocks create cyclical movements in reallocation and productivity as well as time-varying productivity dispersion. Cross-sectional heterogeneity now plays an important role for shaping aggregate dynamics. In the presence of those shocks, capital reallocation endogenously responds and leads to variations in measured aggregate productivity. The

[2]Eisfeldt and Rampini (2006) use dispersion in firm level Tobin's Q, dispersion in firm level investment rates, dispersion in total factor productivity growth rates, and dispersion in capacity utilization. Kehrig (2011) constructs dispersion measures based on TFP estimates.

[3]In Eisfeldt and Rampini (2006) capital reallocation is defined as the sum of sales of PP&E and acquisitions. Since our model does not feature acquisitions we focus on sales of PP&E. The correlation for acquisitions exhibits more cyclicality than for sales of PP&E (0.58).

[4]See Figure 5 in Appendix C.

[5]See Veracierto (2002), Thomas (2002), Khan and Thomas (2003) and Gourio and Kashyap (2007) as well.

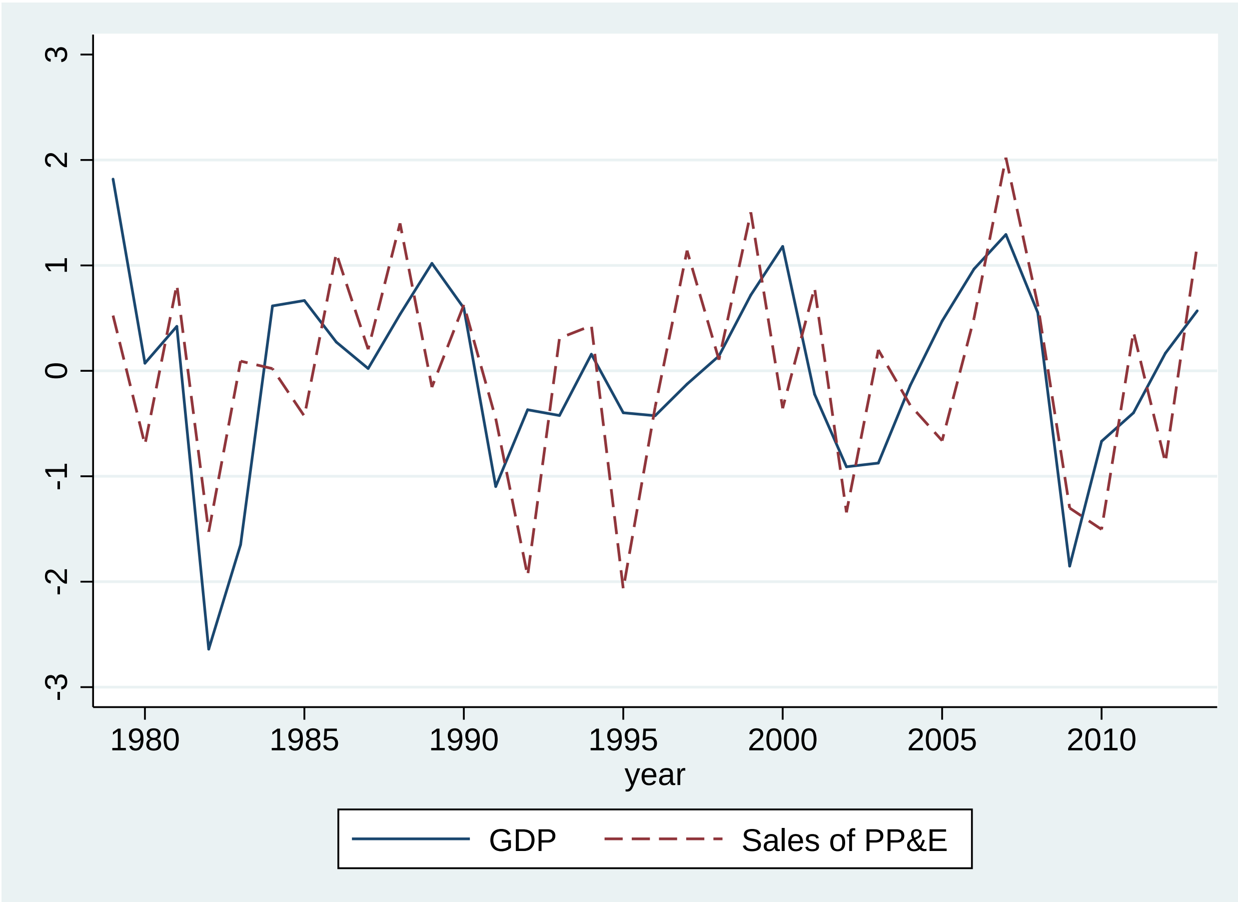

Figure 1: Capital Reallocation over the Business Cycle. The solid blue line denotes real US GDP, the dashed red line denotes capital reallocation. Both series are in logs, have been HP-filtered and normalized by their standard deviation. Source: Compustat.

cross-sectional joint distribution over plants' capital stock and idiosyncratic productivity is a slow-moving object in this environment and tracking its evolution only by its first moment is insufficient: higher order moments are needed to characterize the general equilibrium outcome, in particular the covariance of the cross-sectional distribution between plants' capital stocks and profitability.

Importantly these features of our model are interrelated. The fact that the covariance matters as a moment for determining the optimal allocation is indicative of the significance of reallocation effects. If the covariance was not needed for characterizing optimal allocations, for example because it is constant over time or perfectly correlated with the mean, then reallocation could not have a cyclical effect on aggregate output.[6] Thus the covariance that matters from the perspective of the Krusell and Smith (1998) approach is precisely the moment that reflects cyclical gains to capital reallocation.

This last point is worth stressing. Studies following Krusell and Smith (1998) routinely find that only first moments of distributions are needed to summarize cross sectional distributions. In our economy, the covariance of the cross sectional distribution between a plant's capital and its profitability

[6]As discussed below, even if the covariance is constant, reallocation may be important for average productivity.

is needed in the state space of the problem. When there are shocks either to the capital adjustment process or to the cross sectional distribution, this covariance evolves in response to these shocks. In the presence of such shocks the approximate solution to the planner's problem using only average capital fails: the solution requires higher order moments.

As a final exercise, we study the business cycle properties of an economy driven by shocks to adjustment rates and to the cross sectional distribution of idiosyncratic shocks assuming constant aggregate total factor productivity.[7] This exercise provides a basis for "adverse" aggregate productivity shocks and the serial correlation of the Solow residual. **The aggregate moments produced by this economy are very similar to the moments of the standard stochastic growth model.** In particular: (i) the Solow residual is pro-cyclical and positively serially correlated, (ii) consumption, investment and output are positively correlated, (iii) consumption is smoothed, (iv) reallocation is pro-cyclical and (v) the standard deviation of productivity across plants is counter-cyclical. The first three properties match those of the standard RBC model. The last two properties match those stressed by Eisfeldt and Rampini (2006) and Kehrig (2011). In our setting, reductions in the Solow residual come from variations in the the reallocation process and the distribution of shocks, not an adverse shock to total factor productivity.

2 Frictionless Economy

To fix basic ideas and notation, consider an economy with heterogeneity and no frictions in the accumulation of capital nor in its reallocation. The planner maximizes

$$V(A,K) = max_{K',k(\varepsilon)} \;\; u(c) + \beta E_{A'|A} V(A',K') \tag{1}$$

for all (A,K). The constraints are

$$c + K' = y + (1-\delta)K, \tag{2}$$

$$\int_{\varepsilon} k(\varepsilon) f(\varepsilon) d\varepsilon = K, \tag{3}$$

$$y = A \int_{\varepsilon} \varepsilon k(\varepsilon)^{\alpha} f(\varepsilon) d(\varepsilon). \tag{4}$$

The objective function is the lifetime utility of the representative household. The state vector has two elements: A is aggregate TFP and K is the aggregate stock of capital. There is a distribution of plant specific productivity shocks, $f(\varepsilon)$ which is (provisionally) fixed and hence omitted from the state vector.

At the beginning of the period, A as well as the idiosyncratic productivity shocks ε realize. There

[7]This analysis shares some features with Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) and Bachmann and Bayer (2013). Differences and similarities are made clear in the next sections.

are two controls in (1). The first is the choice of aggregate capital for the next period. The second is the assignment function, $k(\varepsilon)$, which allocates the given stock of capital across the production sites, indexed by their current productivity. While aggregate capital K requires one period time-to-build, the reallocation of existing capital takes place instantaneously and is given by $k(\varepsilon)$.

The resource constraint for the accumulation of aggregate capital is given in (2). The constraint for the allocation of capital across production sites in given in (3). From (4), total output, y, is the sum of the output across production sites. The production function at any site is

$$y(k, A, \varepsilon) = A\varepsilon k^{\alpha} \tag{5}$$

where k is the capital used at the site with productivity ε.[8] Both idiosyncratic and aggregate productivity shocks ε and A can be persistent, parameterized by $\rho_\varepsilon \in [0, 1]$ and $\rho_A \in [0, 1]$. We assume $\alpha < 1$ as in Lucas (1978).[9] In this frictionless environment, a plants' optimal capital stock is entirely determined by ε.

The assumption of diminishing returns to scale, $\alpha < 1$, implies that the allocation of capital across production sites is non-trivial. There are gains to allocating capital to high productivity sites but there are also gains, due to $\alpha < 1$, from spreading capital across production sites.

2.1 Optimal Choices

Within a period, the condition for the optimal allocation of capital across production sites is given by $\alpha A\varepsilon k(\varepsilon)^{\alpha-1} = \eta$ for all ε, where η is the multiplier on (3). This condition is intuitive: absent frictions, the optimal allocation equates the marginal product of capital across production sites.

Working with this condition,

$$k(\varepsilon) = \frac{\eta}{\alpha A\varepsilon}^{\frac{1}{\alpha-1}}. \tag{6}$$

Using (3),

$$\eta = A\alpha K^{\alpha-1}\left(\int_{\varepsilon} \varepsilon^{\frac{1}{1-\alpha}} f(\varepsilon) d\varepsilon\right)^{1-\alpha}. \tag{7}$$

The multiplier is the standard marginal product on an additional unit of capital times the effect of

[8] Labor and other inputs are not made explicit. One interpretation is that these inputs have no adjustment costs and are optimally chosen each period, given the state. In this case, the marginal product of labor (and other inputs) will be equal across production sites. This does not imply equality of the marginal products of capital. Adding labor adjustment, perhaps interactive with capital adjustment, would be a natural extension of our model. Presumably, adding labor frictions would enhance our results. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) include labor adjustment costs while Bachmann and Bayer (2013) assume flexible labor.

[9] As in Cooper and Haltiwanger (2006), estimates of α are routinely below unity. This is interpreted as reflecting both diminishing returns to scale in production and market power due to product differentiation. For simplicity, our model ignores product differentiation and treats the curvature as reflecting diminishing returns. The analysis in Kehrig (2011) includes product differentiation at the level of intermediate goods.

the ε distribution on productivity. Putting these two conditions together,

$$k(\varepsilon) = K \frac{\varepsilon^{\frac{1}{1-\alpha}}}{\int_{\varepsilon} \varepsilon^{\frac{1}{1-\alpha}} f(\varepsilon) d\varepsilon}. \tag{8}$$

Substituting into (4) yields

$$y = AK^{\alpha} \left(\int_{\varepsilon} \varepsilon^{\frac{1}{1-\alpha}} f(\varepsilon) d\varepsilon \right)^{1-\alpha}. \tag{9}$$

This is a standard aggregate production function, AK^{α}, augmented by a term that captures a "love of variety" effect from the optimal allocation of capital across plants. With a given distribution $f(\cdot)$ the idiosyncratic shocks magnify average aggregate productivity as the planner can reallocate inputs to the more productive sites.

The condition for **intertemporal optimality** is $u'(c) = \beta E V_K(A', K')$ so that the marginal cost and expected marginal gains of additional capital are equated. Using (1), this condition becomes

$$u'(c) = \beta E u'(c') \left[(1-\delta) + A' \alpha K'^{\alpha-1} \left(\int_{\varepsilon} \varepsilon^{\frac{1}{1-\alpha}} f(\varepsilon) d\varepsilon \right)^{1-\alpha} \right]. \tag{10}$$

The left side is the marginal cost of accumulating an additional unit of capital. The right side is the discounted marginal gain of capital accumulation. Part of this gain comes from having an extra unit of capital to allocate across production sites in the following period. The productivity from these production sites depend ons two factors, the expected **future** values of aggregate productivity, A', and the cross sectional distribution of idiosyncratic shocks, $f(\varepsilon)$.

The choice of k for each plant within a period is independent of the choice between consumption and saving. The planner optimally allocates capital to maximize the level of output and then allocates output between consumption and capital accumulation. Clearly, once we allow for limits to reallocation, the capital accumulation decision will depend upon the future allocation of capital across production sites.

2.2 Aggregate Output and Productivity

For this economy, there is an interesting way to represent productivity and total output. This is seen from defining $k(\varepsilon) = \xi(\varepsilon)K$, so that $\xi(\varepsilon)$ is the fraction of the capital stock going to a plant with productivity ε. Then (4) becomes:

$$y = AK^{\alpha} \int_{\varepsilon} \varepsilon \xi(\varepsilon)^{\alpha} f(\varepsilon) d\varepsilon \tag{11}$$

By defining a measure of productivity $\tilde{A}$ as

$$\tilde{A} \equiv A \int_{\varepsilon} \varepsilon \xi(\varepsilon)^{\alpha} f(\varepsilon) d\varepsilon \tag{12}$$

total output can be simplified to

$$y = \tilde{A} K^{\alpha}. \tag{13}$$

Researchers interested in measuring TFP from the aggregate data will typically uncover $\tilde{A}$ rather than A. This is the mis-measurement referred to earlier. As the discussion progresses, we will refer to $\tilde{A}$ as the Solow residual, as distinct from aggregate TFP.[10]

From (12) there are three factors which influence $\tilde{A}$. The first one is A. The influence of A, aggregate TFP, on the Solow residual $\tilde{A}$ is direct and has been central to many studies of aggregate fluctuations.

Second, fluctuations in $f(\varepsilon)$ influence $\tilde{A}$ because variations in the cross sectional distribution of the idiosyncratic shocks lead to different marginal productivities of plants and thus changes in the Solow residual. Without any costs of reallocation, a mean-preserving spread in the distribution of idiosyncratic shocks, for example, creates opportunities to assign more capital to higher productivity sites and thus output as well as productivity will increase.

Finally, there is the allocation of factors, ξ. If factors are optimally allocated, then the distribution of capital over plants does not have an independent effect on $\tilde{A}$. However, the presence of frictions may imply that, in a static sense, capital is not efficiently allocated. In that case, even with $f(\varepsilon)$ fixed, the reallocation process will lead to variations in $\tilde{A}$. This is the topic of the next section.

3 Costly Reallocation

The allocation of capital over sites has significant effects on measured total factor productivity in the presence of idiosyncratic productivity shocks. In a frictionless economy with fixed $f(\varepsilon)$ there are no cyclical effects of reallocation on productivity. However, there is ample evidence in the literature for both non-convex and convex adjustment costs associated with changes in plant-level capital. Introducing these adjustment costs will enrich the analysis of productivity and reallocation.[11]

There are two distinct frictions to study, corresponding to the two dimensions of capital adjustment. The first, our focus here, is "costly reallocation" in which the friction is associated with the allocation of capital across the production sites. The second is "costly accumulation" in which the adjustment cost refers to the cost of accumulating rather than allocating capital.

Given the emphasis on reallocation, we study a tractable yet rich model of reallocation costs. Following Calvo (1983) and more recently adopted to study investment decisions by Sveen and Weinke

[10]Thanks for Susanto Basu for urging us to make these terms clear.

[11]In contrast to Midrigan and Xu (2014) there are no borrowing frictions. They argue that these frictions do not create large losses from misallocation between firms, but potentially large losses by deterring entry. In Cui (2014) capital reallocation is pro cyclical because partial irreversibility interacts with financial constraints.

(2005), assume that each period a Bernoulli draw determines the fraction $\pi \in [0,1]$ of plants the planner can costlessly reallocate capital between. The remaining fraction of plants, $1-\pi$, produces with its beginning-of-period capital stock. By assumption, π is not dependent on the state of the plant. Applying a law of large numbers, the plant-specific shocks ε are assumed to be equally distributed over the fractions π and $1-\pi$ of adjustable and non-adjustable plants. This also implies that the mean of ε, denoted $\bar{\varepsilon}$, is time-invariant and identical across adjustable and non-adjustable plants.

There are a number of arguments in favor of this specification of reallocation costs, beyond its tractability. First, the constant hazard assumption allows us to isolate the effects of the cross-sectional distribution through its effects on the allocation of capital and hence output rather than through adjustment costs alone. That is, by construction if the cross sectional distribution influences aggregate outcome, it does so through the determination of aggregate output, as in (12) and not through its interaction with a state dependent adjustment hazard. This does not deny the significance of adjustment costs but rather focuses solely on the output effects of the cross sectional distribution.

Second, following Midrigan (2011), the constant hazard model provides a very good approximation to the adjustment of a plant with multiple inputs, such as different types of capital. Appendix B studies the choice problem of a plant which incurs a cost of adjusting each of its $i = 1, 2, 3, ...I$ inputs. For each of these inputs, it draws a stochastic adjustment costs and decides to adjust or not. A key assumption, as in Midrigan (2011), is that the payment of an adjustment cost for one input, allows free adjustment of all other inputs. Thus there is an extreme complementarity in the adjustment process. A natural interpretation is that members of a production team must meet to make decisions on inputs (and even prices). Once the fixed meeting cost is paid, the marginal cost of adjusting all factors is minimal.

It is possible to gauge how well the constant hazard model approximates the decision rule of a plant with non-convex adjustment costs for multiple inputs. The analysis in the appendix makes clear that even for small I, it is as if the adjustment hazard was nearly flat. The intuition is that with another factors, the likelihood that the adjustment decision depends on the state of a given factor is small. A nice feature of the resulting model is that small adjustments are not incompatible with non-convex adjustment costs. This brings the model closer to the data indicating small investment rates reported in Cooper and Haltiwanger (2006).

Finally, given our emphasis on reallocation of factors, there is another interpretation of π that reflects stochastic trading opportunities in a decentralized search and matching structure. This interpretation rest on the analysis and discussion Ramey and Shapiro (2001) who study the sale of used capital.[12] They emphasize the specificity of capital as well as the thinness of markets. While convenient analytically, the treatment of used capital sales through Walrasian markets does not do justice to the frictions encountered in these trades.[13] These frictions lead to both trading delays and deep discounts on used capital. Thus π can be viewed as a simple device to capture the trading process that underlies the reallocation of capital.

[12] In fact, the working paper version of the published article, Ramey and Shapiro (1998), includes a model of these frictions. See Ottonello (2014) for further analysis of a model with capital market frictions.

[13] See also the discussion in Kurmann and Petrosky-Nadeau (2007).

3.1 The Planner's Problem

For the dynamic program of the planner in the presence of adjustment costs, the state vector includes aggregate productivity A, the aggregate capital stock K, and Γ, the joint distribution over beginning-of-period capital and productivity shocks across plants. Γ is needed in the state vector because the presence of adjustment costs implies that a plant's capital stock may not reflect the current draw of ε. As noted above, frictions to reallocation are introduced in the form of a probability of adjustment π, which we allow to be time-varying. The current value of π therefore becomes part of the state-vector.

Following the discussion above, variations in $f(\varepsilon)$, the distribution of idiosyncratic shocks, influence measured aggregate productivity. To study this effect further, we introduce shocks to the variance of idiosyncratic productivity shocks, parameterized by λ. Such changes can be interpreted as variations in uncertainty. A number of recent papers such as Bloom (2009) and Gilchrist, Sim, and Zakrajsek (2013) find that time-varying uncertainty can have effects on aggregate output, while Bachmann and Bayer (2013) contest the importance of these shocks. Consider a mean-preserving spread (MPS) in the distribution of ε. In a frictionless economy such a spread would incentivize the planner to carry out more reallocation of capital between plants because capital can be employed in highly productive sites.

Let $s = (A, \lambda, \pi; \Gamma, K)$ denote the vector of aggregate state variables. Note the assumed timing: changes in the distribution of idiosyncratic shocks are known in the period they occur, not in advance.[14] The adjustment status of a plant is given by $j = a, n$, where a stands for 'adjustment', while n stands for 'non-adjustment'.

Given the state, the planner makes an investment decision K' and chooses how much capital to reallocate across those plants whose capital stock can be costlessly reallocated, $(k, \varepsilon) \in a$. Let $\tilde{k}_j(k, \varepsilon, s)$ for $j = a, n$ denote the capital allocation to a plant that enters the period with capital k and profitability shock ε in group j after reallocation. The capital of a plant in group $j = a$ is adjusted and is optimally set by the planner to the level $\tilde{k}_a(k, \varepsilon, s)$. The capital of a plant in group $j = n$ is not adjusted so that $\tilde{k}_n(k, \varepsilon, s) = k$.

The choice problem of the planner is:

$$V(A, \lambda, \pi; \Gamma, K) = max_{\tilde{k}_a(k,\varepsilon,s),K'} \; u(c) + \beta E_{[A',\Gamma',\lambda',\pi'|A,\Gamma,\lambda,\pi]} V(A', \lambda', \pi'; \Gamma', K') \tag{14}$$

subject to the resource constraint (2) and

$$y = \int_{(k,\varepsilon)\in F^a} A\varepsilon\tilde{k}_a(k, \varepsilon, s)^\alpha d\Gamma(k, \varepsilon) + \int_{(k,\varepsilon)\in F^n} A\varepsilon\tilde{k}_n(k, \varepsilon, s)^\alpha d\Gamma(k, \varepsilon), \tag{15}$$

which is simply (4) split into adjustable and non-adjustable plants. Here F^j is the set of plants in group $j = a, n$. The fraction of plants whose capital stock can be adjusted is equal to π

[14] Other models, such as Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), include future values of λ in the current state as a way to generate a reduction in activity in the face of greater uncertainty about the future. We include the implications of this alternative timing as part of the results below.

$$\int_{(k,\varepsilon)\in F^a} f(\varepsilon)d\varepsilon = \pi \tag{16}$$

and the amount of capital over all plants must sum to total capital K:

$$\pi \int_{(k,\varepsilon)\in F^a} \tilde{k}_a(k,\varepsilon,s)d\Gamma(k,\varepsilon) + (1-\pi)\int_{(k,\varepsilon)\in F^n} \tilde{k}_n(k,\varepsilon,s)d\Gamma(k,\varepsilon) = K. \tag{17}$$

As the capital is plant specific, it is necessary to specify transition equations at the plant level. Let $i = \frac{K'-K}{K}$ denote the gross investment rate so that $K' = (1-\delta+i)K$ is the aggregate capital accumulation equation. To distinguish reallocation from aggregate capital accumulation, assume that the capital at **all** plants, regardless of their reallocation status, have the same capital accumulation. The transition for the capital this period (after reallocation) and the initial plant-specific capital next period is given by

$$k'_j(k,\varepsilon,s) = (1-\delta+i)\tilde{k}_j(k,\varepsilon,s), \tag{18}$$

for $j = a, n$. Due to the presence of frictions $\tilde{k}_a(k,\varepsilon,s)$ is not given by (8). Notice that A affects unadjustable and adjustable plants in the same way. This implies that the optimal reallocation decision will occur independently of A. The shock to A will have an effect on the mis-measured part of TFP only in the presence of a capital accumulation problem, since the total amount of capital in adjustable and non-adjustable plants may differ.

The quantitative analysis will focus on reallocation of capital, defined as the fraction of total capital that is moved between adjustable plants within a period. Following a new realization of idiosyncratic productivity shocks, the planner will reallocate capital from less productive to more productive sites. Aggregate output is thus increasing in the amount of capital reallocation.

As $\tilde{k}_a(k,\varepsilon,s)$ denotes the post-reallocation capital stock of a plant with initial capital k, the plant-level reallocation rate would be $r(k,\varepsilon,s) = |\frac{\tilde{k}_a(k,\varepsilon,s)-k}{k}|$. Aggregating over all the plants who adjust, the aggregate reallocation rate is

$$R(s) \equiv 0.5 \int_{(k,\varepsilon)\in F^a} r(k,\varepsilon,s)d\Gamma(k,\varepsilon). \tag{19}$$

The multiplication by 0.5 is simply to avoid double counting flows between adjusting plants.

3.2 Joint Distribution of Capital and Productivity

In the presence of reallocation frictions, the state space of the problem includes the cross sectional distribution, Γ. Consequently, when making investment and reallocation decisions the planner needs to forecast Γ'. It is computationally not feasible to follow the joint distribution of capital and profitability shocks over plants, so we represent the joint distribution by several of its moments. These forecast the marginal benefit of investment.

The right set of moments is suggested by rewriting (12) as:[15]

$$\tilde{A} = A\left(\bar{\varepsilon}\int_{\varepsilon}\xi(\varepsilon)^{\alpha}f(\varepsilon)d\varepsilon + cov(\varepsilon, \xi(\varepsilon)^{\alpha})\right). \tag{20}$$

Here $\bar{\varepsilon}$ denotes the time-invariant mean of the plant-specific shock. As is well understood from the Olley and Pakes (1996) analysis of productivity, the level of aggregate output will depend on the covariance between the plant-level productivity and the factor allocation. Define $\mu = \bar{\varepsilon}\int_{\varepsilon}\xi(\varepsilon)^{\alpha}f(\varepsilon)d(\varepsilon)$, and $\phi = cov(\varepsilon, \xi(\varepsilon)^{\alpha})$. Total output from (13) depends on these two moments:

$$y = AK^{\alpha}(\mu + \phi). \tag{21}$$

This analysis holds for a economy without reallocation frictions. For the economy with a reallocation rate of π, aggregate output, taken from (15), becomes

$$y = AK^{\alpha}[\pi(\mu_a + \phi_a) + (1-\pi)(\mu_n + \phi_n)], \tag{22}$$

where $\mu_j \equiv \bar{\varepsilon}E(\tilde{k}_j(k,\varepsilon,s)^{\alpha})$ and $\phi_j \equiv Cov(\varepsilon, \tilde{k}_j(k,\varepsilon,s)^{\alpha})$, for $j = a, n$. Instead of Γ we retain μ_n and ϕ_n in the state vector of (14). These two moments contain *all* the necessary information about the joint distribution of capital and profitability among non-adjustable plants. The information about capital in plants in F^A, captured in μ_a and ϕ_a is not needed since capital in those plants can be freely adjusted, independently of their current capital stock. Each period the planner chooses an allocation of capital over plants, which maps into values of μ_a and ϕ_a. Together, μ_n and ϕ_n are sufficient to compute the output of those plants whose capital cannot be reallocated and thus to solve the planner's optimization problem.

Note that by keeping μ_n and ϕ_n in the state space, we are not *approximating* the joint distribution over capital and productivity since the two moments can account for all the variation of the joint distribution. That is, the covariance appears in (22) precisely because output depends on the assignment of capital to plants, based on the realization of ε. This feature of our choice of moments allows us to compare it with common approximation techniques in the spirit of Krusell and Smith (1998) in Section 5.

The covariance term ϕ_n is crucial for understanding the impact of reallocation on measures of aggregate productivity. If the covariance is indispensable in the state vector of the planner, then the model is not isomorphic to the stochastic growth model. That is, if the covariance is part of the state vector, then the existence of heterogeneous plants along with capital adjustment costs matters for aggregate variables like investment over the business cycle.

[15] This uses $E(XY) = EX \times EY + cov(X,Y)$. This decomposition of productivity taken from Olley and Pakes (1996) highlights the interaction between the distribution of productivity and factors of production across firms. Gourio and Miao (2010) use a version of this argument, see their equation (45), to study the effects of dividend taxes on productivity. Khan and Thomas (2008) study individual choice problems and aggregation in the frictionless model with plant specific shocks. Basu and Fernald (1997) also discuss the role of reallocation for productivity in an aggregate model.

3.3 Laws of Motion and Stationary Equilibria

The evolution of μ_n and ϕ_n can be described analytically by two laws of motion.[16] These are given by

$$\mu_n' = \pi'\mu_a + (1-\pi')\mu_n \tag{23}$$

and

$$\phi_n' = \pi'\rho_\varepsilon\phi_a + (1-\pi')\rho_\varepsilon\phi_n. \tag{24}$$

As noted above, the choice of $\tilde{k}_a$ for adjustable plants, along with the respective ε shocks at these plants, maps into values of the moments μ_a and ϕ_a.

Together, (23) and (24) define the law of motion of the joint distribution Γ, allowing us to follow the evolution of this component of the aggregate state.[17] Equations (22)-(24) permit us to study the trade-off regarding the optimal allocation of capital across sites. The planner can increase contemporaneous output by reallocating capital from low- to high-productivity sites in F^a. This will increase the covariance between profitability and capital, ϕ_a, while at the same time decreasing μ_a because $\alpha < 1$. A fraction $1-\rho_\pi$ of currently adjustable plants will not be able to adjust its capital stock next period. The planner therefore has to trade off the higher instantaneous output from reallocation with the higher probability of a mismatch between $\tilde{k}_n(k,\varepsilon,s) = k$ and the realization of ε' for plants in F^n tomorrow. This is captured in the laws of motion (23) and (24).

To fix ideas we can analyze the stationary economy where π and λ are not varying over time. In this environment a stationary distribution Γ^* exists. Using (23) it follows that

$$\mu_n = \mu_a = \mu^*. \tag{25}$$

Furthermore, the economy converges towards stationary values ϕ_a^* and ϕ_n^*. Using (24) one can show that ϕ_n converges to

$$\phi_n^* = \phi_a^* \frac{\pi\rho_\varepsilon}{1-(1-\pi)\rho_\varepsilon}. \tag{26}$$

Hence total output in (22) becomes

$$y = \bar{\varepsilon}\mu^* + \Lambda\phi_a^*, \tag{27}$$

where $\Lambda \equiv \frac{\pi}{1-(1-\pi)\rho_\varepsilon}$ is a function of parameters. Λ is (weakly) increasing in both π and ρ_ε.[18] Intuitively,

[16] With time-varying uncertainty λ we compute the evolution of ϕ_n numerically. With log-normally distributed shocks the analytics only hold for the evolution of the mean, (23).

[17] Note that $\phi' = Cov(k(\varepsilon)^\alpha, \varepsilon')$ is an expectation. The term ε' is made up of two components, one is the persistent part, and one is an i.i.d. part, denoted η. Rewrite $\varepsilon' = \rho_\varepsilon\varepsilon + (1-\rho_\varepsilon)\eta$ to obtain $\phi' = Cov(k(\varepsilon)^\alpha, \rho_\varepsilon\varepsilon + (1-\rho_\varepsilon)\eta) = \rho_\varepsilon\phi$. To derive equations (23) and (24) note that from the fraction π of adjustable plants today a fraction π' will remain adjustable tomorrow, while a fraction π' of the $(1-\pi)$ non-adjustable plants today will join the pool of adjusters. The overall fraction of adjusters tomorrow is thus $\pi\cdot\pi' + (1-\pi)\cdot\pi' = \pi'$. The same logic applies to the other cases.

[18] Formally, $\frac{\partial\Lambda}{\partial\pi} = \frac{1-\rho_\varepsilon}{[1-(1-\pi)\rho_\varepsilon]^2} \geq 0$, $\frac{\partial\Lambda}{\partial\rho_\varepsilon} = \frac{\pi(1-\pi)}{[1-(1-\pi)\rho_\varepsilon]^2} \geq 0$. The cross-derivatives are given by $\frac{\partial^2\Lambda}{\partial\rho_\varepsilon\partial\pi} = \frac{\partial^2\Lambda}{\partial\pi\partial\rho_\varepsilon} = \frac{1}{[1-(1-\pi)\rho_\varepsilon]^2} - \frac{2\pi}{[1-(1-\pi)\rho_\varepsilon]^3}$.

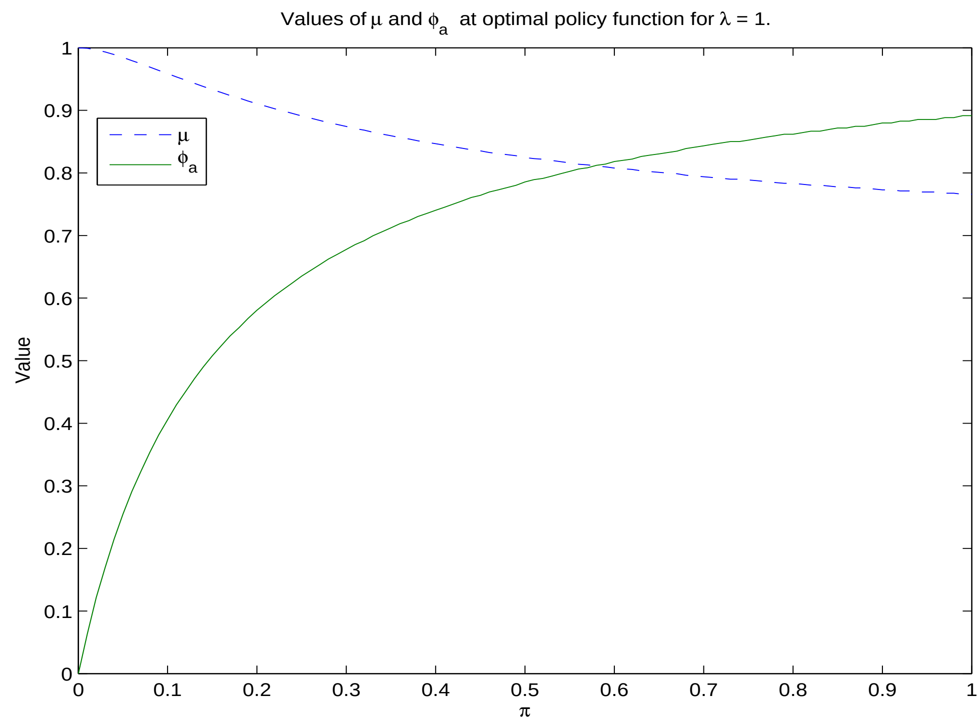

Figure 2: Values of μ and ϕ_a in stationary equilibrium for various π. Economy with $\lambda = 1$ and $\rho_\varepsilon = .9$. Values of ϕ_a are shown as fraction of frictionless value.

an increase in π increases total output because more plants' capital stock can be costlessly adjusted. An increase in ρ_ε, the persistence of idiosyncratic productivity shocks, implies that the probability of a plant switching status and being non-adjustable with a mismatch between ε and k is decreased. In the extreme case of iid shocks to idiosyncratic productivity $\rho_\varepsilon = 0$ so that $\phi_n^* = 0$. The planner would be more reluctant to allocate large amounts of capital to high-productivity sites, decreasing aggregate output.

Figure 2 shows equilibrium values of μ^* and ϕ_a^* in stationary economies for different values of π. As $\pi \to 0$ the planner reallocates less capital between plants. A value of $\mu^* = 1$ implies $\phi_a^* = 0$, because $k(\varepsilon) = 1$ for all sites, meaning that the capital level is independent of ε. On the other hand, as the fraction of adjustable plants increases, ϕ_a^* increases.

4 Quantitative Results

In the stationary economy, reallocation effects only mattered for aggregate **levels**. When are reallocation effects likely to play a role for aggregate **dynamics**? One key prerequisite is that the economy be subject to shocks that cause the distribution Γ to move over time. Without movements in Γ the benefits from reallocation are constant and the covariance term ϕ is not required to forecast Γ'. The reasons why Γ may vary and the **quantitative** implications of its variability will be clear as the analysis proceeds.

In keeping with the distinction noted earlier between reallocation and accumulation, the initial quantitative analysis presented in section 4.1 is for an economy with a fixed capital stock, thus highlighting reallocation. The economy is then enriched to allow for capital accumulation in section 4.2.

We solve the model at a quarterly frequency, using these **baseline** parameters. Following the estimates in Cooper and Haltiwanger (2006), we set $\alpha = 0.6$.[19] We assume log-utility and a depreciation rate $\delta = 0.025$. Assuming an annual interest rate of 4% implies a discount factor $\beta = 0.987$. We set the mean of π to $\bar{\pi} = 0.33$. This implies that plants adjust their capital stock on average every three quarters. Sveen and Weinke (2005) treat changes in the capital stock of under 10% in absolute value as maintenance and hence use $\pi = 0.08$. In our setup, the choice of π mainly affects aggregate *levels*, not transitions. Aggregate profitability takes the form of an AR(1) in logs

$$\ln a_t = \rho_a \ln a_{t-1} + \nu_{a,t}, \qquad \nu_a \sim N(0, \sigma_a), \tag{28}$$

where $\rho_a = 0.9$ and $\sigma_a = 0.007$. Idiosyncratic profitability shocks are log-normal and evolve according to a law of motion with time-varying variance

$$\ln \varepsilon_t = \rho_\varepsilon \ln \varepsilon_{t-1} + \lambda_t \nu_{\varepsilon,t}, \qquad \nu_{\varepsilon,t} \sim N(0, \sigma_\varepsilon). \tag{29}$$

The parameters of the idiosyncratic shock process are $\rho_\varepsilon = 0.9$ and $\sigma_\varepsilon = 0.2$. The parameter λ governs the mean-preserving spread of the normal distribution from which idiosyncratic profitability ε is drawn. It has a mean of 1 and variance σ_λ

$$\lambda_t = \rho_\lambda \lambda_{t-1} + \nu_{\lambda,t}, \qquad \nu_{\lambda,t} \sim N(1, \sigma_\lambda). \tag{30}$$

We set $\rho_\lambda = 0.9$. Finally, the process of π follows

$$\pi_t = \rho_\pi \pi_{t-1} + \nu_{\pi,t}, \qquad \nu_{\pi,t} \sim N(\bar{\pi}, \sigma_\pi), \tag{31}$$

with $\rho_\pi = 0.9$. The parameters of the AR(1) processes have been chosen to approximate the correlations of capital reallocation and productivity dispersion with aggregate output from the US data. The resulting values were $\sigma_\pi = 0.03$ and $\sigma_\lambda = 0.014$. Section 4.3 explores the sensitivity of our findings to this parameterization. All exogenous shocks are discretized using the methodology described in Galindev and Lkhagvasuren (2010). The computational strategy is discussed in further detail in Appendix A.

4.1 Capital Reallocation

Table 1 shows measures of capital reallocation and productivity. The column labeled R/R^* for 'Reallocation' measures the time series average of the cross-sectional reallocation of capital across plants

[19] This curvature is 0.44 in Bachmann and Bayer (2013) and 0.4 in Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012).

as defined in (19), relative to the frictionless benchmark without adjustment costs. The column labeled $E_t(c_v(arpk_{it}))$ measures the time series average of the cross sectional coefficient of variation of the average revenue product of capital. The column labeled G shows the output gap, defined as $G(s) = \frac{y^{FL}(s)-y(s)}{y^{FL}(s)}$, output in state s relative to the frictionless benchmark.[20] The column labeled $\sigma(\tilde{A}/A)$ reports the standard deviation of the Solow residual relative to TFP. This is a key moment as it measures the extent to with the cross-sectional distribution $f(\varepsilon)$ and the allocation affect aggregate productivity; i.e. this measures the cyclicality of productivity which does not come from A alone. The columns $C(R, \tilde{A})$ and $C(c_v(arpk_{it}), \tilde{A})$ show the correlation between the Solow residual and respectively capital reallocation and the coefficient of variation of the average revenue product of capital. These two columns provide a link back to the facts, noted in the introduction, about the cyclical behavior of reallocation and dispersion in productivity.

Case	R/R^*	$E_t(c_v(arpk_{it}))$	G	$\sigma(\tilde{A}/A)$	$C(R,\tilde{A})$	$C(c_v(arpk_{it}),\tilde{A})$
Frictionless						
nonstochastic	1 (0)	0 (0)	0 (0)	0 (0)	*na* (−)	*na* (−)
stochastic A	1 (0)	0 (0)	0 (0)	0 (0)	*na* (−)	*na* (−)
stochastic λ	1 (0)	0 (0)	0 (0)	0.01 (0.0003)	0.73 (0.02)	*na* (−)
Frictions						
nonstochastic	0.29 (0)	0.18 (0)	0.05 (0)	0 (0)	*na* (−)	*na* (−)
stochastic A	0.28 (0)	0.18 (0)	0.05 (0)	0 (0)	*na* (−)	*na* (−)
stochastic π	0.28 (0.01)	0.18 (0.002)	0.05 (0.001)	0.01 (0.0001)	0.98 (0.001)	−0.98 (0.01)
stochastic λ	0.28 (0.0001)	0.18 (0.001)	0.05 (0.0001)	0.01 (0.0002)	0.86 (0.01)	0.92 (0.06)
stochastic A, π, λ	0.29 (0.01)	0.18 (0.01)	0.05 (0.001)	0.03 (0.001)	0.54 (0.02)	−0.12 (0.02)

Table 1: Capital Reallocation Model: Productivity Implications

Results from 100 simulations with T=2000, standard deviations in parentheses below. $\frac{R}{R^*}$ measures the time series average of the cross-sectional reallocation of capital across plants, relative to the frictionless benchmark, R^*. $E_t(c_v(arpk_{it}))$ is the mean coefficient of variation of the average revenue product of capital. G refers to the output gap relative to the frictionless benchmark. The column $\sigma(\tilde{A}/A)$ shows the standard deviation of measured vs. real TFP. The last columns $C(R,\tilde{A})$ and $C(c_v(arpk_{it}),\tilde{A})$ show the correlation between mismeasured TFP and respectively capital reallocation and the coefficient of variation of the average revenue product of capital. The "*na*" entry means that the correlation is not meaningful as one of the variables is constant.

[20] The frictionless output $y^{FL}(s)$ is a function of s because changes in λ affect the output achieved in the frictionless case.

4.1.1 Frictionless Economy

The first row of Table 1 shows the results for the frictionless economy when $\pi = 1$, without time series variations in TFP, the volatility of the idiosyncratic shocks λ, or the fraction of adjustable sites π. This case serves as a benchmark. Without frictions, the marginal product of capital is equalized across plants and our measure of the inefficiency of the capital allocation, $E_t(c_v(arpk_{it}))$, is zero. The first-best output is achieved. The mis-measurement of TFP is constant and there is no time-variation in output. Although capital is being reallocated each period, the total amount is time-invariant and hence plays no role for the cyclicality of aggregate productivity.

Other treatments of adding shocks to productivity, A, the adjustment rate, π, and the cross sectional distribution, λ, are shown in the table. For the moments of interest, only variations in the dispersion of the cross sectional distribution creates cyclical reallocation. The correlation is not equal to one because part of the effect on output comes directly from changes in $f(\varepsilon)$. Shocks to λ cause subsequent changes in the allocation of capital across sites. As there are no frictions, even in this case there is not dispersion in average revenue products.

4.1.2 Costly Capital Reallocation

Setting $\pi < 1$ introduces capital adjustment costs to the frictionless economy. Costly capital reallocation will affect measured productivity and its cyclical properties. The second block of Table 1 reports results for the model with frictions benchmark economy.

When π is non-stochastic and there are no other aggregate shocks, there exists a stationary joint distribution Γ with constant moments (μ_n, ϕ_n), as was shown in Section 3.3. Table 1 shows the results for this case in the row labeled 'nonstochastic'. In this economy the fraction of capital reallocated is far below the frictionless benchmark, as indicated in the second column.

The inefficiency of the allocation when $\pi < 1$ is highlighted by the column labeled $E_t(c_v(arpk_{it}))$. This measure of the inefficiency of the allocation is larger than zero, reflecting frictions in the reallocation process that stem from two sources. First, the planner chooses not to equalize marginal products between adjustable plants, reflecting the tradeoffs discussed above. Second, the marginal products of capital among non-adjustable plants exhibit a high degree of heterogeneity due to the fact that their capital is fixed despite a new realization of idiosyncratic profitability. Because ϕ_n and μ_n converge to their steady-state values output does not vary in this economy.

The output gap is positive, around 5%, directly reflecting the impact of $\pi < 1$. Importantly, reallocation and the mis-measurement in TFP are constant over time. There is only obtain a level-effect on output and productivity.

The row labeled 'stochastic A' allows for randomness in aggregate productivity with constant π. As explained above, the amount of reallocation is independent of variations in A. Output and $\tilde{A}$ vary only with A. Because $\pi < 1$ the allocation is characterized by a positive standard deviation of average revenue products of capital.

At this point, the model is not able to match the motivational observations of comovement in reallocation and the aggregate economy. Cyclical variations in reallocation emerge once either the adjustment rate π and/or the distribution of idiosyncratic shocks, λ, varies. Further, in these cases, the cross sectional dispersion of capital productivity will be cyclical as well.

A stochastic π creates time series variation in the moments μ_n and ϕ_n. Fluctuations in π lead to pro-cyclical capital reallocation patterns, as shown in column $C(R, \tilde{A})$. But this is not *simply* a correlation. In the presence of adjustment frictions, reallocation **causes** the observed time-variations in output. Variations in π therefore also lead to variations in (mis-measured) total factor productivity. The marginal products of capital are not equalized across plants, neither among the adjustable nor the unadjustable sites. This results in a positive output gap which varies with the evolution of μ_n and ϕ_n. This gap is about 5% of real GDP. Additionally, this economy exhibits counter-cyclical productivity dispersion, as seen in the last column. When π is low, less capital can be reallocated between adjustable plants. This decreases output and increases the standard deviation of marginal products between those plants. I λ is held fixed, $c_v(arpk_{it})$ nonetheless varies over time.

The row 'stochastic λ' of Table 1 studies the effects of time-variation in $f(\varepsilon)$ under costly capital reallocation. Due to the presence of adjustment costs, the marginal products of capital cannot be equalized over time. In addition, the variations in λ lead to changes in the optimal allocation decision by the planner and create considerable time-variation in μ_n and ϕ_n. The resulting fluctuations in output stem from different reallocation choices of the planner that show up in variations of the Solow residual. While variations in π affect output directly through the fraction of plants among which capital can be reallocated, the effect of changes in λ is less direct. Variations in λ induce different reallocation choices but a fraction of the effect on output comes from the fact that the marginal revenue product of capital is changed through productivity draws with larger or smaller tails. As the last two columns show, shocks to λ lead to pro-cyclical reallocation and produce a pro-cyclical dispersion in average revenue products of capital. A larger spread in the distribution of shocks leads to more reallocation of capital among adjustable plants and hence higher output. At the same time the increase in dispersion leads to a larger standard deviation of the marginal products of capital, both among adjustable and non-adjustable plants. This result is driven by the probability of a mismatch between k and ε' for plants in F^n.

The joint effects of changes in A, π and λ are presented in the last row of Table 1. Output varies significantly over time, with variations resulting directly from all three shocks. While $\pi < 1$ leads to a positive output gap the presence of a stochastic λ again causes additional variation in this gap. Notably, mis-measured TFP exhibits significantly more time variation than in the cases of varying λ or varying π alone. This is the result of changes in π and λ jointly affecting the slow-moving joint distribution Γ. Both changes in λ and π induce changes in reallocation. The correlation between capital reallocation and output is lower than in the cases of stochastic π or λ alone because the variation in A generates additional volatility. The effect of varying π on reallocation is predominantly an extensive margin effect, as a changing fraction of plants can reallocate capital. The effect of λ is on the intensive margin: more capital is reallocated within a given fraction of adjustable plants.

Overall, adjustment frictions reduce reallocation, generating a non-degenerate distribution of average (and marginal) products of capital across plants. The cost is a reduction in output of about 5%, relative to the frictionless benchmark. In all of the experiments, reallocation is pro-cyclical. For these cases, measured variations in TFP are the consequence of reallocation rather than true variations in aggregate productivity. Variations in π lead to counter-cyclical productivity dispersion across firms.

The economy with variations in both π and λ mimic the patterns of pro-cyclical reallocation and counter-cyclical dispersion emphasized by Eisfeldt and Rampini (2006). This will be a leading case as the analysis proceeds.

Impulse Response Functions Figures 3 and 4 show impulse response functions for negative shocks to π and λ. The shocks occur in period $t = 5$. The x-axes show time, while the y-axes in panels 2-4 shows the % deviation from the unconditional mean. The drop in the exogenous shock of interest is plotted in the first panel, while all other exogenous shocks are set to their unconditional means.

We first discuss the negative shock to π shown in Figure 3. The second panel shows the evolution of the two moments μ_n and ϕ_n. The negative correlation between the two series is very high, as changes in π effect the evolution of μ_n and ϕ_n in very similar ways through the extensive margin effect. The third panel illustrates the co-movement between reallocation 'R' and the Solow residual. Following the shock to π less capital can be reallocated between plants, which directly affects $\tilde{A}$. Because fewer plants' capital stock can be reallocated, the dispersion of marginal products of capital increases, leading to a negative response of output, as the last panel shows.

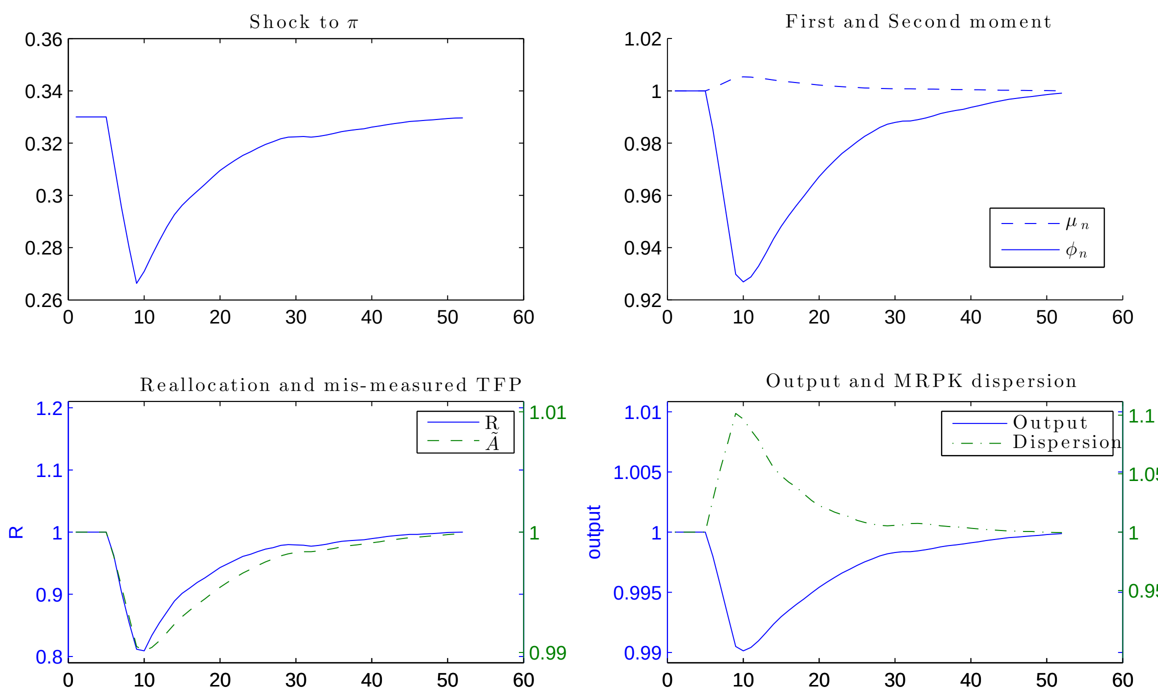

Figure 3: Variations in π: Impulse Response Functions. The y-axes show % deviations from unconditional means.

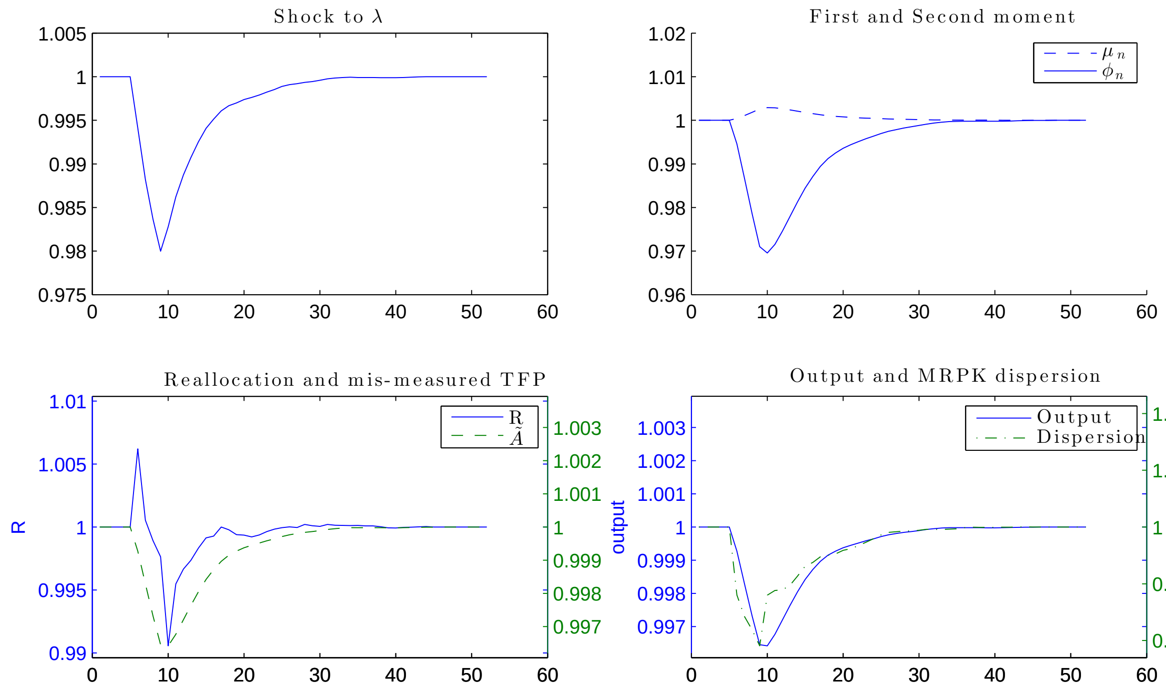

Figure 4: Negative shock to λ: Impulse Response Functions. The y-axes show % deviations from unconditional means.

Figure 4 shows the effects of a negative shock to λ. The second panel shows the evolution of the two moments μ_n and ϕ_n. The drop in ϕ_n is a direct effect of the shock to λ, whereas the increase in μ_n reflects the effects of different reallocation choices.

Panel 3 shows the connection between mis-measured TFP and reallocation. Different from a shock to π reallocation now increases on impact before falling in consecutive periods. The initial spike in reallocation occurs because the planner chooses a different allocation of capital across adjustable plants. Once this initial reallocation has occurred, the level of churning is lower at the lower level of λ. The negative correlation between reallocation and output on impact also explains the lower overall correlation between these two variables in the presence of time-varying λ reported in Table 1. The combined effect of the change in $f(\varepsilon)$ and reallocation produces a pro-cyclical effect on output. In this economy with time-varying idiosyncratic uncertainty in the presence of adjustment costs there is a strong cyclical dimension of capital reallocation. Reallocation is driving time-variations in output.

Output and dispersion both fall in response to a negative shock to λ. The drop in dispersion is a direct effect of the tightening of the distribution of idiosyncratic productivity shocks. The increase in reallocation increases the dispersion, the overall effect is negative, however. Because less capital can be allocated to plants in the upper tail of the distribution, output falls. This effect is driven by the "love of variety" aspect of the production technology. These responses **do not** include the fall in output associated with an *increase* in the dispersion of shocks, as emphasized in Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) and others. As noted above, this reflects a couple of features of our environment: (i) the timing of the shock to λ, (ii) the model of adjustment costs and (iii) the specification of the production function. Nonetheless, as indicated above, the model with both shocks, i.e. the stochastic (π, λ) case, is able to match the two key observations of pro-cyclical reallocation and a counter-cyclical dispersion in capital productivity.

4.2 Endogenous Capital Accumulation

With endogenous capital accumulation, solving (14), the capital *reallocation* process has significant interactions with the capital *accumulation* decision. The frictions exert a level effect on the optimal capital stock and induce different dynamics following an exogenous shock. As we saw above, reallocation behaves cyclically in the presence of time-series variation in π and/or λ. Variations in λ and π affect the instantaneous value of existing capital and, because of persistence, the expected future return to capital, too. This affects the planner's incentives to invest. Even absent any frictions to capital accumulation the dynamics of investment and consumption are considerably altered by the presence of exogenous shocks to reallocation or the variance of the idiosyncratic shock.

Adding endogenous capital accumulation does not alter the results on the reallocation process shown in Table 1. The reason parallels the argument for the independence of reallocation from A. From (9), total output is proportional to AK^α. Thus just as variations in A scale moments, so will variations in K. Consequently, the analysis focuses on the effects of frictions in reallocation on capital accumulation.

Table 2 summarizes results for the endogenous capital accumulation problem, using the baseline parameters defined earlier. The aggregate capital stock is now endogenous and creates additional variation. The average capital stock (relative to the frictionless benchmark) is shown in the $\bar{K}/\bar{K}^*$ column. The other columns report correlations of reallocation with investment and output, $C(R, i)$ and $C(R, y)$ and the correlation of investment and the Solow residual, $C(\tilde{A}, i)$.

From Table 2, the interaction of costly reallocation and accumulation is evident in a number of forms. First, $\bar{K}$, which is the average capital for a particular treatment, depends on the nature and magnitude of the capital adjustment costs. Even in the absence of any aggregate shocks, the capital stock is around 11% lower when there are adjustment frictions compared to the frictionless case. This comparison of the average capital stocks with and without frictions stands regardless of the source of the shocks. The addition of the shocks decreases the average stock of capital. With shocks to both π and λ the coefficient of variation of capital is about twice as large as when there are only exogenous productivity shocks (not shown).

Second, capital accumulation is positively correlated with both reallocation and the Solow residual. An increase in λ, for example, leads to an increase in investment, reallocation and output. The correlation of reallocation and investment, $C(R, i)$, is informative about the effects of frictions on the incentive to accumulate capital. An increase in π say, will imply that more plants are able to adjust and for this reason alone reallocation will increase. With π correlated, it is likely that more plants will be able to adjust in the future, so investment increases too. The magnitude of this correlation is smaller when only λ is random. Though the same fraction of plants adjusts each period, the gains to

Case	$\bar{K}/\bar{K}^*$	$C(R,i)$	$C(R,y)$	$C(\tilde{A},i)$
Frictionless				
nonstochastic	1 (0)	na (–)	na (–)	na (–)
stochastic A	1 (0)	na (–)	na (–)	na (–)
stochastic λ	1 (0)	0.75 (0.01)	0.73 (0.01)	0.99 (0.001)
Frictions				
nonstochastic	0.89 (0)	na (–)	na (–)	na (–)
stochastic A	0.89 (0.003)	na (–)	na (–)	0.98 (0.01)
stochastic π	0.89 (0.004)	0.92 (0.01)	0.85 (0.01)	0.93 (0.01)
stochastic λ	0.90 (0.002)	0.69 (0.04)	0.71 (0.03)	0.81 (0.02)
stochastic A, π, λ	0.89 (0.003)	0.84 (0.02)	0.60 (0.05)	0.95 (0.006)

Table 2: Endogenous Capital Accumulation: Aggregate Moments

Results from 100 simulations with T=2000. Standard deviations in parentheses. Simulations were computed using benchmark parameters. $\bar{K}/\bar{K}^*$ reports the average capital stock relative to the frictionless benchmark. $C(R,i)$ is the correlation between reallocation and investment, $C(R,y)$ is the correlation between reallocation and output, and $C(\tilde{A},i)$ is the correlation between mis-measured TFP and investment. The "*na*" entry means that the correlation is not meaningful as one of the variables is constant.

adjustment are larger when λ is high. This generates a positive correlation between reallocation and investment.

Finally, reallocation is pro-cyclical in the presence of shocks to either π or λ. This returns to one of the themes of the paper. If variations arise from either changes in the fraction of adjusting plants, through π, or by a change in the spread of the shocks, through λ, output responds. The key to this response is reallocation: the effects on output of getting the right amount of capital into its most productive use. This is captured through $\tilde{A}$.

4.3 Robustness

The previous results illustrated a couple of themes. First, variations in either π or λ are necessary to generate cyclical movements in reallocation, with resulting effects on mis-measured TFP. Second, the evolution of the cross sectional distribution generated dynamics only in the stochastic π and/or λ cases. This is illustrated by the fact that higher order moments are relevant in the planner's optimization problem and the evolution of these moments are seen in the impulse response functions.

This sub-section studies the robustness of these findings to alternative values of key parameters. Table 3 reports our findings. It has the same structure as Table 1. The first column indicates the model. The baseline is the case with adjustment costs and stochastic (A, π, λ) taken from Table 1.

In the second row we show the effects of moving α from 0.6 to 0.8. The increase in the curvature

Parameter changes	R/R^*	$E_t(c_v(arpk_{it}))$	G	$\sigma(\tilde{A}/A)$	$C(R,\tilde{A})$	$C(c_v(arpk_{it}),\tilde{A})$
Frictions						
Baseline	0.29 (0.01)	0.18 (0.01)	0.05 (0.001)	0.025 (0.001)	0.54 (0.02)	-0.12 (0.02)
$\alpha = 0.8$	0.30 (0.009)	0.18 (0.001)	0.11 (0.002)	0.054 (0.001)	0.80 (0.02)	-0.03 (0.05)
$\bar{\pi} = 0.8$	0.78 (0.005)	0.05 (0.001)	0.01 (0.0002)	0.02 (0.0003)	0.29 (0.04)	-0.09 (0.06)
$\rho_\pi = 0.5$	0.29 (0.001)	0.19 (0.003)	0.05 (0.0003)	0.02 (0.0001)	0.30 (0.01)	-0.06 (0.04)
$\rho_\varepsilon = 0.5$	0.18 (0.003)	0.13 (0.01)	0.03 (0.001)	0.02 (0.0004)	0.14 (0.07)	-0.18 (0.09)
$\sigma_\lambda = 0.1$	0.28 (0.006)	0.20 (0.005)	0.05 (0.002)	0.07 (0.003)	0.70 (0.02)	0.72 (0.02)
timing	0.29 (0.007)	0.15 (0.01)	0.05 (0.0002)	0.015 (0.006)	0.94 (0.02)	-0.35 (0.02)

Table 3: Capital Reallocation: Robustness

Model with stochastic A, π and λ. Standard deviations in parentheses.

of the revenue function leads to a larger output gap and a higher variability of mis-measured TFP.

The baseline model assumes $\bar{\pi} = 0.33$. The third row of Table 3 studies the implications of a higher adjustment rate. Not surprisingly, the reallocation rate is increasing in π, as frictions are lower. The correlation of reallocation and mis-measured TFP is positive, though lower than in the baseline, at $\pi = 0.8$.

In the next row the serial correlation of π shocks is set to 0.5, lower than their baseline values of $\rho_\pi = 0.9$. Relative to the baseline, this reduction leads to a reduction in the cyclicality of reallocation. With adjustment opportunities less correlated, the costs of reallocating resources that are subsequently mismatched with productivity is higher. Hence reallocation is less correlated with $\tilde{A}$. This will imply that the correlation of reallocation and investment is lower than in the baseline reflecting the costs of accumulating capital when future adjustment costs are less certain.

When ρ_ε is decreased, the planner has fewer incentives to reallocate capital among adjustable plants. Consequently, the amount of capital reallocation falls and the inefficiency of the solution becomes more pronounced. The effect of shocks to λ is stronger, leading to a higher counter-cyclicality of productivity dispersion.

The row labeled $\sigma_\lambda = 0.1$ increases the variability of λ by an order of magnitude relative to the baseline. This spread is closer to that in Bloom (2009) and Gilchrist, Sim, and Zakrajsek (2013). Not surprisingly, this extra volatility in the spread of idiosyncratic shocks leads to more volatility in $\tilde{A}$ relative to the baseline. Reallocation remains pro-cyclical though less compared to the baseline. The correlation between the cross-sectional standard deviation of marginal products of capital and $\tilde{A}$ becomes positive in this scenario, a result of the positive correlation between λ and aggregate output.

The last row is a modification to the model that influences the extent of the "love of variety effect". The row labeled "timing" assumes that the planner knows of a change in the cross sectional distribution of the idiosyncratic shocks one period in advance. That is, the future value of λ is in the current state space. This is the timing used in Bloom (2009) as a way to emphasize the uncertainty effects of a change in the distribution. In our environment, the change in timing has some modest effects relative to the baseline. There is less dispersion in the average product of capital but this dispersion is more negatively correlated with $\tilde{A}$ compared to the baseline. With the alternative timing assumption the planner reallocates more capital when λ is known to remain high, and less capital when λ is known to remain low. This increases the counter-cyclicality of the dispersion and leads to an allocation of capital that is on average closer to the frictionless benchmark.

5 Approximation

The previous sections showed that the covariance ϕ *matters* for determining the optimal capital allocation. The problem in (14) includes Γ, the joint distribution of (k, ε). Using the first two moments of this distribution, μ_n and ϕ_n, the evolution of Γ can be tracked perfectly. This is important for the planner, who has to forecast the expected future output from non-adjustable plants, y'_n. Variations in π and λ generate movements in Γ and hence in y_n. Capital reallocation is tightly linked to changes in the mis-measurement of TFP when stochastic shocks are present.

Movements in Γ may not be captured well by the first moment μ_n alone. In the frictionless case the two moments were perfectly correlated, but this perfect correlation is broken by the existence of time-variation in the adjustment probability π and/or λ. The impulse response functions above showed that both in the case of shocks to π or λ the two moments μ and ϕ were strongly correlated. However, different shocks imply different magnitudes of change in μ, ϕ, and output. A change in λ produces a stronger reaction in ϕ and a smaller reaction in μ compared to a shock in π. Output changes of the *same* magnitude can therefore occur at the same time as *different* changes in μ. This fact is what generates the limited explanatory power of the first moment μ alone. The significance of reallocation effects is related to the forecasting power of ϕ_n.

Relative to the literature starting with Krusell and Smith (1998), this is an important finding. **In particular, this result is distinguished from preceding papers in that for our environment the approximation of the cross sectional distribution requires higher order moments.**

This section emphasizes the importance of including the higher order moments in the state vector. From this we can determine how well the evolution of Γ could be captured by different subsets of its moments under different cases of stochastic π and λ.

Table 4 evaluates the importance of the higher order moments. To understand this table, let "DGP" refer to a data set (and moments) created by solving the baseline model (with stochastic π and/or λ) using (μ, ϕ) in solving the planner's problem. In (14), the planner forecasts y'_n, the output from non-adjustable plants next period. The correctly specified regression model including both moments is given by

$$y_{n,t}^{DGP} = \beta_0 + \beta_1 \mu_{n,t} + \beta_2 \phi_{n,t} + \beta_3 s_t + \varepsilon_t, \tag{32}$$

where s_t includes π_t and λ_t. Estimation results in $\hat{\beta}_0 = 0$, $\hat{\beta}_1 = 1.6487 = \bar{\varepsilon}$, $\hat{\beta}_2 = 1$, and $\hat{\beta}_3 = 0$ with an $R^2 = 1$. The maximum forecast error (MCFE) is zero. As discussed in Den Haan (2010) a problem of

R^2 measures to assess the approximation is that observations generated using the *true* law of motion (instead of the forecast) are used as the explanatory variable. We construct a series $\hat{\hat{y}}_n$ which is using only the approximate law of motion. The forecast error is defined as $\hat{\hat{\varepsilon}}_{t+1} = |\hat{\hat{y}}_{n,t+1} - y_{n,t+1}|$, and the MCFE is the maximum of this series.

Case	R^2	MFCE
Truth, approximated		
Stochastic π	0.991	0.60%
Stochastic λ	0.968	0.25%
Stochastic π, λ	0.940	1.25%
Linear, consistent		
Stochastic π	0.991	0.40%
Stochastic λ	0.696	0.73%
Stochastic π, λ	0.70	1.70%
Linear using DG truth		
Stochastic π	0.94	1.52%
Stochastic λ	0.82	1.339%
Stochastic π, λ	0.945	1.78%

Table 4: Different approximation strategies

The first column shows the R^2 of a regression of output from non-adjustable plants on an intercept and the first moment, μ only. The second column reports the maximum forecast error from such a regression.

Below we study three cases (experiments). The first takes output of the non-adjusting plants from the DGP and regresses it on an intercept, the exogenous state, and the first moment only. Thus this exercise is about approximating the nonlinear solution with a linear representation. The regression model for the linear approximation is given by (32) where we force $\beta_2 = 0$. From Table 4, the linear representation is very accurate if only π is stochastic. When λ is random, the resulting movements in the distribution of shocks leads to much greater significance of the cross sectional distribution in forecasting (decisions do not change in this experiment).

The second case actually solves the planner's problem under the (false) assumption that the model is linear. The resulting decision rules and expectations are model consistent by construction, but not data consistent.[21] The goodness of fit measure in Table 4 is computed from a regression of the output of the non-adjusting plants in the DGP using the model consistent estimators from the linearized approximation. As before, the linear beliefs in the stochastic π case are approximately consistent with the outcome. Again this is not the case when λ is random. For this experiment, the linear forecast rule leads to very different allocative decisions by the planner. Consequently, the R^2 is quite low –

[21] The R^2 from the forecast of μ in the linearized version of the model typically exceeds 0.99. In this sense, the solution is internally consistent.

movements in the cross sectional distribution are very important.

In the third case, the planner uses the DGP to obtain a linear approximation of the law of motion. With this representation, the planner solves the optimization problem. In this case, the expectations about the evolution of the state vector is consistent with the data, but not with the model. Here, none of the experiments generate a good fit. The planner is simply unable to capture the nonlinear movements in the economy with a linear approximation of the law of motion.

6 Aggregate Implications

This section returns to the themes of the introduction: the cyclical properties of reallocation and business cycles.

6.1 Capital Reallocation and Uncertainty

The model points to the importance of uncertainty as a source of variation in reallocation and thus output. This sub-section returns to the data and studies the impact of GDP and uncertainty on reallocation using firm-level data from Compustat.[22] The analysis thus goes beyond that of Eisfeldt and Rampini (2006) by considering an explicit role for uncertainty, independent of its affect on output. The inclusion of GDP controls for other aggregate factors which might impact reallocation. This specification allows us to study uncertainty directly.

The basic panel regression model is

$$\text{Reallocation}_{it} = \beta_0 + \beta_1 \text{GDP}_t + \beta_2 \text{uncertainty}_t + \zeta X_{it} + \varepsilon_{it},$$

where GDP and uncertainty are the cyclical components after HP-filtering. The regressor X_{it} contains other control variables. We estimate panel regressions with standard errors clustered at the firm level. Reallocation is defined as sales of plant, property and equipment (PP&E) over a firm's total PP&E.

The estimation results are reported in Table 5. Reallocation co-moves positively with GDP regardless of the model specification. The impact of aggregate uncertainty on capital reallocation is significantly larger than zero in all specifications. This continues to be the case for the subset of plants that exhibit positive capital reallocation (column 2) and once we control for a firm's share price variation, a measure of idiosyncratic uncertainty (column 3). Using an alternative definition of capital reallocation - sales of PP&E plus acquisitions over total investment - does not affect our conclusions. Using an aggregate time series instead of a panel regression results in the same patterns (not shown). The last column of Table 5 shows marginal effects from a Probit model with the dependent variable be-

[22] Annual real GDP data was obtained from the Bureau of Economic Analysis. Aggregate uncertainty is measured using the index proposed by Baker, Bloom, and Davis (2013). Results are qualitatively unchanged by using an alternative indicator, such as the VIX index. Both series have been HP filtered and divided by their respective standard deviations. Compustat data was obtained annually for the years 1979-2012. Firms with fewer than 3 observations were removed from the sample. Appendix C provides further information on the data.

ing whether or not a firm engaged in positive sales of PP&E. Both higher GDP and higher uncertainty increase a firm's adjustment probability.

	R	R	R	R+Acq	R+Acq	ADJ
GDP	0.0380***	0.0603***	0.0345***	0.0459***	0.0423***	0.0719***
	(10.92)	(8.07)	(9.56)	(12.46)	(11.05)	(8.71)
Uncertainty	0.0240***	0.0195**	0.0185***	0.0170***	0.0125***	0.0627***
	(7.24)	(2.87)	(5.33)	(4.88)	(3.42)	(8.57)
_cons	15.71***	19.10***	12.41***	0.2900	-3.7181***	-
	(16.44)	(9.28)	(12.39)	(-0.29)	(-3.49)	
Time trend & SIC	Yes	Yes	Yes	Yes	Yes	Yes
Adjusters only	No	Yes	No	No	No	No
c_v(Share Price)	No	No	Yes	No	Yes	No
N	150,617	64,612	130,813	138,154	119,739	153,998

Table 5: Reallocation and Uncertainty: Regression Results

The dependent variables are capital reallocation (R) defined as sales of PP&E over PP&E, total reallocation (R+Acq) - sales of PP&E plus Acquisitions over total investment - and ADJ, which takes the value of 1 if a firm exhibits positive sales of PP&E and 0 otherwise. Both the GDP and uncertainty measures are detrended using an HP filter. All variables except ADJ are divided by their standard deviation. The column ADJ present marginal effects from a Panel Probit regression. t-statistics are in parentheses.* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

6.2 Business Cycles

This sub-section compares the aggregate properties of our model with those of the RBC model. There are two motivations for this exercise.

First, one of the key findings of Thomas (2002) and the literature that followed was the near equivalence between the **aggregate moments** of a model with lumpy investment and the aggregate implications of a real business cycle model with quadratic adjustment costs at the plant-level. This sub-section returns to that theme in a model that stresses reallocation rather than the accumulation of capital.

Second, a standard criticism of the RBC model is technological regress: i.e. apparent reductions in total factor productivity. As emphasized in Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) as well, model economies which induce variations in the Solow residual have the potential to explain technological regress and can potentially match other correlation patterns.

Our environment is different from Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) in a couple of important ways. First, our model includes shocks to both the distribution of idiosyncratic shocks and to adjustment costs. Second, as emphasized earlier, a mean preserving spread increases investment. This reflects the timing in our model as well as the structure of adjustment costs. In contrast to models with irreversibility and other forms of non-convexities, there is no option-to-wait in our model with Calvo style adjustment costs. Third, there are no adjustment costs to labor. Finally, as already emphasized, higher order moments matter for the planner and generate an underlying dynamic. In contrast, Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) exclude higher order moments in their approximation. As indicated earlier, there is a dynamic to these higher order moments that underlies the serial correlation in the Solow residual.

Case	$C(y,c)$	$C(y,i)$	$C(y,\tilde{A})$	$C(i,c)$	ρ_c	ρ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\sigma_y}$
Frictions								
stochastic A	0.98 (0.01)	0.98 (0.01)	0.96 (0.01)	0.93 (0.001)	0.93 (0.02)	0.90 (0.02)	0.93 (0.03)	0.98 (0.01)
stochastic π	0.67 (0.06)	0.87 (0.01)	0.84 (0.002)	0.22 (0.04)	0.93 (0.01)	0.88 (0.01)	0.44 (0.04)	0.84 (0.03)
stochastic λ	0.68 (0.04)	0.75 (0.03)	0.90 (0.03)	0.42 (0.03)	0.82 (0.04)	0.81 (0.03)	0.59 (0.03)	0.66 (0.05)
stochastic A,π,λ	0.88 (0.01)	0.66 (0.04)	0.66 (0.04)	0.45 (0.06)	0.88 (0.03)	0.81 (0.02)	0.47 (0.04)	0.81 (0.03)
RBC	0.98 (0.002)	0.91 (0.01)	0.98 (0.002)	0.82 (0.01)	0.95 (0.01)	0.89 (0.013)	0.63 (0.04)	0.92 (0.02)

Table 6: Endogenous Capital Accumulation - Macroeconomic Moments

Results from 1000 simulations are reported with standard deviations in parentheses below. Here $C(x,y)$ are correlations, ρ_x is an autocorrelation and σ_x is a standard deviation. The variables are: output (y), consumption (c), investment (i) and the Solow residual (mis-measured TFP) ($\tilde{A}$).

Table 6 presents standard aggregate moments for a number of cases. These are the traditional macroeconomic moments: the correlations of output (y), consumption (c), investment (i) and TFP($\tilde{A}$). Here the TFP measure is the one constructed from the data as if plants were homogeneous, i.e. mis-measured TFP. The serial correlations of consumption and output as well as relative standard deviations are reported, too.

The rows are the various cases explored before, using the baseline parameters. The last row, "RBC" is the standard stochastic growth model with productivity shocks and without adjustment costs.[23] Here the productivity shocks come from fitting an AR(1) process to the mis-measured TFP series, $\tilde{A}$, generated by the stochastic (π,λ) case. We obtain an AR(1) parameter $\rho_{\tilde{A}} = 0.9183$ and standard deviation of the residual $\sigma_{\tilde{A}} = 0.0132$. This process is fed into the model without adjustment frictions to produce the "RBC" moments.

All of the models match the standard business cycle properties of positively correlated movements of consumption and investment with output. All of these variables are positively correlated with (mis-measured) TFP. The models exhibit consumption smoothing. The aggregate moments are all positively serially correlated. These properties are not surprising in the presence of TFP shocks. But, these same patterns emerge for shocks to (π,λ) as well.

The models with stochastic π and/or λ create considerably lower comovement between consumption and investment compared to the RBC case. As in models with intermediation shocks, such as Cooper

[23]The RBC moments are produced using our model without adjustment frictions. The only stochastic shocks occur to A.

and Ejarque (2000), and discussed further for the case of stochastic λ in Bachmann and Bayer (2013), when returns to investment are large, say due to a high value of λ, consumption is reduced to finance capital accumulation.

The key to this lower correlation is the immediate inverse relationship between consumption and investment when there is a shock to λ. After the impact, consumption and investment move together in the transition dynamics. So, overall there is a positive correlation but one that is reduced due to the negative comovement in response to the innovation. This can be see in the impulse response functions for our model, Figures 3 and 4.

This effect appears in other models of shocks to the variance of productivity shocks. Looking at the impulse response functions in Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), Figures 7 and 8, this negative comovement at impact is apparent. Further, though this negative comovement is not evident in unconditional data moments, it does appear in impulse response functions. In Figure 3 of Bachmann and Bayer (2013), the immediate response in the data to an increase in idiosyncratic risk is for output and investment to increase and consumption to fall.[24] Output and investment fall subsequently.

The aggregate moments of the model with stochastic (A, π, λ) share many of the characteristics of the RBC model. The Solow residual, driven by reallocation, has a serial correlation of nearly 0.92. Consumption, investment and output are positively correlated with the Solow residual and the model exhibits consumption smoothing. In our environment, the puzzle of "What causes a reduction in the Solow residual?" is easily resolved: measured productivity is low when reallocation is low, either due to lower adjustment rates or a contraction in the distribution of profitability shocks.

7 Conclusion

The goal of this paper was to understand the productivity gains from capital reallocation in the presence of frictions. To study this we have looked at the optimization problem of a planner facing frictions in capital accumulation and shocks to productivity, adjustment costs and the distribution of plant specific shocks.

The heterogeneity in plant-level productivity provides the basis for reallocation. The frictions in adjustment prevent the full realization of these gains. The model can generate cyclical movements in reallocation and in the cross sectional distribution of the average productivity of capital.

There are three key findings in this paper. The first is the cyclical behavior of reallocation and the distribution of capital productivity. When shocks to either adjustment frictions or the distribution of plant-level shocks are present, then reallocation is pro-cyclical. In fact, even if there are no direct shocks to TFP, the reallocation process creates fluctuations in output and investment. These effects are not present when the only shock is to TFP. Further the standard deviation of the cross sectional distribution of average capital productivity is counter-cyclical, as in Eisfeldt and Rampini (2006) and

[24] These results are for German data. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) do not report impulse response functions to uncertainty shocks in US data.

Kehrig (2011).

Second, in some, though not all environments, the plant-level covariance of capital and profitability shocks matters for characterizing the planner's solution. This is important for a few reasons. It is indicative of state dependent gains to reallocation and our economy is an example of one where moments other than means are needed in the planner's problem.

Third, the model with shocks to adjustment costs and the cross sectional distribution of productivity shocks can reproduce many features of the aggregate economy. A researcher would interpret the data as generated by a model with TFP shocks even though it is actually constant. That is, the researcher could certainly misinterpret the variations in the Solow residual driven by the reallocation of capital as variations in TFP.

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A Solution Algorithm for Planner's Problem

This section describes the solution of the planner's problem in more detail. The key for the computation is to define a grid for k, which then implies values for μ_A and ϕ_A. The starting point is the non-stochastic environment. Here the planner chooses an allocation of capital over plants whose value is the discounted present value of the implied output.

$$V = \max_k \frac{\bar{\varepsilon}\mu^* + \Lambda\phi_A}{1-\beta} \tag{33}$$

We proceed by computing this vector for any non-stochastic value of π and/or λ. Using the fact that an adjustable plant with idiosyncratic shock $\varepsilon^j > \varepsilon^i$ must have $k(eps^j) > k(\varepsilon^i)$ we create a grid for capital by interpolating between the vectors for the stationary cases. As a lower bound for the grid the vector where

$$k^{MIN} \equiv k(\varepsilon) = 1 \tag{34}$$

can be used. As an upper bound we use the frictionless benchmark computed in (8).

How good is the k-grid? We propose the following measure to check whether for a given value of $\pi < 1$ and λ the vector of capital across plants is indeed optimal. The first robustness check is to add random Gaussian noise to the policy function. We draw Gaussian i.i.d. shocks from a distribution with $N \backsim (0, \sigma_G)$. Applying 1'000 such perturbations to each of our computed optimal k-vectors we find that the throughout the model simulation the maximum increase in output which can be achieved is in the order of 0.01%. Generally we find that the at the tails of the k-vector the largest percentage deviations can occur as a result of our interpolation technique.

B Multi-factor Adjustment

A firm produces output Y using I *inputs*, denotes as k_i with an associated depreciation rate δ_i, where $i = 1, \ldots, I$.[25] The revenue function is given by

$$y = A\left(\sum_{i=1}^{I} k_i\right)^{\alpha},$$

where A describes an aggregate state. We study a multi-input machine replacement problem, building on Cooper and Haltiwanger (1993). Each period the firm decides whether to replace the ith input, i.e. to adjust k_i to its optimal value (normalized to one) or to let the input depreciate. Adjusting an input is costly and subject to a non-convex adjustment cost. We assume that this cost is given by a fixed cost denoted as F_i. Each period the firm draws an F_i for each of its inputs. The draws come from a uniform distribution and are identically and independently distributed across time and inputs. Following Midrigan (2011) we introduce economies of scope in adjusting input levels: the fixed cost of

[25] The depreciation rates can be allowed to vary across inputs.

adjusting inputs is given by the lowest of the I cost draws. The fixed cost of adjusting input levels is thus independent of the number of inputs that deviate from their optimal level. The cost of adjusting *all* inputs is given by

$$F = \min\{F_1, \ldots, F_I\}$$

The state vector at a given point in time consists of the time since the last input adjustments, t, and the current cost F of adjusting all inputs to their optimal level. The value of being in state (t, F) is described by the maximum of the value of adjusting the inputs, V^A, and the value of non-adjustment, V^N.

$$V(t, F) = \max\{V^A(t, F), V^N(t, F)\},$$

with

$$V^A(t, F) = A - F + \beta E_{F'} V(0, F');$$

and

$$V^N(t, F) = A \left(\sum_{i=1}^{I} (1-\delta)^t \right)^\alpha + \beta E_{F'} V(t+1, F').$$

The solution is described by a cutoff value of F for each level of t:

$$\bar{F}(t) = A(1 - \left(\sum_{i=1}^{I} (1-\delta)^t \right)^\alpha) + \beta E_{F'}(V(0, F') - V(t+1, F')).$$

We can now use the model to understand how an individual input i's adjustment decision is shaped by its own state, F_i or by the state of all other inputs F_j for $j = 1, \ldots, I$ with $j \neq i$. We use numerical simulations of the model to illustrate the results and summarize them in three tables.

Table 7 shows the importance of state-dependent vs. non-state dependent adjustments at the input level. We link episodes of adjustments at the factor level with that input's cost draw. For various levels of I the first row shows the percentage of cases in which an adjustment would have taken place in the single-input factor case. In other words, the first row reports in how many cases adjustment occurred at the same time that an individual input i's adjustment cost draw was below its cutoff $\bar{F}_i(t)$.[26] When $I = 1$ this is 100% by definition. Row 2 shows the percentage of positive adjustments where the *single* plant would not have adjusted (since $F_i > \bar{F}_i(t)$). The fact that the firm adjusted although $F_i > \bar{F}_i(t)$ can come about because an individual input's adjustment threshold increases as I increases. This is expressed in row 3, where the cost draw of F_i was the min of all cost draws, but $\bar{F}_i(t) < F_i \leq \bar{F}(t)$. Alternatively the adjustment can occur because another input j had a low enough cost draw realization. Row 4 shows that this becomes the dominant reason for adjusting as I increases.

[26] Analogously to the multi-factor plant the cutoff of a single plant is given by $\bar{F}_i(t) = A((1-\delta)^t)^\alpha) + \beta E_{F'_i}(V(0, F'_i) - V(t+1, F'_i))$.

To sum up, Table 7 shows that the probability that a plant adjusts because it's own cost draw was low enough falls sharply as I increases and then slowly converges towards zero.

Reason	$I=1$	$I=2$	$I=3$	$I=5$	$I=10$	$I=20$
$F_i < \bar{F}_i(t)$	100%	37.1%	27.6%	12.7%	7.2%	3.5%
$F_i > \bar{F}_i(t)$	0%	62.9%	72.3%	87.3%	92.8%	96.5%
$\bar{F}_i(t) < F_i \leq \bar{F}(t)$	-	23.6%	12.3%	10.5%	4.9%	2.2%
$F_j \leq \bar{F}(t)$	-	76.4%	87.7%	89.5%	95.1%	97.8%

Table 7: Adjustments. Simulation results from 10'000 simulations for each case.

Another way to show the importance of state-dependent adjustment rules is by running the following regression model on the simulated data:

$$Adjust_{it} = \beta_0 + \beta_1(F_i \leq \bar{F}_i(t)) \tag{35}$$

The regression assesses the predictive power of input i drawing an adjustment cost F_i below the cutoff from the one-input model, $\bar{F}_i(t)$, for adjusting. Table 8 shows the R^2 of the above regression for various values of I in the first row. As I increases the R^2 rapidly converges towards zero, as does the estimated coefficient $\hat{\beta}_1$. With multiple inputs, plants increasingly adjust due to other plants' input cost draws.

Row 2 shows a comparison with a Calvo model. The regression in (35) included a time-dependent cutoff $\bar{F}_i(t)$. A Calvo-style representation of the problem would feature a time-*independent* adjustment threshold, $\bar{F}^{Calvo}$. For row 2 of Table 8, we set the adjustment probability $\bar{F}^{Calvo}$ to be the average adjustment probability from the time-dependent model and report the R^2 from the following regression:

$$Adjust_{it} = \beta_0 + \beta_1(F_{it} \leq \bar{F}^{Calvo}) \tag{36}$$

The results show that as I increases the time- and state-independent adjustment cost is an increasingly good approximation of the adjustment behaviour of a single plant.

R^2	$I=1$	$I=2$	$I=3$	$I=5$	$I=10$	$I=20$
$F_i \leq \bar{F}_i(t)$	1	0.34	0.18	0.07	0.02	0.01
$F_i \leq \bar{F}^{Calvo}$	0.51	0.50	0.46	0.59	0.82	0.99

Table 8: The importance of the state-dependent cutoff. Simulation results from 10,000 simulations for each case.

Finally, for various values of I we compare several moments generates by the state-dependent model (SD) and compare them to the moments generated by the Calvo version of the model, which does not allow for any selection effects. The results are shown in Table 9. Although the autocorrelation of capital adjustments can, by definition, not be replicated by a Calvo model, the results show that the other relevant moments are very well approximated, even for low values of I.

	$I = 1$		$I = 2$		$I = 5$		$I = 10$	
Moment/Model	SD	Calvo	SD	Calvo	SD	Calvo	SD	Calvo
Autocorrelation of adjustments	-0.20	-0.001	-0.29	0.001	-0.33	-0.006	-0.18	-0.01
Coeff of variation of output	0.17	0.19	0.11	0.12	0.05	0.06	0.03	0.03
Avg. time between adjustments	2.20	3.22	1.17	1.70	0.45	0.60	0.17	0.18
Inaction rate	0.02	0.06	0.08	0.17	0.48	0.54	0.73	0.75

Table 9: Adjustments. Simulation results from 10'000 simulations for each case. 'B' represents the benchmark model described above. 'Calvo' represents the Calvo-model with an exogenous adjustment hazard chosen to match the benchmark model. Inaction is defined as changes in the capital stock of less than 2.5%.

C Data on Capital Reallocation

Here we present additional information about the data on capital reallocation, GDP, and uncertainty. Table 10 provides summary statistics of the Compustat firm-level variables. These are sales of PP&E (#107), acquisitions (#129), total assets (#6), PP&E (#8), capital expenditures (#30). Tobin's Q was created as the market to book value (#60, #74, and #6). We deflated all series to 1984 dollars using the CPI from the BLS. Following Eisfeldt and Rampini (2006) we compute turnover ratios - reallocation normalized by the subset of the capital stock included in our data - to account for the fact that Compustat only includes a subset of all firms. Figure 5 shows the time series of dispersion in Tobin's Q and real GDP.

Variable	**Mean**	**Median**	**Std. Dev.**
(A) Summary Statistics			
Sales of PP&E	6.66	0	101.92
Acquisitions	21.13	0	246.014
Total Assets	1227.24	60.37	5956.71
PP&E	517.96	12.47	2631.88
Capex	85.18	2.80	502.42
(B) Reallocation Ratios			
Reallocation/Capex	26.8%		
Acquisitions/Capex	24.8%		
Sales of PP&E/Capex	7.8%		
Sales of PP&E/Reallocation	29.2%		
(C) Turnover Rates			
Acqusitions/Assets$_{t-1}$	1.7%		
Sales of PP&E/PP&E$_{t-1}$	1.3%		
Reallocation/Assets$_{t-1}$	2.0%		
Reallocation/PP&E$_{t-1}$	4.5%		

Source: Compustat

Table 10: Summary statistics for Compustat capital reallocation data

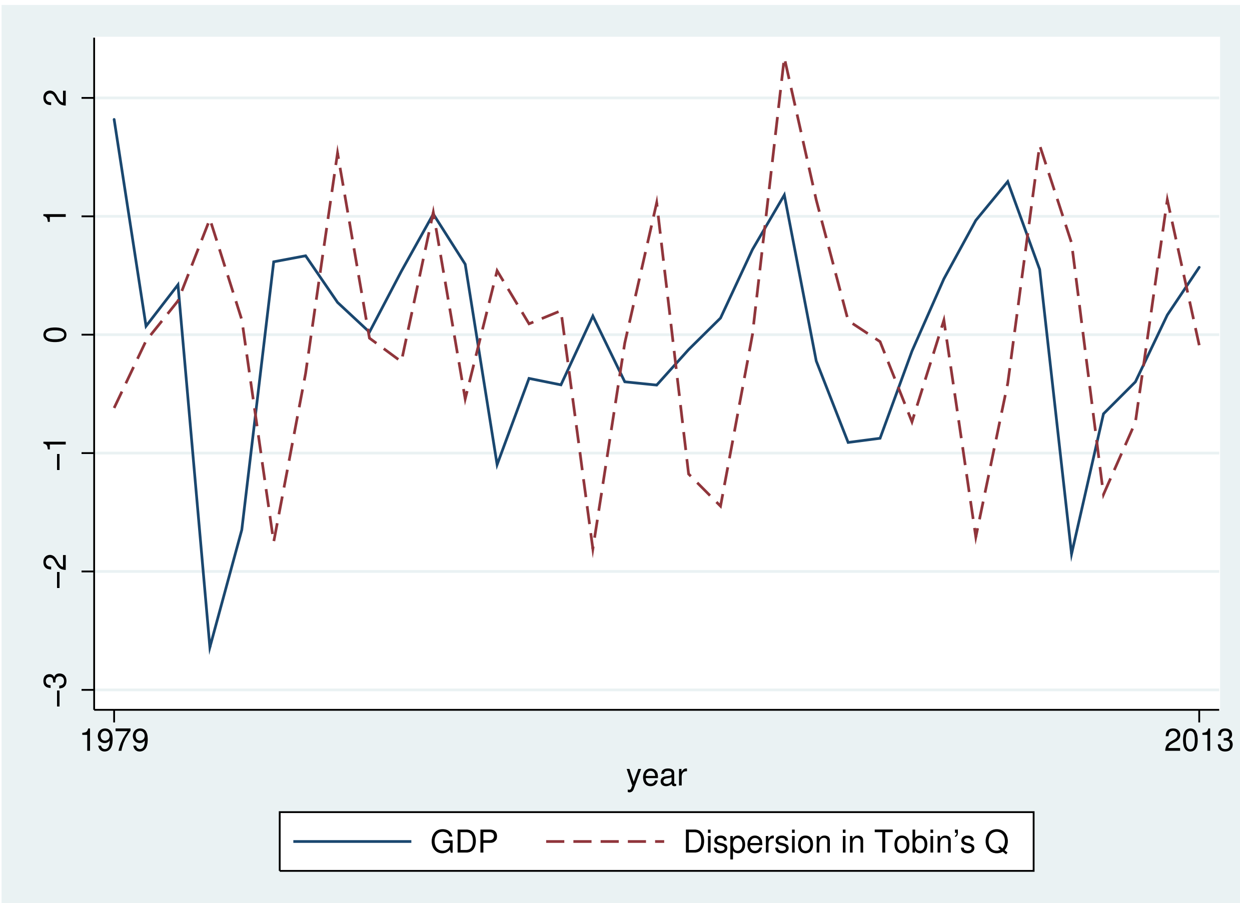

Figure 5: Dispersion in Tobin's Q over the Business Cycle. Data are HP filtered and normalized by standard deviation. The solid blue line denotes log US GDP, the dashed red line denotes the standard deviation in Tobin's Q across firms. Source: Compustat.