Earnings Inequality And Income Redistribution Through Social Security

Job market paper

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Abstract

Despite increasing earnings inequality and aging population, Social Security replacement rates in the U.S. have not been changed since 1977. In this paper, I ask what an optimal Social Security policy might look like in 2017. Using a general equilibrium overlapping generations model with heterogeneous agents and incomplete markets, I recover the Pareto weights consistent with the Social Security Amendments of 1977. I find that the government in 1977 must have put a large weight on the young and middle-aged workers, with the largest weight on the middle-aged with medium earnings records. Applying the same weights in 2017, the optimal Social Security policy differs substantially from the current system. Conversely, assuming the existing policy is optimal throughout the period, the changes in the implied Pareto weights during 1977-2017 reveal a shift from favoring young and middle-aged workers with medium income to favoring earnings-rich households.

Key words: Economic Inequality, Heterogeneous Agents, Political Inequality, Social Security.

JEL codes: D3, D52, H21, H55, E62.

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1 Introduction

In 1977 the U.S. Congress introduced the Social Security (henceforth, SS) Amendments, whose main purpose was to "stabilize future replacement rates", defined as the ratio of an individual's pension benefit to their average lifetime earnings. Upon signing the Amendments, President Carter announced that "the provisions of this Law are tremendous achievements and represent the most important SS legislation since the program was established." The statutory replacement rates have not been adjusted since then, despite the fact that the U.S. economy has changed a lot since the adoption of the Amendments. In particular, earnings inequality has increased sharply and the U.S. population has continued to age. These developments are likely to change the optimal degree of income redistribution across and within generations, as well as the optimal level of risk sharing by U.S. households. In this paper I ask: Given these changes in the U.S. economy, what is the optimal SS policy now? I find that it is very different from the existing one.

A vast strand of the literature has analyzed the optimal design of government policies (e.g. Rios-Rull & Krusell 1999, Hassler et al. 2007, Corbae et al. 2009). Following this literature, I will assume that the SS policy is chosen optimally by a utilitarian government (a government who maximizes the total welfare of all households). There is, however, one novel difference in my approach. Instead of assuming that the government weighs all households equally, I will identify the Pareto weights which are consistent with the SS Amendments of 1977. Then I will apply the identified weights to find the optimal SS policy in 2017, accounting for the realized changes in inequality and aging.

Pareto weights are important for a normative analysis. The distribution of weights across households reflects the degree of political inequality in the economy. The aforementioned papers abstract from political inequality and, not surprisingly, over-predict the amount of income redistribution. But there is large empirical evidence that political inequality in the U.S. is indeed substantial.[1] Therefore, the optimal Social Security policy in response to rising earnings inequality and population aging will crucially depend, both qualitatively and quantitatively, on a particular distribution of Pareto weights in the economy.

I use a general equilibrium overlapping generations model with incomplete markets and labor-augmenting technological progress in the style of Huggett (1996). The model features no aggregate risk. The birth rate in the economy is constant. Agents enter the model as workers with a given ability (high or low). Workers decide on how much to consume,

[1] Numerous empirical studies document that richer people may have stronger power in politics than the poor in the U.S. (Rosenstone & Hansen 1993, Page et al. 2013, Campante 2011).

work and save in a risk-free asset. Worker's idiosyncratic labor productivity is composed of a deterministic component, which depends on agent's ability, and a random component, which consists of a persistent auto-regressive shock and a transitory shock. At a mandatory retirement age, all agents retire and decide on consumption and saving. Agents live up to a maximum age but may decease earlier due to stochastic mortality.

There is a government, which decides upon the replacement rate schedule to maximize the weighted sum of the expected discounted lifetime utilities of all generations alive. I model the replacement rate schedule flexibly via two parameters. This representation encompasses a broad class of potential pension systems. One parameter controls the overall *level* of pension benefits, while the other determines the *curvature* of the schedule, i.e. the progressiveness of the pension system. Whenever the Social Security system is re-optimized, the government sets the replacement rate schedule once-and-for-all. In the baseline model, the SS tax rate adjusts to balance the government budget in each period. Apart from running a pension system, the government distributes lump-sum transfers across all agents financed by progressive income taxes and the wealth collected from deceased households.

I first calibrate the benchmark model to the U.S. data in 1977. One of the key model parameters is the variance of the persistent productivity shock, which I calibrate to match the P80/P50 ratio of the earnings distribution, while I use empirical estimates for the remaining parameters of the productivity process. The calibrated model matches the key moments of the pre-tax earnings and income distributions at that time. I then calibrate the parameters of the Pareto weight function so that the government in 1977 optimally chooses the replacement rate schedule implied by the SS Amendments. When calibrating the weights, I assume that the government and households expected no changes in the U.S. economy after 1977.

I specify the Pareto weights as a Cobb-Douglas function of individual's average lifetime earnings and individual's age. This choice is motivated by numerous empirical studies which find that participation in almost any form of political activities in the U.S. differs across households' age and income. The calibration strategy for the Pareto weight function is based upon the distributional conflict among agents in the economy. First, I argue that the *level* of the replacement rate schedule in the data helps identify the parameter of the Pareto weight function attached to age. This is because the young prefer a low level of replacement rates and therefore a low SS tax rate, since they face increasing wage profiles and would suffer welfare losses from paying higher SS contributions. The middle-aged expect to receive labor income for a shorter period than the young and therefore they are more willing to sacrifice falling after-tax wages for higher future replacement rates. Retirees do not pay any contributions, which makes their most preferred replacement rate even higher. Second, the *curvature* of the pension schedule across different levels of earnings in the data helps identify the parameter of the Pareto weight function attached to average lifetime earnings. Conditional on age, workers with lower lifetime earnings opt for a more redistributive pension system. Poor young prefer a progressive system because they value a lot the ex-ante insurance of the public pension system. Poor retirees also prefer a progressive system, though for pure ex-post redistribution reasons. For poor middle-aged, it's a combination of the two reasons.

Having calibrated the model, I simulate it during 1977-2017, feeding into the model the exogenous paths of the key model parameters, which reflect the major economic and demographic changes in the U.S. economy during this period. The first change refers to a drastic rise in cross-sectional earnings inequality. In line with Heathcote et al. (2010), I capture this through a rise in the skill premium and an increase in the dispersion of life-cycle earnings. The second change refers to population aging. I account for this change by reducing the birth rate and increasing (age-dependent) survival rates. The third change is a sharp reduction in the progressiveness of the income tax schedule during the 1980s, mostly due to the adoption of the Economic and Recovery Tax Act of 1981.

There are several findings. First, the government in 1977 must have put a disproportionately large weight on the young (age 20-23) and middle-aged workers (age 24-64), with the largest weight on the middle-aged with medium earnings records. At the same time, retirees of all earnings classes must have had an insignificant weight. The intuition for the result is the following. The only group of agents in the model whose most preferred level of pension benefits lies slightly below the observed one in the data in 1977 are the young. Since every other group of agents in the model prefers pension levels far above the one observed in the data, only a large weight on the young would be consistent with the *level* of replacement rates in the data. But the young opt for a more progressive system than in the data. On the other hand, the middle-aged with medium earnings prefer an intermediate degree of tightness between average lifetime earnings and pensions, very close to the one seen in the data. Thus, a large weight must be assigned to these agents, for the optimal *curvature* of the replacement rate schedule to be consistent with the data.

Second, the optimal SS policy in 2017 (with the identified Pareto weights from 1977) looks very different from the prevailing one. The optimal replacement rate for poor individuals is more than 6 times higher than in the status quo due to an increased demand for insurance by the young. The replacement rate for individuals with medium lifetime earnings rises from 50 percent to 88 percent due to an increased demand for ex-post (i.e. after realization of idiosyncratic productivities) income redistribution by the middle-aged workers. On the contrary, earnings-rich individuals face falling replacement rates. The implied SS tax rate goes up from the status quo level of 11 percent in 2017 to more than 17 percent in the long run. Would this reform gain majority in a *one-man-one-vote* political system? It turns out, no: The reform has support of 46 percent of the voters alive in 2017.

Third, I find that the optimal SS policy in 2017 generates an aggregate welfare gain (in consumption equivalent terms) as high as 9 percent relative to the status quo policy. However, the unequal distribution of Pareto weights leads to an unequal distribution of welfare gains across age and income groups. Large welfare gains are recorded for the young and the middle-aged. Retirees with low earnings records have the largest welfare gains, since they immediately benefit from spiking replacement rates without having to pay any additional funds into the pension system. Even though the government puts insignificant weights on retirees, it is the ex-ante insurance for the young that creates huge ex-post (i.e. after realization of idiosyncratic productivities) benefits for the poor retirees.

Fourth, I recompute the Pareto weights in 1977 assuming that the government and households had a perfect foresight about the future changes in the U.S. economy. The calibrated Pareto weights are qualitatively similar, except that the weight on the middle-aged with medium earnings deteriorates in favor of the young and earnings-rich middle-aged. Anticipating a widening gap in labor productivities, the middle-aged with medium earnings opt for a level of pension benefits that is too high and a system that is far more progressive than in the data. For these agents, the chances of ending at the bottom of the lifetime earnings distribution at retirement are higher now. Therefore, a larger weight is required on the young and earnings-rich workers, for the optimal level and the curvature of the replacement rate schedule to be consistent with the data. In terms of the optimal SS policy in 2017 under these weights, the replacement rate rises only for earnings-poor individuals but decreases for all other earnings groups. This allows the government to reduce the SS tax rate from the status quo level of 11 percent in 2017 to 8 percent in the long run.

Finally, I explore one potential reason for why the SS system has not been adjusted since 1977. So far, I have assumed that the SS Amendments were optimal (under the calibrated weights) in 1977 but suboptimal afterwards given the changes in the economic environment and demographics. But what if the SS policy of 1977 has in fact been an optimal response to the changes in earnings inequality and population aging, due to changes in the Pareto weights? I re-compute the Pareto weights, such that the government in the model optimally chooses the SS Amendments along the transition. I detect several trends in the Pareto weights (normalized to sum up to 1 at each point in time). Compared to 1977, the weight on the young has declined by almost three times. This drop has been accompanied by the rise in the weights for earnings-rich workers and retirees: While the weight on earnings-rich middle-aged workers more than doubled, the weight on earnings-rich retirees almost quadrupled. Overall, assuming that the SS policy has been optimal during 1977-2017, it no longer reflects the preferences of young and middle-aged but rather rich individuals.

2 Related literature

The paper builds upon several strands of the literature.

First, the paper relates to the macroeconomic literature, which develops politico-economic models to rationalize the size of the observed welfare programs in the U.S. Rios-Rull & Krusell (1999), Hassler et al. (2007), Corbae et al. (2009), Song et al. (2012) introduce a social welfare function into a general equilibrium model with production in order to account for the observed amount of income redistribution through income taxation. The first three papers assume a utilitarian social planner who puts equal weights on all households, so it is not surprising that the resulting equilibrium income tax rate exceeds the empirical rate in these models. The last paper parametrizes the relative Pareto weight on retired households to account for the level of public good provision and public debt in the U.S. As opposed to these studies, the Pareto weights in my paper are calibrated within the model. Moreover, the focus of my paper is on SS, not income taxation.

The second strand of the literature takes SS as given and studies macroeconomic implications of an (exogenous) transition from the publicly provided to a fully funded pension system. This analysis is done in a general equilibrium overlapping generations framework with production, incomplete financial markets and idiosyncratic labor productivity risk in the style of Huggett (1996). Conesa & Krueger (1999) find that quantitatively SS plays an important role as a partial insurance device against idiosyncratic risk. Krueger & Kubler (2006) study the risk sharing properties of SS and find that the introduction of an unfunded SS system can lead to a Pareto improvement in a model with stochastic aggregate production shocks if markets are incomplete and households are fairly risk averse. Fuster et al. (2007) acknowledge the importance of SS as an insurance device but show that household members can provide valuable insurance to each other privately. Then, privatizing the pension system can generate significant welfare gains. In my paper, I confirm the importance of SS as a partial insurance device. However, in my model SS arises endogenously. The equilibrium distribution of Pareto weights, skewed towards young households, reflects the demand for insurance by these agents.

This paper also builds upon the growing literature on the *inverse optimum* (or *revealed* *preference*) approach. Bourguignon & Spadaro (2012), Lockwood & Weinzierl (2016), Saez & Stantcheva (2016) combine analytical results from the optimal tax theory in the Mirrleesian framework together with the assumptions on economic parameters to infer the marginal social welfare weights prevailing in the data. Bai & Lagunoff (2013) assume a political system that produces policies as if they resulted from a weighted majority voting process, in which an individual's vote share depended on her wealth. This implied vote share is assumed to be a power function of agent's wealth holdings. The authors provide regularity conditions for a unique mapping between the equilibrium income tax rate and the wealth bias parameter. Similar to my work, Lockwood & Weinzierl (2016) not only recover but also use the positive, empirical estimates of the weights in order to provide normative assessments of the income tax policies in the U.S. All of these studies, however, are based on stylized static model economies, which is not the case in my paper.

Bachmann & Bai (2013) is a noteworthy exception. The authors introduce Pareto weights, assuming the same functional form as in Bai & Lagunoff (2013), into a general equilibrium dynastic framework with production, aggregate and idiosyncratic productivity risk. The authors recover the parameter of the Pareto weight function, which makes the equilibrium contemporaneous correlation between output and government purchases in their model consistent with the U.S. data. Note that their model misses any form of income redistribution, which is the key aspect of my paper. Moreover, I analyse changes in the Pareto weights over time.

3 Pension benefit formula

The SS Amendments of 1977 fixed a specific formula and its parameters, which the SS Administration applies to compute the pension benefit. The key variable of this formula are the average indexed monthly earnings (AIME). Essentially, these are the average monthly earnings (adjusted for inflation and growth in wages) over individual's 35 highest years of working career. I will refer to the AIME as the average lifetime earnings. Only earnings below a certain threshold (*earnings cap*) flow into the computation of average lifetime earnings. Earnings below the earnings cap are referred to as *annual maximum taxable earnings*. The pension benefit formula then maps the average lifetime earnings into a pension benefit. More specifically, the formula multiplies a 90, 32, or 15 percent factor by the portion of worker's average lifetime earnings that fall within the three respective ranges, and then adds the resulting products together. These ranges are determined by two *bend points*. Since the

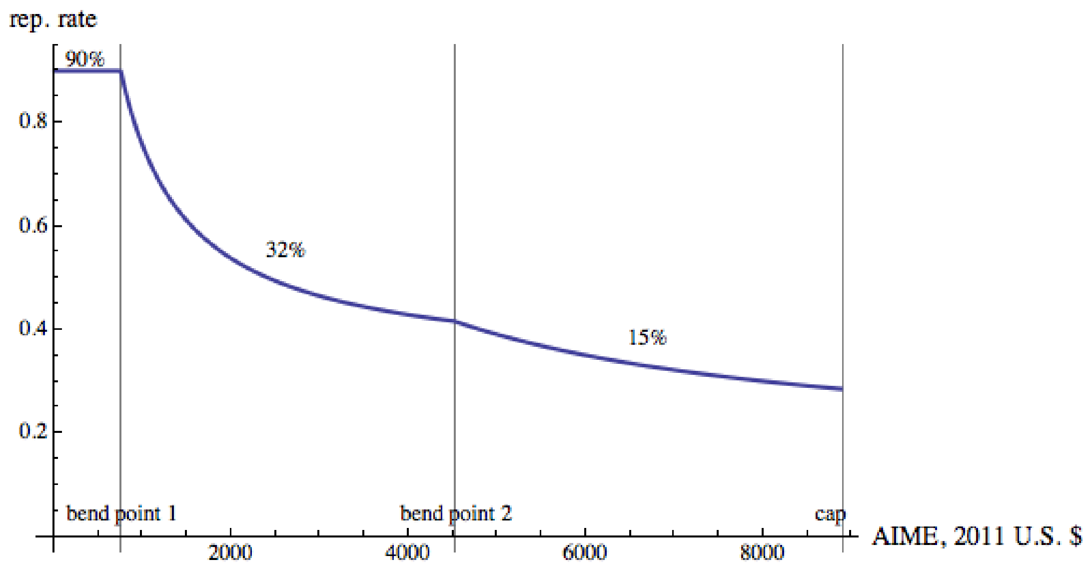

Figure 1: Statutory replacement rate for workers entering retirement in 2011

earnings cap sets the upper bound on the average lifetime earnings, it also sets the upper bound on the amount of the pension benefit.[2]

Figure 1 plots the schedule of replacement rates implied by the pension benefit formula for those individuals, who entered retirement in 2011. As an example, consider a worker, whose AIME are equal to \$6,000. Her replacement rate is then ca. 35 percent, which implies a monthly pension benefit of \$2,100. This amount is computed as follows: $90\% \times 749 + 32\% \times (4,517 - 749) + 15\% \times (6,000 - 4,517)$, where \$749 is the first and \$4,517 – the second bend point as of 2011.

The SS Amendments of 1977 introduced automatic indexation of the two bend points and the earnings cap to account for inflation and growth in wages. The 90, 32, and 15 percent factors have been kept fixed since adoption of the Amendments. [3]

4 Model

The model is based on Huggett (1996), which is a general equilibrium overlapping generations model with production, incomplete financial markets and idiosyncratic labor productivity risk. Among several others, I make two important departures from this environment. First,

[2]In the figure I mark the cap in *monthly* terms by dividing the annual cap of \$106,800 in 2011 by 12.

[3]See Appendix D in the *Annual Statistical Supplement for 2012* by the SS Administration for a detailed explanation on wage indexing, https://www.ssa.gov/policy/docs/statcomps/supplement/2012/.

I introduce earnings-dependent pension benefits. Second, I endogenize the SS policy.

4.1 Demographics and endowments

The economy is populated by overlapping generations of households. Each period a new generation of agents is born. The birth rate is constant and equals n. Each generation lives for J periods. Age is denoted by $j \in \{1, 2, .., J\}$. Agents enter the economy and start working at age $j = 1$. The mandatory retirement age is J^R. Agents die with probability 1 at age J. Denote ψ_{jt} the probability that an agent survives up to age $j + 1$, conditional on surviving up to age j at time t .

Each agent is endowed with one unit of productive time in each period, which she supplies inelastically to a competitive labor market. Agents are born with zero assets but can accumulate savings over time, supplying capital to a competitive capital market.

At birth, each individual receives a realization of a random variable $z \in Z = \{H, L\}$, where H stands for high and L – for low ability. Abilities are drawn from a stationary distribution λ_z, which is assumed to be unique. The ability remains constant during the entire working stage of the agent. The ability determines agent's labor productivity during the working stage.

4.2 Labor productivity process

The productivity of type-z agent at age j is given by $\zeta_{zjt} \times \exp(y_{jt})$. The first term, ζ_{zjt}, is a deterministic component; it captures the returns to experience over the life-cycle shared by each ability group. For retired agents, $\zeta_{zjt} = 0$. The second term, y_{jt}, is a random individual-specific component of log labor productivity. It is composed of a persistent auto-regressive shock and a transitory shock:

$$\begin{aligned} y_{jt} &= \eta_{jt} + \nu_t, \\ \eta_{jt} &= \rho\eta_{j-1,t-1} + \gamma_t \text{ with } \eta_1 = 0, \end{aligned} \tag{1}$$

where $\nu_t \sim \mathcal{N}(0, \sigma^2_{\nu_t})$ and $\gamma_t \sim \mathcal{N}(0, \sigma^2_{\gamma_t})$. The auto-regressive specification for η captures mean-reverting shocks, such as human capital innovations that depreciate over the life-cycle. The transitory component ν represents short-term variations in individual productivity. To simplify notation below, I stack the realizations of η_{jt} and ν_t into a vector $\mathbf{y}_{jt} \in \mathcal{Y}$ and denote agent's total efficiency units per unit of raw labor by $\epsilon_{zjt}(\mathbf{y}_{jt})$. The stochastic process for $\mathbf{y}$ is identical and independent across agents and follows a finite-state Markov process with stationary transitions over time, i.e.:

$$\pi(y, \mathcal{Y}) = \text{Prob}(y_{j+1,t+1} \in \mathcal{Y} \mid y_{jt} = y)$$

Let Π_y denote the invariant probability measure of newborn agents with productivity y, which I assume to be unique.

4.3 Labor-augmented technology growth

The aggregate output good is produced using the production function $Y_t = K_t^\theta (A_t N_t)^{1-\theta}$, where A is the labor-augmented technology index that grows at an exogenous rate g, K – the aggregate capital stock, N – the aggregate labor input and $\theta \in (0, 1)$ – the capital share in production. The output can be consumed or invested in capital. The depreciation rate of capital is $\delta \in (0, 1)$. The firm produces output goods and sells them in a competitive market at a price that is normalized to one. As standard with a constant returns to scale technology and perfect competition, I assume that there exists a representative firm, which operates this technology. The rental price of capital, r_t, and the wage per *effective* unit of labor, w_t, are determined competitively:

$$r_t = \theta \left(K_t / A_t N_t \right)^{\theta - 1} - \delta \text{ and } w_t = (1 - \theta) \left(K_t / A_t N_t \right)^{\theta}. \tag{2}$$

4.4 Households

A worker supplies raw labor l to the competitive labor market and receives gross earnings equal to

$$e_{zjt} = w_t \epsilon_{zjt}(y_{jt}) l$$

Then, agent's earnings net of SS contributions amount to:

$$e_{zjt} - \tau_t \min(cap_t, e_{zjt}),$$

where the linear SS tax rate, τ, applies to the portion of gross earnings below the maximum taxable earnings, cap. During working time, agent's average lifetime earnings evolve according to:

$$\bar{e}_{z,j+1,t+1} = \begin{cases} \left[(j-1) \bar{e}_{zjt} E_t / E_{t-1} + \min(cap_t, e_{zjt}) \right] / j & \text{for } 1 \leqslant j < J^R \\ \bar{e}_{zjt} & \text{for } j \geqslant J^R \end{cases} \tag{3}$$

with $\bar{e}_{z1t} = 0$. Consistent with how the SS Administration computes the AIME, agent's average lifetime earnings in the model are adjusted for the growth in average earnings among workers, E_t. To adjust for labor-augmenting productivity growth, the variables e_{zjt}, cap_t and $\bar{e}_{zjt}$ are normalized by A_t at each point in time (recall that the wage is defined in per effective units of labor). When agent with average lifetime earnings $\bar{e}_{zjt}$ retires, she receives a pension benefit, $B_t(\bar{e}_{zjt}; \boldsymbol{\Psi}_t)$. The pension benefit function will be extensively described below; for now, let $\boldsymbol{\Psi}_t$ be a vector of variables, which characterize the pension benefit rule at time t.

Using the dynamic programming notation, agent's problem can be written as follows. Let $\boldsymbol{x}$ denote the individual state of the agent in period t:

$$\boldsymbol{x} = (z, j, \boldsymbol{y}, a, \bar{e}),$$

where $a \in A = [0, a^{max}]$ – asset holdings and $\bar{e} \in \bar{E} = [0, \bar{e}^{max}]$ – average lifetime earnings.[4][5] Furthermore, denote $F_t(\boldsymbol{x})$ the cumulative population density function of agents at the beginning of period t; the corresponding density function is denoted $f_t(\boldsymbol{x})$. Finally, let $V(\boldsymbol{x}; \boldsymbol{\Psi}_t, F_t)$ be the discounted lifetime indirect utility of agent in state $\boldsymbol{x}$ at time t.

Taking $(\boldsymbol{\Psi}_t, \boldsymbol{\Psi}_{t+1}, \tau_t)$ as given, agents solve:

$$V(\boldsymbol{x}; \boldsymbol{\Psi}_t, F_t) = \max_{c,l,a'} \left\{ \frac{(c^{\gamma}(1-l)^{1-\gamma})^{1-\sigma}}{1-\sigma} + \beta\psi_j \mathbb{E}\left[V(\boldsymbol{x}'; \boldsymbol{\Psi}_{t+1}, F_{t+1}) \mid (\boldsymbol{x}, t)\right] \right\} \tag{4}$$

subject to:

$$\begin{aligned} \boldsymbol{x}' &= (z, j+1, \boldsymbol{y}', a', \bar{e}') && (5) \\ (1+g)a' &= \mathbb{1}_{1 \leqslant j < J^R}\left[e_{zjt} - \tau_t \min(cap_t, e_{zjt}) - \tau_{I,t}(r_t a + e_{zjt})\right] && (6) \\ &+ \mathbb{1}_{J^R \leqslant j \leqslant J}\left[B_t(\bar{e}; \boldsymbol{\Psi}_t) - \tau_{I,t}(r_t a + B_t(\bar{e}; \boldsymbol{\Psi}_t))\right] \\ &+ (1+r_t)a + T_t - c \end{aligned}$$

including the law of motion for the average lifetime earnings in eq. (3), and the non-negativity constraints:

$$c \geqslant 0, \ a' \geqslant 0 \text{ and } 0 \leqslant l \leqslant 1. \tag{7}$$

In eq. (4), c denotes consumption and σ controls the degree of relative risk aversion; γ

[4]Since there will be only one type of asset in the economy, I will refer to a as capital, wealth and assets interchangeably.

[5]Note that I drop the subscripts for the variables $\bar{e}_{zjt}$ and $\boldsymbol{y}_{jt}$.

is the relative weight on consumption; β is the growth-adjusted discount factor (explained in the calibration section) and $\mathbb{E}$ is a conditional expectation operator. The CRRA utility function is consistent with the assumed balanced growth. Eq. (5) is a law of motion for the individual state; note that the ability z doesn't change during agent's life.

Eq. (6) is a budget constraint of a worker and a retiree. A working agent receives gross earnings e_{zjt}, pays SS contributions and income taxes on gross earnings and earned interest, ra, according to the function, τ_I. A retired agent receives a pension benefit B and pays income taxes on earned interest and pension.[6] Both a worker and a retiree receive gross interest on their savings, $(1+r)a$, and a lump-sum income transfer, T. I exclude agents from borrowing, which explains the non-negativity constraint on assets in eq. (7). In eq. (6), consumption, asset holdings, earnings, earnings cap, pension benefit and lump-sum transfers are adjusted for the labor-augmenting productivity growth. While agent's pension benefit stays constant during retirement (since her average lifetime earnings remain constant), the technology grows at a rate g; therefore, the pension benefit per effective labor decreases during retirement at a rate g.

The solution to the household's problem generates the decision rules $c(x; \boldsymbol{\Psi}_t, F_t)$, $l(x; \boldsymbol{\Psi}_t, F_t)$ and $a'(x; \boldsymbol{\Psi}_t, F_t)$ as well as the law of motion for average lifetime earnings $\bar{e}'(x; \boldsymbol{\Psi}_t, F_t)$.

4.5 Government

The government is involved in two activities. First, it runs a pay-as-you-go SS system: it collects payroll contributions from workers and redistributes them among retirees. Second, the government taxes earnings, capital interest and pensions based on the income tax function τ_I, confiscates the wealth left by deceased agents at the end of the year and redistributes tax proceeds as lump-sum benefits, T, among all individuals in the same year. The government runs a balanced budget in each of these activities.[7]

4.6 Recursive competitive equilibrium

I define the recursive competitive equilibrium in two steps. In the first step, I set up the equilibrium for an exogenous SS policy: at time t, all agents observe the current SS policy $\boldsymbol{\Psi}_t$ and take the future (constant) SS policy as given. In the second step, I make the SS policy itself consistent with the solution to the optimization problem of the social planner.

[6] According to the U.S. law, payroll taxes withheld from earnings are not deductible from federal or any state income tax. Furthermore, pension benefits are subject to income taxation since 1984.

[7] I relax this assumption and discuss the results in section 8.3.

Definition 1. Given F_t, $\mathbf{\Psi}_t$, and a constant future pension policy $\mathbf{\Psi}_{t+1} = \mathbf{\Psi}_{t+2} = \mathbf{\Psi}'$, a recursive competitive equilibrium with an **exogenous** SS policy is a set of functions $(V, c, l, a', w, r, T, F, \tau)$, such that the following statements hold:

- the functions (c, l, a') solve agent's optimization problem in eq. (4);
- the factor prices r and w are determined competitively from (2);
- the capital and labor markets clear:

$$\begin{aligned} K_t &= \sum_{j=1}^{J} \int_{A\times\bar{E}\times\mathcal{Y}} a\, dF_t(\boldsymbol{x}), \\ N_t &= \sum_{j=1}^{J^{R-1}} \int_{A\times\bar{E}\times\mathcal{Y}} \epsilon_{zjt}(\boldsymbol{y}) l(\boldsymbol{x}; \mathbf{\Psi}_t, F_t) dF_t(\boldsymbol{x}); \end{aligned}$$

- the SS system runs a balanced budget:

$$\tau_t \sum_{j=1}^{J^R-1} \int_{A\times\bar{E}\times\mathcal{Y}} \min(cap_t, w_t \epsilon_{zjt}(\boldsymbol{y}) l(\boldsymbol{x}; \mathbf{\Psi}_t, F_t)) dF_t(\boldsymbol{x}) = \sum_{j=J^R}^{J} \int_{A\times\bar{E}\times\mathcal{Y}} B_t(\bar{e}; \mathbf{\Psi}_t) dF_t(\boldsymbol{x}); \tag{8}$$

- the income transfer program runs a balanced budget:

$$\begin{aligned} T_t &= \sum_{j=1}^{J^R-1} \int_{A\times\bar{E}\times\mathcal{Y}} \tau_{I,t}(r_t a + w_t \epsilon_{zjt}(\boldsymbol{y}) l(\boldsymbol{x}; \mathbf{\Psi}_t, F_t)) dF_t(\boldsymbol{x}) & (9) \\ &+ \sum_{j=J^R}^{J} \int_{A\times\bar{E}\times\mathcal{Y}} \tau_{I,t}(r_t a + B_t(\bar{e}; \mathbf{\Psi}_t)) dF_t(\boldsymbol{x}) & (10) \\ &+ \sum_{j=1}^{J} \int_{A\times\bar{E}\times\mathcal{Y}} (1-\psi_j)(1+g) a'(\boldsymbol{x}; \mathbf{\Psi}_t, F_t) dF_t(\boldsymbol{x}) & (11) \end{aligned}$$

- the law of motion for the population density is, for $j = 1, ..., J-1$:

$$f_{t+1}(\boldsymbol{x}') = f_{t+1}(z, j+1, \boldsymbol{y}', a', \bar{e}') = \frac{\psi_{jt}}{1+n} \int_{A\times\bar{E}\times\mathcal{Y}} \mathbb{1}_{a'=a'(\boldsymbol{x};\mathbf{\Psi}_t,F_t), \bar{e}'=\bar{e}'(\boldsymbol{x};\mathbf{\Psi}_t,F_t)} \pi(\boldsymbol{y}' \mid \boldsymbol{y}) dF_t(\boldsymbol{x})$$

  together with the distribution for age 1 households:

$$f_t(z, 1, \boldsymbol{y}, 0, 0) = \lambda_z \Pi_y,$$

where λ_z is the measure of newborn agents with ability z and Π_y is the measure of newborn agents with productivity y. Recall that agents are assumed to enter the economy at age $j = 1$ without any assets and working histories.

Definition 2. A **steady-state** recursive competitive equilibrium with an **exogenous** SS policy is a recursive competitive equilibrium based on definition 1 with $F_t = F_{t+1} = F$ for all t. This implies that the economy is on the balanced-growth path with labor-augmenting productivity and population growth rates, g and n, respectively. Also, the pension benefit policy is constant with $\boldsymbol{\Psi}' = \boldsymbol{\Psi}_t$ for all t.

4.7 Replacement rate schedule

I specify the replacement rate schedule, $R_t(\bar{e}; \boldsymbol{\Psi}_t)$, as a power function of agent's average lifetime earnings normalized by the average (growth-adjusted) earnings among working agents:

$$R_t(\bar{e}; \boldsymbol{\Psi}_t) = \begin{cases} \alpha_{1t} \left(b_t / E_t \right)^{\alpha_{2t}} & \text{for } \bar{e} \leqslant b_t \\ \alpha_{1t} \left(\bar{e} / E_t \right)^{\alpha_{2t}} & \text{for } \bar{e} > b_t, \end{cases} \tag{12}$$

with $\alpha_1 \in \mathcal{R}_+$, $\alpha_2 \in \mathcal{R}$ and $b \in \mathcal{R}_+$. The first line of (12) formalizes the idea that the replacement rate for agents with average lifetime earnings below some threshold b_t is constant and equals to $\alpha_1 \left(b_t / E_t \right)^{\alpha_2}$ (otherwise agents with sufficiently low earnings would be eligible for infinite replacement rates in the model). The second line represents the replacement rate for those agents whose average lifetime earnings exceed b_t.

Given the replacement rate schedule $R_t(\bar{e}; \boldsymbol{\Psi}_t)$, agent's pension reads:

$$B_t(\bar{e}; \boldsymbol{\Psi}_t) = R_t(\bar{e}; \boldsymbol{\Psi}_t) \times \bar{e} / (1 + g)^{j - J^R}. \tag{13}$$

In the model, $\bar{e}$ is adjusted for productivity growth, while agent's pension stays constant during retirement (consistent with the SS policy in the U.S.); this necessitates the adjustment of $\bar{e}$ by $(1 + g)$ in the model.[8]

Figures 3-2 plot the replacement rate as a function of average lifetime earnings for different values of α_1 and α_2 (with $b = 0.5$). An increase in α_1, everything else equal, implies

[8]SS Administration adjusts household's pension benefit during retirement based on increases in the cost of living, as measured by the Consumer Price Index. This adjustment is referred to as Cost-Of-Living-Adjustment *COLA*). Since my model abstracts away from inflation, agent's pension benefit remains constant in the model.

an increase in replacement rates across all retirees (as the replacement rate curve shifts upwards). The variable α_2 determines progressiveness of the pension system, i.e. the degree to which pension benefits are proportional to average lifetime earnings (i.e. the curvature of the replacement rate schedule). The replacement rate is constant for average lifetime earnings below b. Above b, the replacement rate strictly decreases (increases) in average lifetime earnings if $\alpha_2 < 1$ ($\alpha_2 > 1$). If $\alpha_2 = 1$, every retiree receives the same fraction of her average lifetime earnings as a pension benefit.

The threshold b_t is adjusted by A_t at each point in time; the same is true for $\bar{e}$ and E_t. Therefore, the replacement rate formulation above is consistent with the intention of the SS Amendments to "stabilize future replacement rates": As long as earnings of successive cohorts of workers grow at the same rate as the average earnings among workers, all cohorts should be subject to the same replacement rates. The normalization of the average lifetime earnings by E_t is necessary for the analysis of this paper and will be justified below. The chosen functional form turns out to be flexible enough to capture accurately the replacement rate schedule implied by the pension benefit formula in the U.S. (figure 1).

The specified replacement rate schedule is a function of α_{1t}, α_{2t} and b_t. Since throughout the paper I treat b_t as an exogenous variable, I drop it from the SS policy vector $\mathbf{\Psi}_t$, which then leads to[9]:

$$\mathbf{\Psi}_t = (\alpha_{1t}, \alpha_{2t}).$$

4.8 Government's problem

I use a social welfare approach to endogenize the pension benefit policy, $\mathbf{\Psi}$: the government maximizes the weighted sum of expected discounted lifetime utilities of all generations, who are alive in the period, when the change to SS is made. Given $\mathbf{\Psi}_t$ and F_t, the government solves in period t:

$$\mathbf{\Psi}^\star = \arg\max_{\mathbf{\Psi}'} \sum_{j=1}^{J} \int_{A\times\bar{E}\times\mathcal{Y}} \omega_t(\mathbf{x}) V(\mathbf{x}; \mathbf{\Psi}_t, \mathbf{\Psi}', F_t) dF_t, \tag{14}$$

subject to the balanced SS budget constraint in eq. (8). In the expression above, $V(\mathbf{x}; \mathbf{\Psi}_t, \mathbf{\Psi}', F_t)$ denotes a value function of agents, who are alive at time t. These agents face the policy $\mathbf{\Psi}_t$ at time t and a constant policy $\mathbf{\Psi}'$ in the future. In the equation above, $\omega_t(\cdot)$ is a Pareto

[9] I also treat the maximum taxable earnings threshold, cap, as an exogenous variable to overcome a significant computational burden associated with letting b_t and cap_t be additional choice variables in government's maximization problem.

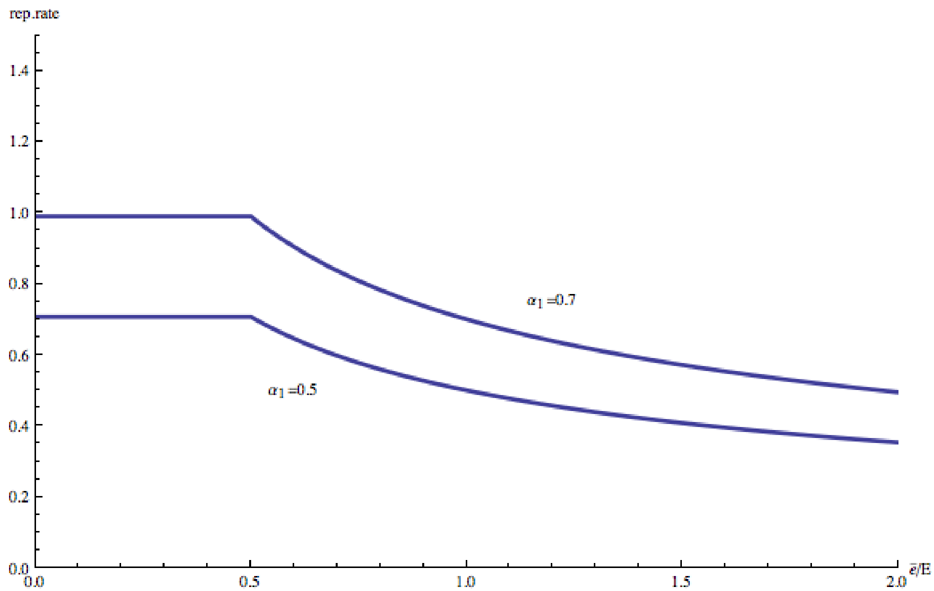

Figure 2: Replacement rate $\mathsf{R}_t(\bar{e}; \boldsymbol{\Psi}_t)$ as a function of α_1

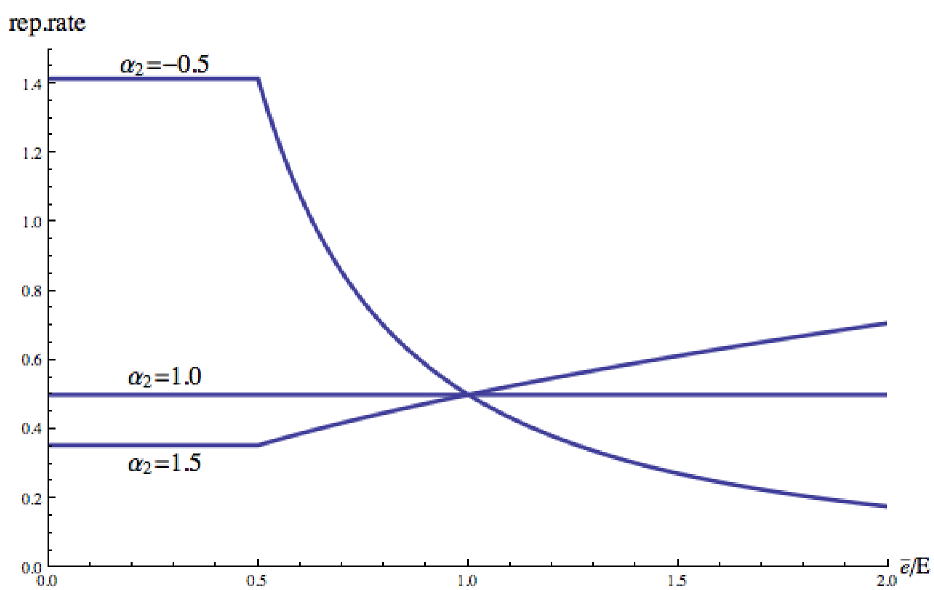

Figure 3: Replacement rate $\mathsf{R}_t(\bar{e}; \boldsymbol{\Psi}_t)$ as a function of α_2

weight function (to be specified in section 5.2). The government chooses the policy under the rational expectation about the effects of this policy on future equilibrium outcomes and welfare of each agent during her entire lifetime.

A few comments are in order. At time t, the government maximizes the welfare of all generations *alive at time* t. This assumption reflects the fact that in the real-world the governments seek reelection and propose policies to gain support of current voters. The government chooses (α_1, α_2) once-and-for-all. Such a specification (as opposed to the one, in which the government sets a *constant* tax rate, while pension benefits adjust in each period to balance the budget) is consistent with the SS Amendments of 1977, which fixed the replacement rates, not the tax rate. Finally, a change in the pension benefit schedule at time t alters the pension entitlements of those, who have entered retirement prior to period t.

Definition 3. A recursive competitive equilibrium with an **endogenous** SS policy is

- a set of functions $(V, c, l, a', w, r, T, F)$ and policies $(\mathbf{\Psi}_t, \mathbf{\Psi}')$, which satisfy definition 1;
- a Pareto-weight function ω_t, such that $\mathbf{\Psi}' = \mathbf{\Psi}^\star$ given by (14).

Definition 4. A **steady state** recursive competitive equilibrium with an **endogenous** SS policy adds to definition 1 the condition that the Pareto-weight function ω_t is such that $\mathbf{\Psi}' = \mathbf{\Psi}^\star = \mathbf{\Psi}$.

5 Calibration of the benchmark model economy in 1977

The parameters of the model can be grouped into four different sets: (i) demographics $\{J, J^R, n, \psi_j\}$; (ii) preferences and technology $\{\sigma, \gamma, \beta, \theta, \delta, g\}$; (iii) productivity parameters $\{\zeta_{j,z}, \sigma_\nu^2, \sigma_\gamma^2, \rho, \lambda_z\}$; (iv) government parameters, which refer to the SS policy, $\{\tau, \alpha_1, \alpha_2, cap, b\}$, the income tax function, τ_I, and the Pareto weight function, $\omega(\cdot)$.

5.1 Calibration with an exogenous Social Security policy

I parameterize and calibrate the model to match the key target moments from the U.S. data on the evolution of earnings and income inequality, while holding the SS variables exogenously given. I assume that in $t = 1977$ the U.S. economy is in a steady state recursive competitive equilibrium (definition 1) with a stationary distribution of agents across states, F_{1977}, and a given SS policy, $\mathbf{\Psi}_{1977}$. One model period equals to one year. All dollar amounts in this

Parameter	Description	Value	Source
- Demographics:			
J	max. life span	65	real life age 85
J^R	retirement age	45	SS Administration
n	birth rate	0.65 %	calibrated
ψ_j	cond. surv. prob.	vector	Bell et al. (1992)
- Preferences and technology:			
σ	degree of risk aversion	2.0	Conesa & Krueger (2009)
θ	capital share	0.36	Cooley & Prescott (1995)
δ	depreciation	6 %	Cooley & Prescott (1995)
g	technology growth	1.4 %	Fuster et al. (2007)
- Income tax and transfer systems:			
m_0	average tax rate	0.48	Gouveia & Strauss (1994)
m_1	progressivity	0.21	Gouveia & Strauss (1994)
- Social Security:			
cap	earnings cap	\$61,215	SS Administration
b	bend point	\$8,014	SS Administration
α_1	level of rep. rate	0.45	estimated
α_2	curvature of rep. rates	-0.43	estimated
- Labor productivity:			
λ_H	share college degree	22 %	CPS
ζ_{zj}	age-efficiency profiles	vectors	CPS
ρ	AR(1) coeffecient	0.97	Heathcote et al. (2010)
σ_ν^2	var. temp. shock	0.05	Heathcote et al. (2010)

Table 1: Parameters of the benchmark model obtained outside the model

section are in terms of 2011 U.S. dollars. An agent in the model corresponds to a household in the data.

Table 1 shows all the parameter values obtained outside the model; table 2 shows the parameters calibrated using the model. In section 5.2, I specify and calibrate the Pareto weight function, so that the SS policy arises endogenously.

5.1.1 Demographics

The maximum possible age, J, is set to 65 periods, which corresponds to a real life age of 84; therefore, agents enter the model at a real life age of 20. The retirement age, J^R, is 45 (real life age 64). I take the series of conditional survival probabilities for males for 1970 from Bell et al. (1992). The birth rate, n, is set such that, given the conditional survival probabilities,

Parameter	Description	Value	Target
$\tilde{\beta}$	subjective discount factor	1.01	$K/Y = 3.3$
γ	weight on consumption	0.49	$\bar{l} = 0.33$
σ_{γ}^{2}	var. persistent shock	0.01	P80/P50 (pre-tax earnings)=1.64
m_2	income tax scaling factor	0.21	T/Y=15%

Table 2: Calibrated parameters of the model

dependency ratio (i.e. ratio of retired to the working age population) equals 0.185 in 1977.[10] A high ability type in the model corresponds to a head of the household with at least 16 years of schooling in the CPS; a low ability type is a household with fewer years of schooling.[11]

5.1.2 Preferences and technology

The degree of risk aversion, σ, is fixed at 2.0. I calibrate the weight on consumption, γ, so that workers spend on average $\bar{l} = 1/3$ of their discretionary time to market work. The implied elasticity of inter-temporal substitution of consumption, $[1 - \gamma(1 - \sigma)]^{-1}$, equals 0.67. The implied Frisch elasticity of working hours of the average household (a household whose working hours are the average of those of all working-age households) is given by $(1-\bar{l})[1-\gamma(1-\sigma)]/\bar{l}\sigma$ and equals 1.5.[12] The growth-adjusted discount factor, β, is calibrated to match the capital to output ratio of around 3.3; this implies a subjective discount factor of $\tilde{\beta} = \beta/(1+g)^{\gamma(1-\sigma)} = 1.01$.

The capital share, θ, is chosen to match the labor share of 64 percent, while the depreciation rate of capital, δ, is calibrated to match the investment to capital ratio of ca. 0.06. All these values are consistent with Cooley & Prescott (1995).

5.1.3 Labor productivity

A household's labor productivity depends on three components: a deterministic ability dependent age-efficiency profile, ζ, a persistent shock, η, and a transitory shock, ν. I construct the age-efficiency profiles from the CPS following the procedure by Hansen (1993). More

[10] See "The 2014 Annual Report Of The Board Of Trustees Of The Federal Old-Age And Survivors Insurance And Federal Disability Insurance Trust Fund" on the homepage of the SS Administration.

[11] For all moments from the CPS dataset, I drop a household from the sample if no household member is of working age (20 and 64). I keep households with a male head only, where the head is the oldest age member of working age and working at least 260 hours in a year.

[12] For a discussion of estimates of the Frisch elasticity of labor supply, see Reichling, F. & Whalen, C. (2012). 'Review of Estimates of the Frisch Elasticity of Labor Supply', *Congressional Budget Office*, Working Paper 2012-13.

details are provided in appendix A.1. The share of high ability agents, λ_H, corresponds to the share of male college graduate heads in CPS in 1977.

I use the estimates of the autoregressive coefficient, ρ, and the variance of the temporary shock, σ^2_ν, from Heathcote et al. (2010).[13] With $\sigma^2_\nu = 0.0472$, I build an i.i.d. two-state Markov chain with equal probabilities. I calibrate the variance of the persistent shock, σ^2_γ, to match the $P80/P50$ ratio of the distribution of pre-government earnings in 1977 in the CPS equal to 1.64. Given the estimate $\sigma^2_\gamma = 0.0038$, I construct an age-dependent Markov chain for the autoregressive process using six equally-spaced nodes at each age: with $\eta_1 = 0$, the conditional variance of η_j increases with the age. More details can be found in appendix A.1.

5.1.4 Government parameters

Social Security

In order to estimate parameters (α_1, α_2) of the replacement rate schedule in (12), I use the statutory pension benefit formula by the SS Administration to generate a sample of AIME and PIA. I parametrize the formula as of 1977 with the bend points and the earnings cap converted to 2011 dollars.[14] This is the same formula, as the one I used to plot figure 1. Then I conduct two data transformations on the obtained sample. First, I convert the average indexed *monthly* earnings and the *monthly* pension benefits into their *annual* counterparts (multiplying each by 12), since in the model one period corresponds to one year. Second, I normalize both series by the average household earnings of $32,156 in 1977 (converted to 2011 dollars) from the CPS.[15] The reason for this normalization is important for further analysis and will be discussed in section 5.2. Finally, I apply a non-linear least squares estimator to estimate parameters (α_1, α_2) on the subsample of average lifetime earnings above the lowest bend point of $6,694 (annualized) and below the earnings cap of $61,256 (in 2011 dollars).

[13] The authors compute annual estimates of these parameters using PSID for 1967-2000. The authors restrict attention to married households, in which the husband is between 25 and 59 years old and works at least 260 hours per year. These sample selection criteria are different from the ones I apply in CPS. In particular, my sample includes not only married but also single households, as long as the head is a male. Also, I include the households, whose head is between 20 and 64 years old. This discrepancy in sample selection criteria can potentially be problematic. However, Guvenen (2009), who uses PSID for 1968-1993 applying the same selection criteria as me, obtains annual estimates, which are very similar to the ones by Heathcote et al. (2010). I prefer to use the estimates by Heathcote et al. (2010) because of a larger time span.

[14] See Table 2.A11 of the *2014 Annual Report Of The Board Of Trustees Of The Federal Old-Age And Survivors Insurance And Federal Disability Insurance Trust Funds.*

[15] See footnote 13 for the sample selection criteria.

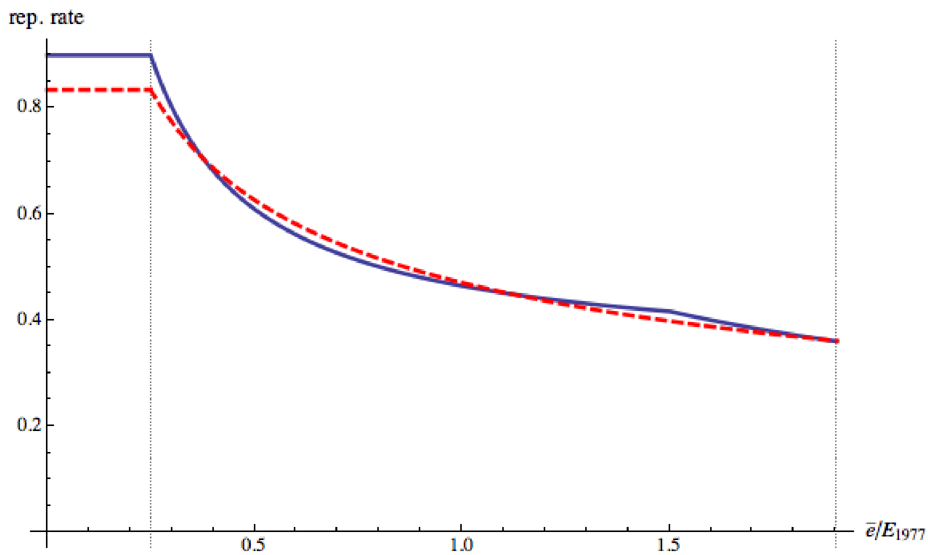

Figure 4: Estimated statutory replacement rate schedule for workers entering retirement in 1977

This procedure results in the estimates $\hat{\alpha}_1, \hat{\alpha}_2$ shown in table 1; the estimated replacement rate schedule is depicted in figure 4 (with the first vertical dashed line located at the first bend point and the second line located at the earnings cap).

The annual maximum taxable earnings threshold, cap, was \$61,215, while the lowest bend point (in annual terms) equaled \$8,014 in 1977 (both dollar amounts are converted to 2011 dollars). These dollar values have to be converted to model units. To establish the relationship between model and data units, I use the average pre-government earnings of working-age households in 1977 in the CPS equal to \$32,156, which I assume to equal to the average pre-government earnings among working agents in the benchmark model economy.

Income tax and transfer system

The individual income tax function follows Gouveia & Strauss (1994). It is a commonly used specification in the empirical macroeconomic literature. This progressive taxation rule reads:

$$\tau_{I,t}(\iota_t) = m_{0,t}[\iota_t - (\iota_t^{-m_1} + m_2)^{-1/m_1}],$$

where $\tau_{I,t}(\iota)$ is the amount of taxes the agent has to pay if her taxable income equals ι at time t.

In the schedule above, the parameter $m_0 \in [0,1]$ is the marginal (and average) tax rate as income goes to infinity. The parameter $m_1 \in [-1, \infty)$ determines the curvature of the marginal tax function $\tau_I'(\iota)$. I set m_0 to 0.48 and m_1 to 0.82, which are the estimates obtained by Gouveia & Strauss (1994) for 1979. I calibrate m_2 to match the share of government transfers in GDP.[16]

5.1.5 Model fit

Table 3 evaluates the performance of the calibrated benchmark model economy by showing some of important moments that were not targeted in the calibration.[17] It can be seen that the level of the SS tax rate in the model is slightly lower than the one in the data. The discrepancy in the tax rates can be explained by the fact the SS Trust Fund generated slight surpluses with SS contributions exceeding benefit expenditures in the 1970s, while the government is assumed to run a balanced budget in the model. The model slightly underestimates the percentage of (male) workers above the maximum taxable earnings threshold because the model lacks some of the features (such as entrepreneurship) to account for the upper tail of the earnings distribution. Introducing these features would unnecessarily complicate the model. The model achieves a fairly good overall fit of pre-government earnings and pre-government income inequality, yet underestimates the incomes held by the bottom quintile. The model share of agents with non-positive net worth is below the one in the data because I rule out borrowing. Since my model lacks some of the real life features which account for wealth accumulation, such as bequests, it is not surprising to see that the mean-to-median ratio for the net worth is lower than the corresponding empirical ratio.

[16] I use the data from "Historical Effective Tax Rates, 1979 to 2005" (Table 5. "Total Income and Total Federal Tax Liabilities for All Households, by Household Income Category, 1979 to 2005") by the Congressional Budget Office, which are available at http://www.cbo.gov/sites/default/files/cbofiles/ftpdocs/98xx/doc9884/12-23-effectivetaxrates_letter.pdf. To find the aggregate amount of transfers, I sum up cash transfers (excluding pensions) and in-kind income. To find the aggregate output, I sum up pre-tax wages, proprietor's income, other business income, interest and dividends, and other income. I exclude capital gains because they are not part of the model.

[17] The SS tax rate and the percentage of male (self-employed and employed) workers above the maximum taxable earnings threshold in 1977 are taken from Table 2.A3 and Table 4.B4, respectively, of the SS Administration report (see footnote 14). The share of SS in GDP is taken from http://www.usgovernmentspending.com/social_security_spending_by_year. The moments of the income and earnings distribution are taken from the CPS for 1977 with the same sample selection criteria applied as before in this paper (see footnote 13). In the CPS, I restrict attention to those households of age 20-64, who work more than 260 hours annually; this corresponds to supplying more than 5% of a unitary time endowment in the model assuming the discretionary time per year is 5,096 hours. The moments of the net worth distribution are taken from the Survey of Consumer Finances for 1983. Since my model lacks some of the real life features (entrepreneurship), which are necessary for the model to account for the upper tail of the net worth distribution, I drop all households located in the top decile of the net worth distribution in the SCF data set.

Moment	Model	Data
Social Security:		
- tax rate τ, %	7.43	8.66
- pensions/GDP, %	4.01	4.23
- workers above cap, %	19.61	23.70
Pre-govt. earnings distribution:		
- Q1	1.32	4.97
- Q2	7.91	12.21
- Q3	19.85	17.37
- Q4	28.12	23.91
- Q5	42.80	41.54
Pre-govt. income distribution:		
- P80/P50	1.67	1.64
- Q1	1.92	5.47
- Q2	8.64	11.94
- Q3	19.38	16.72
- Q4	27.73	22.90
- Q5	42.33	42.96
Net worth distribution:		
- net worth$\leqslant 0$, %	5.69	9.05
- mean-to-median	1.22	1.64

Table 3: Untargeted moments in the benchmark model and in the data

5.2 Calibration with an endogenous Social Security policy

In this section, I specify the Pareto weight function, $\omega_t(x)$, from (15). Instead of allowing the weights to depend on a full state space x, I assume that the weight at time t is a function of agent's *age*, j, and her (normalized) *average lifetime earnings*, $\bar{e}/E_{1977}$, only. There are at least two reasons for this choice. First, as I will argue below, agent's age and her average lifetime earnings are the key variables that determine agent's preferences over the policy variables α_1 and α_2. In other words, agent's age and her average lifetime earnings turn out to be informative statistics about this agent's most preferred SS policy. This fact will allow me to uniquely pin down the weights, such that the SS policy in the data coincides with the one that emerges as a solution to the government's optimization problem in the model.

Second, the choice of age and lifetime earnings is reasonable from the political-economy point of view. To see this, note that the social welfare function in eq. (14) is equivalent to a micro-founded probabilistic voting environment introduced by Lindbeck & Weibull (1987). In this environment, there are two candidates, who are maximizing the probability of winning the election. Voters differ not only in terms of the most preferred policy but also in terms of their propensity to participate in political activities (vote, contribute money to elections, etc.). In equilibrium, both candidates propose the same policy, which maximizes the weighted sum of welfare of all voters. The weights in that environment correspond to the weights implied by the weighting function ω in my model.

Numerous empirical studies find that participation in almost any form of political activities in the U.S. differs across households' age and income (Bartels 2009, Benabou (2000), Rosenstone & Hansen (1993)). Therefore, it is reasonable to let the weighting function depend on age and average lifetime earnings (which are highly correlated with income but are easier to deal with computationally because this is a state variable).

I employ a parametric approach with the following specification of the Pareto weight function:

$$\omega_t(x) = \omega_t(\bar{e}, j) = (\bar{e}/E_t)^{\kappa_{1t}} j^{\kappa_{2t}}, \tag{15}$$

where $\kappa_{1t}, \kappa_{2t} \in \mathcal{R}$ are parameters to be calibrated. I will refer to parameter κ_1 as *earnings bias* and κ_2 – as *age bias* below.[18]

The chosen specification of the weighting function is surely ad-hoc but it allows me to feasibly identify the parameters (κ_1, κ_2) consistent with the SS Amendments of 1977. The values of α_1 and α_2 implied by the SS Amendments are the moments conditions for the two unknown parameters. It turns out that α_1 is an informative moment for the parameter κ_2, while α_2 is an informative moment for the parameter κ_1.

Intuitively, young agents, regardless of their lifetime earnings, prefer a smaller size of the SS system (smaller τ or, equivalently, smaller α_1) than older agents, since the prospect of lower taxes on earnings for the rest of their working career outweighs the cost of having contributed for a short period of time to the pension system without receiving as much benefits in retirement.

For a given level of α_1, retired agents with lower average lifetime earnings prefer a more redistributive pension system (lower α_2) than agents with higher average lifetime earnings. As can be seen from the normalized replacement rate function (13), the pension benefit of a retiree with average lifetime earnings at retirement above the economy-wide average earnings E_{1977} strictly increases in α_2, while the opposite is true for a retiree, whose average lifetime earnings at retirement turn out to be below the economy-wide average earnings. As a matter

[18] I also experimented with a non-parametric approach by directly estimating the Pareto weights ω_t. The key limitation of this approach is that identification requires the number of moment conditions (the estimates $\hat{\alpha}_1$ and $\hat{\alpha}_2$ in the data) to be at least as large as the number of unknown parameters (the Pareto weights). With an additional requirement that weights sum up to 1, only weights for three groups of agents can be estimated.

of fact, ignoring any general equilibrium effects, a former (latter) retiree would opt for an infinitely large (small) α_2. Of course, preferences of retired agents over the policy variable α_2 also depend on its impact on the interest rate and transfers through the general equilibrium.

Preferences of a working-age household over α_2 are similar with two important distinctions. First, workers have to form expectations over their average lifetime earnings at retirement. Second, they have to consider the additional impact of α_2 on after-tax wages through the general equilibrium. More specifically, an increase in α_2 may increase or decrease the equilibrium SS tax rate depending on the relative sizes of retirees with (normalized) average lifetime earnings above and below 1.0 as well as the magnitudes of their (normalized) average lifetime earnings. For example, an increase in α_2 lowers the SS tax rate if the fraction of retirees with average lifetime earnings above 1.0 is sufficiently low or/and their average lifetime earnings are sufficiently small (i.e. close to 1.0).

6 Exogenous changes in model parameters (1977-2017)

I assume that starting from 1978 some parameters of the economic environment and demographics change in a way consistent with the estimates from the data. The changes in these parameters are summarized in table 4; below I discuss them in detail.

Along the transition, all endogenous variables in the model (which, recall, are adjusted for the growth in technology) grow at a rate $(E_t/E_{t-1}) - 1$. This includes the variables of the SS system, cap_t and b_t. Thus, the unadjusted growth rate of these variables is $g + (E_t/E_{t-1}) - 1$. [19] All the remaining parameters in the model stay constant.

Labor productivity process

To account for the rise in cross-sectional earnings inequality, I change four model parameters. First, I account for the widening college premium by updating the age-efficiency profiles of college and high school graduates, ζ_j, from the CPS for 2005 following the same procedure as

[19] Since nominal pensions, once the agent retires, should stay constant along the transition, the pension benefit $B_t(\bar{e};\boldsymbol{\Psi}_t)$ in (13) must be transformed to:

$$B_t(\bar{e};\boldsymbol{\Psi}_t) = \begin{cases} R_t(\bar{e};\boldsymbol{\Psi}_t)\frac{\bar{e}}{(1+g)^{j-J^R} E_t/E_{t-(j-J^R)}} & \text{for } \bar{e} \leqslant b_t \\ R_t(\bar{e};\boldsymbol{\Psi}_t)\frac{\bar{e}}{(1+g)^{j-J^R} E_t/E_{t-(j-J^R)}} & \text{for } \bar{e} > b_t \end{cases}$$

where $E_{t-(j-J^R)}$ are the average earnings among workers at the time when an agent, who is of age j at time t, entered retirement.

Parameter	Description	Benchmark	Simulation	Source
1. Earnings inequality:				
a) Idiosyncratic risk:				
σ_ν^2	var. temp. shock	0.0472	0.0735	Heathcote et al. (2010)
σ_γ^2	var. pers. shock	0.0038	0.012	calibrated
b) College premium and participation:				
ζ_{zj}	age-efficiency profiles	vectors		CPS (Appendix A.1)
λ_H	share of high ability	22%	32%	CPS
2. Population aging:				
n	birth rate	0.65%	0%	calibrated
ψ_j	cond. surv. prob.	vectors		Bell et al. (1992)
3. Income tax progressivity:				
m_0	progressivity	0.48	0.28	Gouveia & Strauss (1994)

Table 4: Parameters changed in the experiment

for 1977. The old and the updated age-efficiency profiles are plotted in appendix A.1. Second, I account for an increasing share of college graduates by updating the share of newborn high ability agents, λ_H. The new age-efficiency profiles and the share of newborn high ability agents are both effective from 1978 on.

Finally, I update the variance of the temporary component of the idiosyncratic labor productivity process, σ_ν^2. Based on the time estimates by Heathcote et al. (2010), σ_ν^2 increased from 0.0472 in 1977 to 0.0735 in 2000. I assume that σ_ν^2 increases linearly during this period and stays at the level of 2000 afterwards until the economy settles down in the new steady state. Fourth, I calibrate the variance of the persistent shock, $\sigma_\gamma^2 = 0.012$, to match the $P80/P50$ ratio of the distribution of pre-tax earnings in the data in 2005 (CPS) which equals ca. 1.80 in 2005. I assume that σ_γ^2 increases linearly between 1977-2005 and then stays constant until the economy reaches a new steady state. In each year along the transition, I construct an age-dependent Markov chain for the autoregressive process using four equally-spaced nodes at each age. More details can be found in appendix A.1.

Demographics

Falling fertility and increased life expectancy have resulted in a higher old-age dependency ratio in the data. I feed into the model the conditional survival probabilities for males for 2060 from Bell et al. (1992) and adjust birth rate n, such that in the model the dependency ratio equals 0.399 in 2060.[20] Both the updated survival probabilities and the birth rate are

[20]See the report of the SS Administration mentioned in footnote 14.

effective from 1978 on.

Income tax and transfer systems

The progressiveness of the income tax schedule declined sharply during 1980s due to adoption of the Economic and Recovery Tax Act of 1981. For example, the top marginal income tax rate was cut from 70 to 50 percent. Gouveia & Strauss (1994) estimate that the parameter $\mathfrak{m}_0$ dropped from 0.48 in 1979 to 0.33 in 1981 and remained (on average) at the level of 0.28 during 1980s. Guner et al. (2014) extend the estimates of Gouveia & Strauss (1994) for 2000 and estimate $\mathfrak{m}_0$ to be 0.26. I account for these changes in progressiveness of income taxes by setting $\mathfrak{m}_0$ to 0.28 effective from 1978 on.[21] The earnings cap and the bend point are earnings indexed along the transition. I assume that all the remaining parameters stay at the level of the benchmark model economy.

Transitional dynamics

Figure 5 shows the dynamics of some aggregate variables and prices during the transition from the initial steady state in 1977 to a new steady state. The paths for the interest rate and the SS tax rate are given in percentage points, while the paths for other variables – in deviation from the benchmark model economy. The dashed vertical lines correspond to the year 2017. As one can see, in response to an increased share of college graduates and the upward shift in the age-efficiency profiles both for college and high school graduates, the total supply of effective labor rises by more than 25 percent by 2017. The surplus in SS contributions due to rising total effective labor supply and pre-tax wages turns out to be insufficient to cover pension entitlements, which grow in size due to falling fertility rates and increased life expectancy. To balance the government budget constraint, the SS tax rate has to rise to almost 13 percent by 2017. The sharp reduction in the progressivity of income taxes effective in 1978 accounts for the collapse in the share of government transfers in output by almost 50 percent on impact.

7 Results

[21] The values of parameters $\mathfrak{m}_1$ and $\mathfrak{m}_2$ stay at the level of the calibrated benchmark model economy.

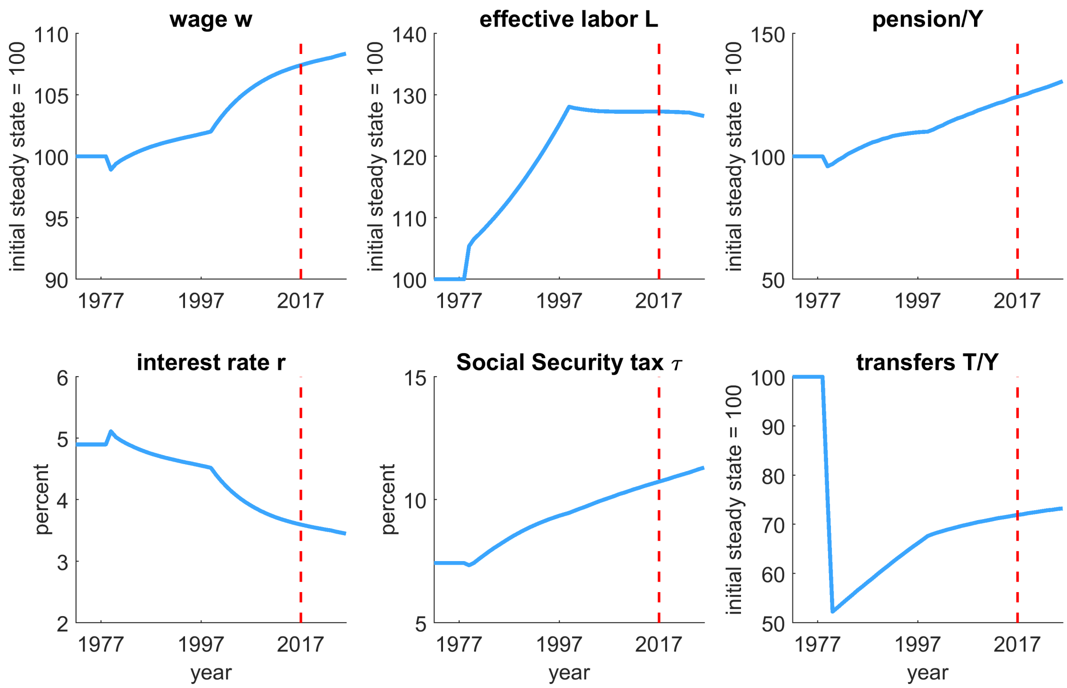

Figure 5: Transitional dynamics of some key model variables in the benchmark model economy

7.1 Pareto weights in 1977

Suppose that at the beginning of 1977 the economy finds itself with an (exogenously given) SS policy – the SS Amendments of 1977 – and the distribution of agents given by the solution to a steady state benchmark model economy with an exogenous SS policy (section 5).[22] I assume that both the government and the households expect all the economic and demographic parameters of the model to stay constant in the future at the level of the benchmark model economy. Following the notation introduced above, the model economy is in a *steady state* recursive competitive equilibrium according to definition 4 with $\boldsymbol{\Psi}' = \boldsymbol{\Psi}^{\star} = \boldsymbol{\Psi}_{1977}$ and F_{1977}. I label this scenario *Weights 1977 (unanticipated change)* in the tables and graphs below. In section 8.1, I will recompute all the results assuming that the government and the households perfectly foresaw the exact future time paths of all model parameters in 1977. The key results of the paper, however, remain unchanged.

The calibrated parameters κ_{1t} and κ_{2t} of the Pareto weight function ω_t for $t = 1977$ are

[22]Observe that my model does not rationalize the adoption of the Amendments in the first place. Instead, I take the Amendments as given but recover the weights, which make this policy optimal from the perspective of maximizing the weighted welfare of the generations alive at that time.

shown in table 5. In order to give an interpretation to the obtained parameters, I compute the implied Pareto weights for several groups of agents defined by age and average lifetime earnings (as a multiple of average earnings among workers in 1977). Based on age, I build three groups of agents: the young (real life age 20-24), the middle-aged (25-64) and the retirees (65-85). Additionally, I normalize the Pareto weights to sum up to one:

$$\sum_{j=1}^{J} \int_{A \times \bar{E} \times \mathcal{Y}} \omega_t(\mathbf{x}) = 1. \tag{16}$$

The normalized Pareto weights are presented in table 6.[23]

	Change	
	unanticipated	**anticipated**
κ_1 (earnings bias)	1.39	5.55
κ_2 (age bias)	-1.80	-3.92

Table 5: Calibrated parameters κ_{1t} and κ_{2t} of the weighting function ω_t for $t = 1977$

	Lifetime earnings (multiples of E_{1977})				
	$\bar{e} < 0.25$	$0.25 \leqslant \bar{e} < 0.75$	$0.75 \leqslant \bar{e} < 1$	$1 \leqslant \bar{e} < 1.25$	$\bar{e} \geqslant 1.25$
young workers (20-23)	13.38	19.20			
middle-aged workers (24-64)	5.71	41.18	8.16	4.31	1.06
retirees (65-84)	0.03	1.56	0.96	2.69	1.76

Table 6: Normalized Pareto weights ω_t (in percent) implied by the estimated parameters κ_{1t} and κ_{2t} for $t = 1977$

To gain some intuition for the obtained result, let's assume that there are only 3 broadly defined groups in the economy: the young, the poor-old and the rich-old. The group of the young is defined as before. The last two groups comprise the middle-aged workers and the retirees, who have average lifetime earnings below or above 1.0, respectively. The solution to the government's maximization problem can be better understood if we first find the most preferred SS policies of each of the three groups. This amounts to re-solving the government optimization problem in (14), assigning some positive weight to one of the three agent groups and a zero weight to the remaining groups. Figure 6 shows the transitional dynamics of some

[23]The mass of the young with average lifetime earnings above 3/4 of E_{1977} is zero in the benchmark model economy, which explains the missing values for the young in the tables.

key real variables (including the SS tax rate) under the most preferred policy of each group.[24] Figure 7 plots the most preferred statutory replacement rates of each group and compares them to the SS Amendments of 1977.

As can be seen from the figures, the young opt for a SS tax rate below the one in the data. The choice of the tax rate by the young is not surprising. These agents face long spans of working career with an increasing wage profile (due to the increasing age-efficiency profile and highly persistent productivity shocks), so SS taxes hurt them a lot. Thus, the young would rather benefit from rising after-tax wages and self-insure against the longevity and idiosyncratic risks instead of paying higher SS contributions throughout their working career. Their most preferred tax rate as a group is nevertheless strictly positive, since they do benefit from some minimal publicly provided insurance.

The groups of the poor-old and the rich-old prefer a SS tax rate that is too high as compared to the data: while the former group prefers a tax rate of 45 percent (in the long-run), the latter group opts for a tax rate of 16 percent (in the long-run). Recall that these two groups consist mostly of middle-aged workers and retirees. The middle-aged expect to receive labor income for a shorter period than the young and therefore they are more willing to sacrifice falling after-tax wages for larger future pensions; the retirees do not pay any contributions at all, which makes their most preferred tax rate as a group even higher. The rich-old additionally benefit from spiking interest rates. Observe that the most preferred tax rate of the rich-old as a group is lower than the one of the poor-old. This shouldn't be surprising, since earnings-rich workers, who constitute the majority of the rich-old, suffer huge welfare losses from falling after-tax wages.

Consider now the most preferred policy α_2 of each group. The young prefer a replacement rate schedule, which very closely resembles the SS Amendments of 1977 (figure 7). Uncertainty about future wages (and therefore uncertainty about the average lifetime earnings at retirement) hasn't fully realized at this age, so the young value the insurance nature of the intra-generational income redistribution through SS. Both groups of the old prefer pension systems with a too high degree of intra-cohort income redistribution: while the poor-old choose a system which redistributes incomes from earnings-rich to earnings-poor agents, the opposite is true for the rich-old. Since these agents face relatively little (middle-aged workers) to no (retirees) uncertainty regarding their lifetime earnings at retirement, their choice of α_2

[24] Along the transition, all endogenous variables grow at a rate $(E_t/E_{t-1}) - 1$. This includes the variables of the SS system, cap_t and b_t. Thus, the unadjusted growth rate of these variables is $g + (E_t/E_{t-1}) - 1$. Furthermore, see footnote 19.

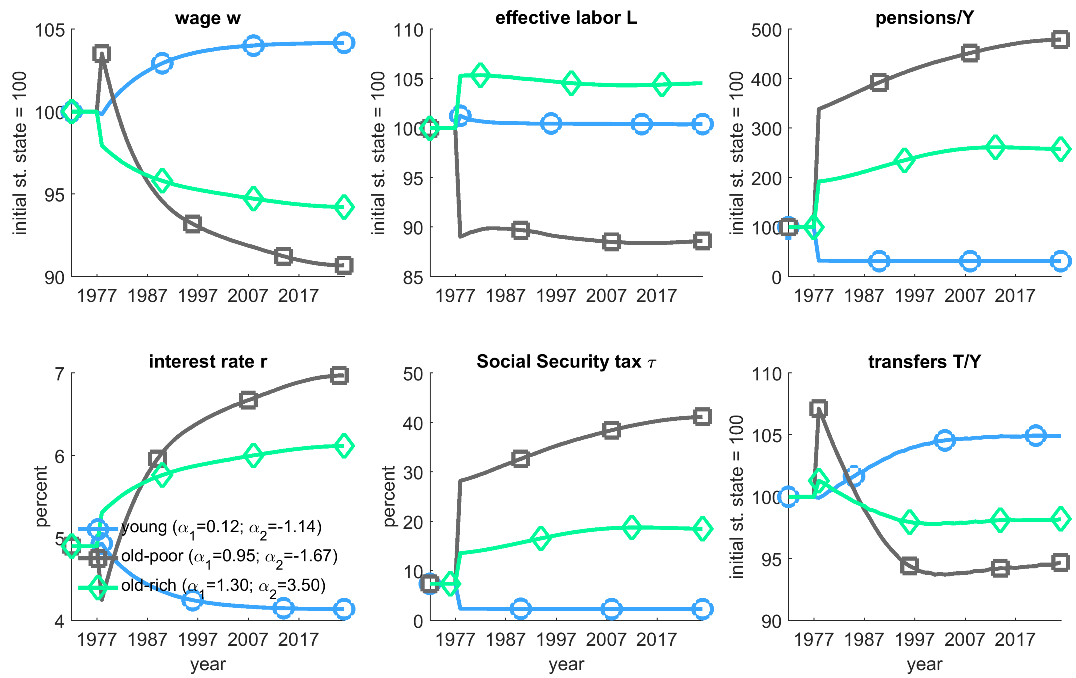

Figure 6: Transitional dynamics of some key model variables under the most preferred policies of each agent groups in 1977

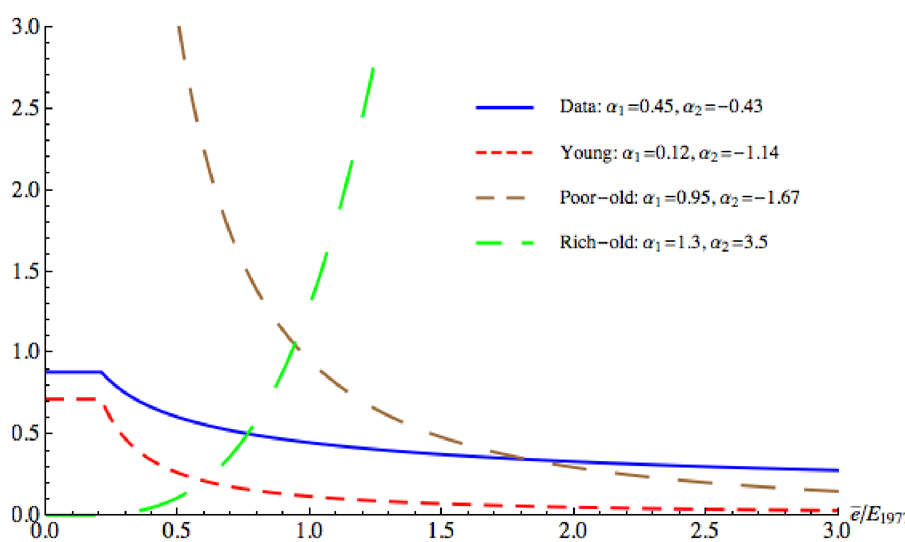

Figure 7: Most preferred statutory replacement rate schedules of each agent group in 1977

is dominated by the income redistribution rather than the insurance motive.

Working age households suffer enormously from such a high SS tax rate, which – due to labor supply distortions – also results into falling pre-tax wages. So it might seem surprising that the most preferred tax rate of the poor-old as a group is that high. These agents must have been unlucky during their working careers. They enter retirement without significant assets and rely mostly on government pensions and lump-sum transfers. Since their marginal utilities are extremely high, their most preferred policies dominate in the solution of the government maximization problem.

By visually inspecting the most preferred and the actual replacement rate schedules in figure 7, one could expect the government in the model to put a large weight on the young for the overall level of pension benefits (and therefore the level of SS taxes) to be consistent with the data. At the same time, for the model to match the curvature of the schedule of replacement rates, the government must have put a larger weight on the rich-old. And this is in fact what we saw in table 6.

7.2 Optimal Social Security policy in 2017

Given the changes in the U.S. economy, what is the optimal SS policy in 2017? To answer this question, I simulate the benchmark model starting from 1977 feeding into the model the exogenous paths of the key model parameters that I described in section 6. At time $t = 2017$, I solve for the optimal SS policy $\mathbf{\Psi}^{\star}$ using the notion of a recursive competitive equilibrium with an endogenous SS policy (definition 3) with $\mathbf{\Psi}_t = \mathbf{\Psi}_{1977}$, the cumulative population density function, F_{2017}, and the weights ω_{1977} (see column *Weights 1977 (unanticipated change)* in table 5).

The optimal SS policy vector $\mathbf{\Psi}_{2017}$ is shown in table 7. The policy is contrasted with the status quo policy under the SS Amendments of 1977. The implied optimal statutory replacement rates are presented in table 8. In this table, the average lifetime earnings are given as multiples of the average earnings among workers in 2017, E_{2017}. The path of SS taxes associated with the optimal policy is depicted in figure 8.

It can be seen that the optimal SS policy in 2017 is much more progressive than the currently prevailing policy. In fact, the replacement rate for earnings-poor individuals rises from 73 percent under the current system to more than 600 percent. Furthermore, the government finds it optimal to raise the replacement rate for households with medium lifetime earnings from 50 percent to 88 percent. At the same time, the replacement rates for earnings-rich individuals drop. The SS tax rate has to rise from 11 to more than 16 percent (in the long run). This is because the optimal pension system has become more progressive and there is a large mass of households subject to increased replacement rates.

	Status quo	**Weights 1977 (unanticipated)**	**Weights 1977 (anticipated)**
α_1	0.45	0.46	0.21
α_2	-0.43	-2.24	-2.87

Table 7: Optimal SS policy, $\mathbf{\Psi}_t^\star$, for $t = 2017$

Multiples of E_{2017}	**Status quo**	**Weights 1977 (unanticipated)**	**Weights 1977 (anticipated)**
0.25	0.73	6.04	5.72
0.75	0.50	0.88	0.48
1.00	0.45	0.46	0.21
1.25	0.40	0.28	0.11
2.00	0.33	0.10	0.03

Table 8: Statutory replacement rates, $R_t(\bar{e}; \mathbf{\Psi}_t)$, implied by the optimal SS policy $\mathbf{\Psi}_t^\star$ for $t = 2017$

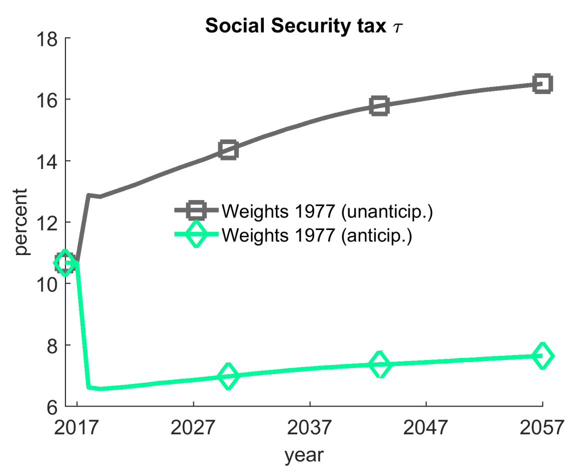

Figure 8: Transitional dynamics of SS tax rate, τ_t, under optimal SS policy $\mathbf{\Psi}_t^\star$

	Lifetime earnings (multiples of E_{2017})				
	$\bar{e} < 0.25$	$0.25 \leqslant \bar{e} < 0.75$	$0.75 \leqslant \bar{e} < 1$	$1 \leqslant \bar{e} < 1.25$	$\bar{e} \geqslant 1.25$
Weights of 1977 (unanticipated change):					
young workers (20-23)	0.56				
middle-aged workers (24-64)	11.58	12.15	1.22	-3.62	-7.34
retirees (65-84)	22.73	55.30	7.02	-4.79	-10.35
All	8.88				
Weights of 1977 (anticipated change):					
young workers (20-23)	1.95				
middle-aged workers (24-64)	8.75	6.64	-5.47	-10.69	-15.13
retirees (65-84)	12.89	25.16	-11.24	-17.25	-18.97
All	0.07				

Table 9: Average welfare change (CEV, percent) by age and average lifetime earnings relative to the status quo policy in 2017

7.3 Welfare effects of the optimal Social Security policy

Table 9 shows average welfare changes in consumption-equivalent variation (CEV) by age and average lifetime earnings of the SS reform relative to the status quo policy in 2017. To compute these welfare effects, I first quantify the welfare change for each individual of type $\boldsymbol{x}$ by asking: By how much (in percent) has this individual's consumption to be increased in all future periods and contingencies (keeping leisure constant) in the benchmark model economy, so that her expected future utility equals that under the policy reform in 2017. More precisely, the welfare change for an agent in state $\boldsymbol{x}$ is given by:

$$\left[\left(\frac{V(\boldsymbol{x};\boldsymbol{\Psi}^{\star}_{2017},F_{2017})}{V(\boldsymbol{x};\boldsymbol{\Psi}_{1977},F_{2017})}\right)^{1/\gamma(1-\sigma)}-1\right]\times 100.$$

Then I average these welfare effects within each age/average lifetime earnings group using the cumulative density function F_{2017}.

It can be seen that the reform raises the total welfare of all generations who are alive in 2017 by as much as 8.9 percent (row *All*). Differences in welfare gains and losses across age and lifetime earnings groups due to different pension system arrangements are substantial. The reform advantages first of all young workers, whose welfare increases by 0.6 percent. Young workers are better off, as the reform provides valuable insurance against idiosyncratic productivity risk. Retirees with low earnings records ($\bar{e} < 0.25$) receive the largest welfare gain because they immediately benefit from spiking replacement rates for the poorest without having to pay any additional funds into the pension system. Even though the government puts an insignificant weight on retirees, it is the ex-ante insurance for the young that creates huge ex-post (i.e. after realization of idiosyncratic productivities) benefits for the poor retirees because the system redistributes incomes from retirees with high to the retirees with low average lifetime earnings. Middle-aged workers with low-to-medium average lifetime earnings ($\bar{e} < 0.75$) are among winners too, since they expect larger pensions at retirement. Largest welfare losses are recorded for rich middle-aged workers and retirees with high earnings records: these agents suffer from having paid into the pension system during earlier years of their lives and suddenly facing an abrupt drop in the level of their future pensions.

Finally, I find political support for the reform assuming a *one-man-one-vote* rule. It turns out that the reform gains support of only 32 percent of the households alive in 2017.

8 Discussion

8.1 Anticipated change in the environment

So far, I have assumed that the government didn't expect the economic environment and demographics to change, when it adopted the SS Amendments in 1977. In this section, I relax this assumption and assume instead that the government and households perfectly foresaw the exact time paths of all model parameters described in section 6. The Pareto weight function is then calibrated in such a way that the government who chooses the SS policy once-and-for-all at time $t = 1977$ (to become effective starting from 1978) prefers not to deviate from the status quo policy, $\mathbf{\Psi}_{1977}$. As opposed to the case of an unanticipated change, the model is no longer in a steady state but rather in a recursive competitive equilibrium with an endogenous SS policy according to definition 3 with $F_t = F_{1977}$ and $\mathbf{\Psi}' = \mathbf{\Psi}^\star = \mathbf{\Psi}_{1977}$. Even though the SS policy remains constant (by construction), the aggregate variables and prices move in response to the changes in the economic environment and demographics. I label this scenario *Weights 1977 (anticipated change)* in tables and graphs.

The calibrated parameters κ_{1t} and κ_{2t} of the Pareto weighting function ω_t for $t = 1977$ are shown in table 5. The normalized Pareto weights are presented in table 10. Comparing the latter table to table 6, one can see that the distributions of Pareto weights are quantitatively very similar, with the difference that the weight on the young and the middle-aged with high earnings ($\bar{e} \geqslant 0.75E_{1977}$) is larger in the case of an anticipated change. This difference can

	Lifetime earnings (multiples of E_{1977})				
	$\bar{e} < 0.25$	$0.25 \leqslant \bar{e} < 0.75$	$0.75 \leqslant \bar{e} < 1$	$1 \leqslant \bar{e} < 1.25$	$\bar{e} \geqslant 1.25$
young workers (20-23)	5.43	42.10			
middle-aged workers (24-64)	0.45	30.92	8.61	6.33	1.97
retirees (65-84)	0.00	0.09	0.20	1.64	2.26

Table 10: Normalized Pareto weights ω_t (in percent) implied by the estimated parameters κ_{1t} and κ_{2t} for $t = 1977$ (anticipated change)

be explained by the discrepancy in the most preferred policies of the young across the two scenarios.[25] In the case of an anticipated change, the young foresee that they are going to be facing a significantly higher risk in idiosyncratic productivities throughout their working career. While the young prefer a very similar level of taxes across two scenarios, they opt for a more redistributive replacement rate schedule in the case of an anticipated change because they value more the ex-ante nature of insurance of the pension system. Thus, for the equilibrium replacement rate schedule to be consistent with the SS Amendments, a larger weight has to be assigned to earnings-rich individuals, while still keeping a significant weight on the young.

The optimal SS policy vector $\mathbf{\Psi}^{\star}_{2017}$ is shown in table 7. The implied path of SS taxes is depicted in figure 8. As before, the optimal SS policy drastically differs from the currently prevailing policy. Similar to the case of an unanticipated change, the replacement rate for earnings-poor individuals skyrockets, while the replacement rate for the earnings-rich decrease. As opposed to the case of an unanticipated change, the replacement rates for individuals with medium lifetime earnings fall. This almost universal reduction in statutory replacement rates allows the SS tax rate to go down from 11 to 7 percent in the long run.

The distribution of welfare gains and losses under this reform is shown in table 9. It can be seen that the total welfare gain is as low as 0.1 percent. The gains for the young are larger in the case of an anticipated change, since the reform reduces the SS tax burden. Middle-aged workers and retirees with low-to-medium earnings benefit from the reform; however, the gains are significantly lower than in the case of an unanticipated change. This is because the replacement rates are lower. For the same reason, the rich middle-aged workers and the rich retirees suffer large losses. Finally, the political support for the reform with a *one-man-one-vote* rule is 46 percent.

[25] For the sake of brevity, I do not plot agents' most preferred policies and the associated paths of aggregate variables in the case of an anticipated change.

8.2 Counterfactual analysis

In this section, I assess the quantitative importance of the changes in the economic environment and demographics for the optimal Social Security policy in 2017. For this purpose, I first simulate the model during 1977-2017 four times, keeping at each time the parameters related to idiosyncratic risk, college premium and college participation, population aging, and income tax progressivity, respectively, constant at the level of 1977 and adjusting all other model parameters as described in section 6. Figure 9 compares the transitional dynamics of some key endogenous model variables in each counterfactual with the benchmark case. For example, in a counterfactual *No change in aging* the birth rate and the conditional survival probabilities are fixed at the level of 1977, while the parameters related to earnings inequality and income tax progressivity adjust along the transition. The paths for the interest rate and the SS tax rate in the figure are given in percentage points, while the paths for all other variables – in deviation from the benchmark model economy.

It can be seen that the counterfactual *No change in idiosyncratic risk* has the most pronounced impact on the paths of all endogenous variables along the transition. Under this scenario, the effective labor rises only slightly (due to an increase in college premium and participation) inducing a steeper rise in pre-tax wages as compared to the benchmark model. In order to assess the quantitative importance of the changes in the idiosyncratic risk, I solve for the optimal SS policy $\mathbf{\Psi}^{\star}$ at time $\mathsf{t} = 2017$ with $\mathbf{\Psi}_{\mathsf{t}} = \mathbf{\Psi}_{1977}$, the cumulative population density function, F_{2017} (obtained from the counterfactual *No change in idiosyncratic risk*), and assuming the weights from the anticipated change scenario (table 5). The optimal SS policy vector $\mathbf{\Psi}^{\star}_{2017}$ is shown in table 11; the implied optimal statutory replacement rates are presented in table 12.

It can be seen that the optimal replacement rate schedule shifts down for all earnings groups and the pension benefit becomes almost linear in average lifetime earnings as compered to the optimal policy in the benchmark case. This suggests that the sharp rise in the replacement rate for the poorest earners in the benchmark scenario is largely driven by the changes in volatility of the idiosyncratic component along the transition.

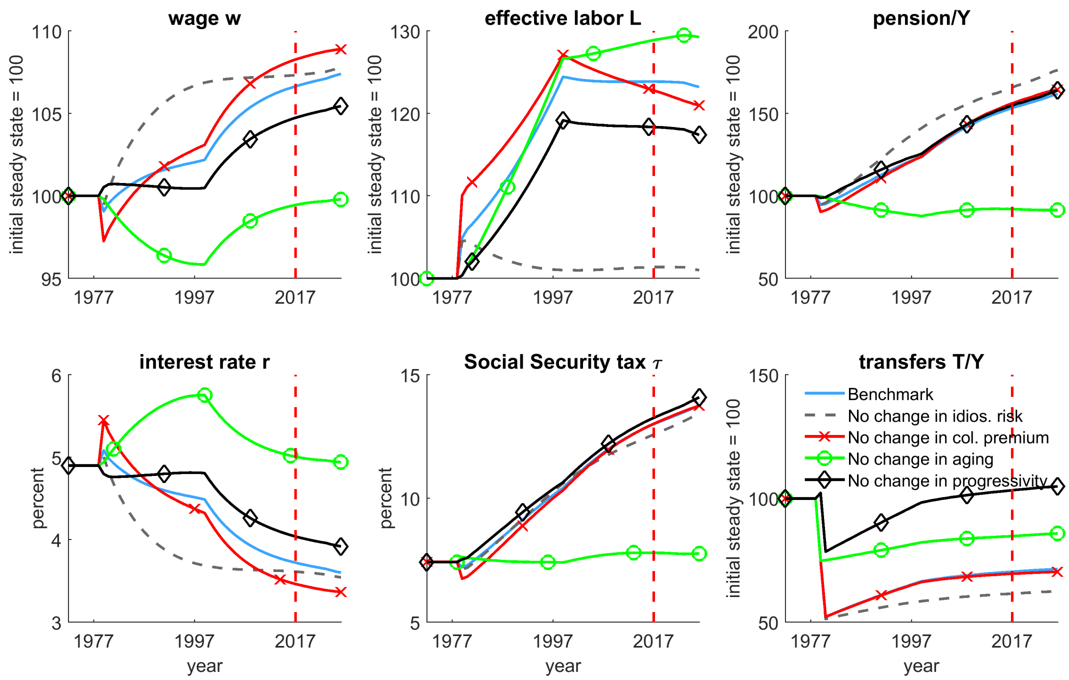

Figure 9: Transitional dynamics of some key model variables in the counterfactuals

	Status quo	Weights 1977 (anticipated) Benchmark	No change in idios. risk
α_1	0.45	0.21	0.21
α_2	-0.43	-2.87	-0.10

Table 11: Optimal SS policy, $\Psi^{\star}_{2017}$, for $t = 2017$ in a counterfactual

Multiples of E_{2017}	Status quo	Weights 1977 (anticipated) Benchmark	No change in idios. risk
0.25	0.73	5.72	0.24
0.75	0.50	0.48	0.22
1.00	0.45	0.21	0.21
1.25	0.40	0.11	0.21
2.00	0.33	0.03	0.20

Table 12: Statutory replacement rates, $R_t(\bar{e}; \Psi_t)$, implied by the optimal SS policy $\Psi^{\star}_t$ for $t = 2017$ in a counterfactual

8.3 Relaxing the balanced budget assumption

So far, I have assumed that the SS system runs a balanced budget in every period. In this section I analyze to what extent this assumption is crucial for the obtained results.

In figure 10, I plot the time path of deficits and surpluses of the OASI Trust Fund (as a percentage of GDP) during 1977-2016.[26] Based on whether the system was running a deficit or a surplus, the whole period can be divided into three subsamples. During 1977-1984, the OASI Trust fund reported an annual deficit in the SS system equal on average to 0.19 percent of GDP (the dashed line in the figure). During 1985-2009, there was an annual surplus equal to less than 0.5 percent. Since 2010, the system started to run an annual deficit equal to 0.47 percent.

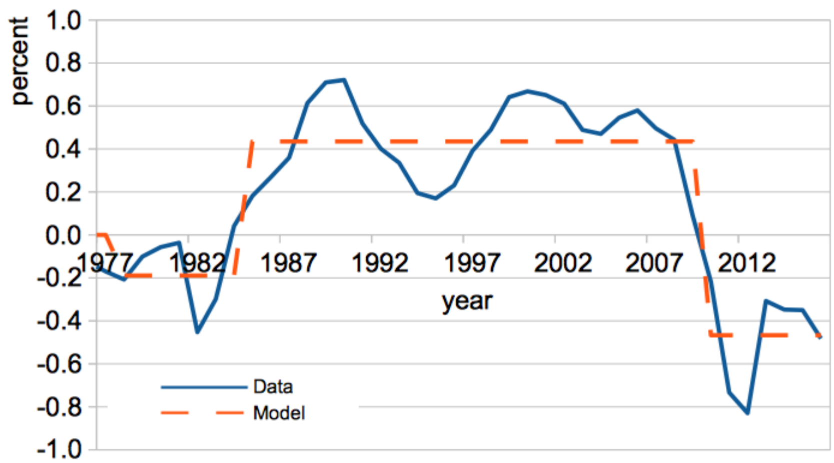

Figure 10: Time series of surpluses and deficits in the OASI Trust Fund in the data and in the model (as a percentage of GDP)

While I continue to assume that the government ran a balanced budget in 1977, which is consistent with the data, I am going to allow the government to run a surplus/deficit in subsequent years. Let $d_t < 0$ denote a deficit and $d_t > 0$ a surplus in the SS system as a fraction of GDP, Y_t, at time t. The government budget constraint in eq. (8) becomes:

$$\tau_t \sum_{j=1}^{J^R-1} \int_{A \times \bar{E} \times \mathcal{Y}} \min(cap_t, w_t \epsilon_{zjt}(\mathbf{y}) l(\mathbf{x}; \mathbf{\Psi}_t, F_t)) dF_t(\mathbf{x}) = \sum_{j=J^R}^{J} \int_{A \times \bar{E} \times \mathcal{Y}} B_t(\bar{e}; \mathbf{\Psi}_t) dF_t(\mathbf{x}) + d_t Y_t$$

[26]See https://www.ssa.gov/oact/STATS/table4a1.html. At each year, I subtract from *Net payroll tax contributions* the variable *Benefit payments* and divide the result by the real GDP obtained from https://fred.stlouisfed.org/series/GDPC1.

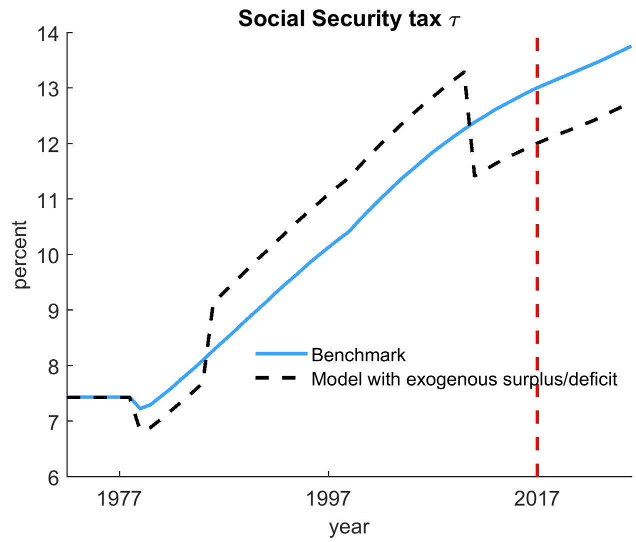

Figure 11: Transitional dynamics of the SS tax rate in the benchmark model and the model with an exogenous surplus/deficit

For the unanticipated change scenario, I assume that the government and households expected future surpluses/deficits to stay at the level of the benchmark model economy, i.e. zero. Thus, the calibrated Pareto weights for 1977 do not differ across the model specifications with and without debt. In the case of an anticipated change, agents perfectly foresaw the future time path of surpluses and deficits during 1977-2017. Together with the previously described changes in the economic environment and demographics, I feed into the model the empirical time path of surpluses and deficits. For simplicity, I let the time path in the model equal to the empirical averages for each of the subsample periods (i.e. the dashed line in figure 10). For the period after 2017, I assume the deficit to stay constant at the level of 0.47 percent. It turns out that relaxing the government budget constraint has only a negligible impact on the path of SS tax rates throughout the period (see figure 11). For this reason, the calibrated parameters of the Pareto weight function $(\kappa_{1t}, \kappa_{2t})$ for $t = 1977$ are quantitatively very close to the ones in the model specification without debt. For the same reason, the optimal SS policy in 2017 is quantitatively very similar to the one obtained under a balanced budget assumption.

8.4 Changes in Pareto weights over time

In this section, I explore one potential reason for why the SS system has not been adjusted since 1977. So far, I have assumed that the SS Amendments were optimal (under the calibrated weights) in 1977 but suboptimal in the period 1977-2017 given the changes in the economic environment and demographics. But what if the SS Amendments of 1977 have in fact been an optimal response to the changes in earnings inequality and population aging during the period? Through the lens of my model, this means that the underlying Pareto weights must have been adjusting during 1977-2017.

Below I calibrate the Pareto weights for the time period 1977-2017, such that the government optimally chooses not to deviate from the status quo policy $\boldsymbol{\Psi}_{1977}$ because this policy maximizes the welfare of all agents alive at time t according to eq. (14). In this equation, the distribution of agents, $F_t(\boldsymbol{x})$, as well as the value function, $V(\cdot, F_t)$, follow from the solution to the benchmark model transiting from the initial steady state in 1977 to a new steady state in response to changes in the economic environment and demographics (the transition paths of some key variables were described in section 6). I assume that at each $t > 1977$ both the government and the agents perfectly foresee the future paths of all model parameters.

The exercise of recovering the Pareto weights in a given year requires resolving the government optimization problem, which is computationally intense. For this reason, I calibrate the weights for the following years only: 1987, 1997, 2007, 2017. The estimated parameters $(\kappa_{1t}, \kappa_{2t})$ of the Pareto weight function are shown in the left-hand panel of table 13.[27]

	Distribution F_t					**Distribution** F_{1977}			
	1977	1987	1997	2007	2017	1987	1997	2007	2017
κ_{1t}	5.55	5.29	5.40	5.41	5.80	5.24	6.00	5.34	5.93
κ_{2t}	-3.92	-3.74	-3.40	-3.09	-3.32	-2.93	-3.00	-2.62	-2.65

Table 13: Calibrated parameters κ_{1t} and κ_{2t} of the weighting function ω_t for selected t

The calibrated parameters $(\kappa_{1t}, \kappa_{2t})$ imply a set of Pareto weights at each time t. To simplify discussion, I define several groups of agents by age and average lifetime earnings and report a weighted sum of weights for each group. The age groups are: young (real-life age 20-23), middle-aged (24-64) and retired (65-84). Based on average lifetime earnings, I define three groups: low ($\bar{e} < 0.25E_t$), medium ($0.25E_t \leqslant \bar{e} < 0.75E_t$), and high ($\bar{e} \geqslant 0.75E_t$). At each time t, I normalize the total sum of weights across all groups to 1 according to eq. (16).

The time series of normalized Pareto weights (in percent) is shown in the left-hand panel of figure 12. It turns out that the total weight on agents with low earnings (and all

[27]For $t = 1977$, I display the parameters $(\kappa_{1t}, \kappa_{2t})$ obtained in the case of an anticipated change. This allows for a better comparison with the parameters computed for later years, since for $t > 1977$ all the changes in the environment are anticipated by assumption.

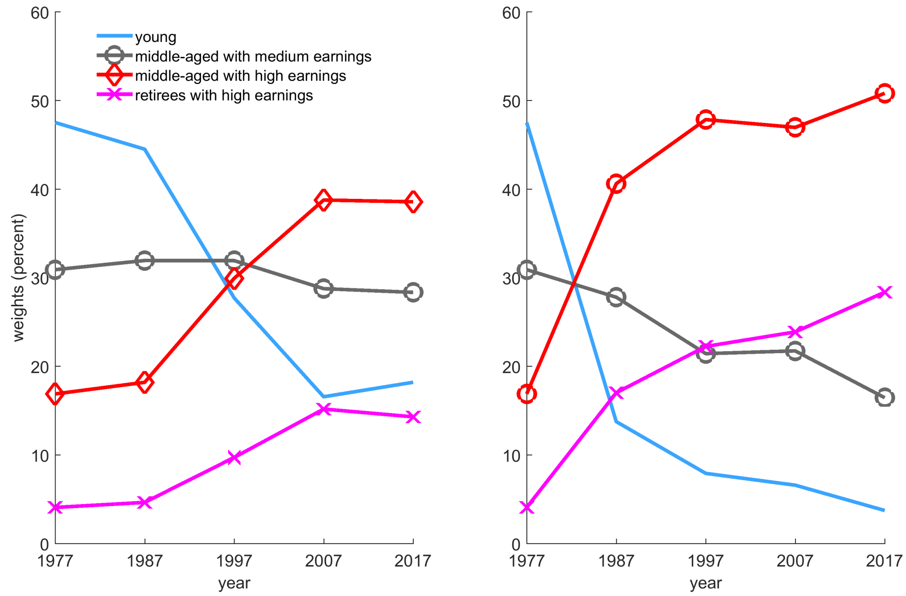

Figure 12: Time series of normalized Pareto weights ω_t (in percent) implied by the estimated parameters κ_{1t} and κ_{2t} for selected t given the actual cdf $F_t(\mathbf{x})$ (left-hand panel) and $F_{1977}(\mathbf{x})$ (right-hand panel)

age groups) as well as the retirees with medium earnings is below 1 percent, so I omit these groups in the figure.

Three observations are worth noting from the figure. First, the weight on the young has been rapidly declining from 48 percent in 1977 to 18 percent in 2017. Second, the drop in the weights for the young have been compensated by the rise in the weights for earnings-rich individuals. More specifically, the weight on rich middle-aged workers more than doubled from 17 to 39 percent, while the weight on rich retirees almost quadrupled from 4 to 14 percent. Finally, the weight on the middle-aged workers with medium earnings records has basically stayed constant at 30 percent.

From the government optimization problem in (14), it can be seen that with a constant optimal policy $\mathbf{\Psi}^{\star} = \mathbf{\Psi}_{1977}$, the Pareto weights ω_t at time t absorb any change in the value function $V(\cdot, F_t)$ or in the distribution of agents $F_t(\mathbf{x})$. In order to isolate the two effects on the weights, I recompute the weights keeping the distribution of agents constant at the level of the steady state benchmark model in 1977, i.e. $F_t = F_{1977}$. The calibrated parameters $(\kappa_{1t}, \kappa_{2t})$ for this exercise are presented in the right-hand panel of table 13, while the implied Pareto weights for the previously defined groups of agents are plotted in the right-hand panel of figure 12.

It can be seen that the identified rising trends in the weights for the rich and a falling trend for the young are amplified, once one controls for the changes in the distribution. Additionally, there is now a declining trend in the weight on the middle-aged workers with medium earnings. Overall, assuming that the SS policy has been optimal during 1977-2017, it no longer reflects the preferences of the young and the middle-aged (of all earnings groups) but rather the preferences of earnings-rich individuals.

9 Conclusions and outlook

This paper offers a rich set of opportunities for future research. First, my model abstracts away from risky investments in private pension plans whose volume has been rapidly growing since the 1980s. Therefore, my model mutes out the intergenerational risk-sharing aspect of Social Security which Krueger & Kubler (2006), Gottardi & Kubler (2011) show to be important. Accounting for risky investments in the portfolio choice problem of a household is likely to change the optimal degree of income redistribution and the value of risk sharing through Social Security.

Second, the U.S. government has alternative instruments to redistribute incomes across households apart from Social Security and progressive income taxation. The redistributive designs of the Medicare has been changing a lot in recent years. Analyzing Social Security, progressive income taxation and the Medicare in a joint framework will promote our understanding of the key trade-offs faced by the government.

Third, I assume that each agent in the model corresponds to a *household* in the data. In the U.S., the pension benefits vary a lot by the *family* structure, since SS pays benefits not only to retired workers but also to their spouses, dependents and survivors. For example, a spouse can claim up to 50 percent of spouse's SS benefits, even if the spouse has never worked. Pronounced changes have occurred in family patterns since 1977: divorce rates have increased, while marriage rates have fallen.

Finally, I focus on households composed of *male* heads only. It is well-known, though, that labor force participation rates among women have increased significantly since 1970s. Hence, fewer wives receive pensions as a function of their husbands' lifetime earnings but rather their own earnings histories. As suggested by Kopczuk et al. (2010), including females would reduce the overall inequality in average lifetime earnings and potentially dampen the incentives of the government to redistribute incomes.

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Appendix

A Calibration

A.1 Labor productivity and lifetime earnings

Age-efficiency profiles

Following Hansen (1993), I compute mean hourly earnings of high school and male college graduates of age 20-64 in CPS in 1977 and 2005. Then I normalize the mean hourly earnings of both types by the average mean hourly earnings across both types and all age groups in each year. Finally, I fit a quadratic polynomial curve to obtain a smoother approximation of the two age-efficiency profiles. Figure 13 plots the obtained profiles for 1977 and 2005.

Idiosyncratic shocks

For the temporary shock ν_t, I apply Tauchen (1986) to build an iid two-state Markov chain (ν_t^1, ν_t^2) with $\nu_t^2 > \nu_t^1$, each occurring with equal probabilities. The value calibrated for the initial benchmark model is $\sigma^2_{\nu,1977} = 0.05$, which leads to $\nu^1_{1977} = -0.21$ and $\nu^2_{1977} = 0.21$, and for the final steady state $\sigma^2_{\nu,new} = 0.07$, which gives: $\nu^1_{new} = -0.382$ and $\nu^2_{new} = 0.382$.

Modeling the persistent shock η_{jt} is a bit more involved. Recall that $\eta_{jt} = \rho\eta_{j-1,t-1} + \gamma_t$ with $\eta_{0t} = 0$. The conditional variance of η_{jt} increases with the age according to:

$$\sigma^2_{\eta_{jt}} = \sigma^2_{\gamma t} \sum_{h=0}^{j-1} \rho^{2h}.$$

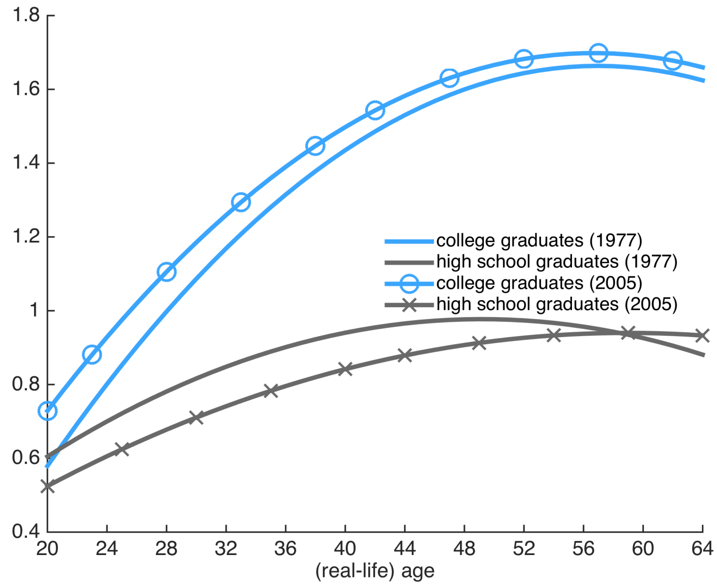

Figure 13: Deterministic age-efficiency profiles in the initial and final steady states

Given the estimates of the persistence ρ and the variance $\sigma^2_{\gamma t}$, I apply Tauchen (1986) to discretize η_{jt} with four nodes $(\eta^1_{jt}, \eta^2_{jt}, \eta^3_{jt}, \eta^4_{jt})$ with $\eta^1_{jt} < \eta^2_{jt} < \eta^3_{jt} < \eta^4_{jt}$ at each age. For the initial steady state, I use $\sigma^2_{\gamma,1977} = 0.01$, and for the final steady state $\sigma^2_{\gamma,new} = 0.02$.

Given the values of the persistent and the temporary shocks at each age, I construct a composed shock $y^{m,k}_{jt} = \eta^m_{jt} + \nu^k_t$ with $m = (1, 2, 3, 4)$ and $k = (1, 2)$. In figure 14 I plot the profiles of $\exp(y^{1,1}_{jt})$ (denoted *lowest*) and $\exp(y^{4,2}_{jt})$ (denoted *highest*) for the initial and the final steady states.

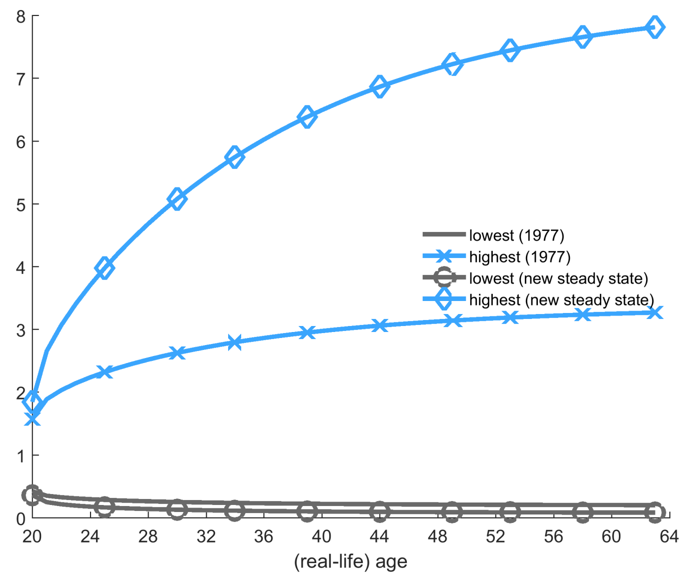

Figure 14: Discretized idiosyncratic shocks (highest and lowest) as a function of age in the initial and final steady states