Optimal Long-Run Inflation with Occasionally-Binding Financial Constraints

(Job Market Paper)

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Salem Abo-Zaid †
University of Maryland

Abstract
This paper studies the optimal inflation rate in a simple New Keynesian model with occasionally-binding collateral constraints that intermediate-good firms face on hiring labor. For empirically-relevant degrees of price rigidity, the optimal long-run annual inflation rate is in the range of half a percent to 2 percent, depending on whether it is TFP risk or markup risk or both that is the source of uncertainty in the economy. A positive inflation rate is a result of an endogenous markup shifter when the collateral constraint binds. A binding collateral constraint resembles a time-varying tax on labor, which the monetary authority can smooth by setting a positive inflation rate. More generally, the basic result is related to standard Ramsey theory in that optimal policy smooths distortions over time. Optimal policy reflects precautionary behavior by the monetary authority that meant to mitigate the effects of potential adverse shocks on the economy. Under optimal policy, the likelihood of hitting the zero-lower bound on the nominal interest rate is decreased markedly relative to zero-inflation policy, particularly if exogenous markup shocks are the source of uncertainty in the economy.

Key Words: Optimal long-run inflation rate; Precautionary motive; Financial frictions; Occasionally-binding collateral constraints.
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† Email address: abozaid@econ.umd.edu.
1 Introduction

Recent economic events have revived interest in the optimal long-run inflation rate. This paper studies the optimal long-run inflation rate in a simple calibrated New Keynesian (NK) framework with occasionally-binding financial constraints. For empirically-plausible sizes of exogenous shocks, optimal monetary policy entails a strictly positive inflation rate in the long-run. In particular, the optimal annual long-run inflation rate in the benchmark calibration of the model is about 0.5 percent when the economy is only subject to total factor productivity (TFP) shocks and slightly above 1 percent when the economy is hit by only markup shocks. When the economy is subject to both shocks simultaneously, the optimal long-run inflation rate is about 2 percent annually. Occasionally-binding financial frictions thus induce monetary policy to move away from the zero-inflation policy that is inherent in NK models due to price rigidities.

The baseline setup assumes three types of agents in the economy: households, entrepreneurs (or intermediate-good firms), and sticky-price firms that produce final goods. Financial frictions arise because hiring labor services by an entrepreneur is constrained by the level of her net worth. The collateral constraint is motivated by a type of the hold-up problem. Prior to supplying their labor services, households require the entrepreneur to show collateral that can be seized if needed. The accumulation of net worth is via purchases of shares that are claims on the profits of final-good firms. These shares pay out the profits of final-good firms as dividends to shareholders.

There are two main differences between this paper and typical papers that study optimal monetary policy within a New Keynesian framework featuring financial frictions. First, this paper assumes an occasionally-binding collateral constraint rather than always-binding collateral constraints as usually assumed in this literature. Second, this paper focuses on the optimal long-run inflation rate (i.e. the mean of the inflation rate in the “stochastic steady state” of the model), whereas the focus of most existing literature on monetary policy and financial frictions is on short-run dynamics and/or the inflation rate in the deterministic steady state.

The assumption of an occasionally-binding collateral constraint not only renders the environment more realistic, but it generates asymmetry in the behavior of the economy in response to favorable vs. adverse shocks. This asymmetry of outcomes in the private sector is the source of the precautionary motive for a positive long-run inflation rate. The computational approach that I use to deal with occasionally-binding constraints is a penalty-function algorithm within a second-order approximation. This approach has been extensively used recently (e.g. Kim, Kollmann and Kim, 2010; Den Haan and Ocaktan, 2009; De Wind, 2008 and Preston and Roca, 2007). A detailed description of this methodology can be found in Judd (1998).
When the collateral constraint binds, the shadow value of relaxing the collateral constraint is akin to an endogenous markup shifter that generates inflation. From the normative point of view, the results of this paper highlight the role of inflation in buffering the economy from adverse shocks, and they are driven by precautionary considerations on the part of monetary policy makers (and hence, the positive inflation rate can also be referred to as the “precautionary inflation rate”). A binding collateral constraint distorts the choice of labor by entrepreneurs, and thus it magnifies the wedge between the marginal rate of substitution between labor and consumption and the marginal product of labor (which exists due to the monopolistic power of final-good firms). This implies a deviation from the first-best level of output. The wedge (to which we refer as the “labor wedge”) resembles a labor-income tax, and it increases with the shadow value of relaxing the collateral constraint. The analyses show that, by setting a positive inflation rate, the monetary authority counteracts the effects of a binding collateral constraint, and it thus smoothes the “tax rate” on labor. Since the collateral constraint is more likely to bind during downturns, monetary policy makers aim for, at least, avoiding excessive increase in the “tax rate” during such episodes.

The ability of the monetary authority to smooth the “labor-income tax” (and more generally, the “labor wedge”) is limited due to the monopolistic power of final-good firms and the price rigidity. Put differently, the monetary authority does not have enough instruments to completely and simultaneously close the three distortions in the economy - the nominal distortion due to price rigidity, the monopolistic power of final-good firms, and the financial distortion. Policy makers choose to spread the distortions across margins. Spreading distortions across all margins is well-known in the literature (Dupor, 2002).

In light of the ongoing discussion about the Zero-Lower Bound (ZLB) on the nominal interest rate, the paper also presents the frequency of hitting the ZLB under both optimal policy and a policy that commits to zero-inflation at all dates and states. Under optimal policy, the ZLB is reached in about 1 and 2 percent of the time, depending on the type of the shock. In contrast, the nominal interest rate hits the ZLB in about 2 percent of the time (under a TFP shock only) and between 12 percent and 15 percent of the time if the economy is subject to an exogenous markup shock and both shocks, respectively.

There is a long literature debating the optimal inflation rate. Friedman (1969) argued that inflation should be negative in order to eliminate monetary distortions. Standard New Keynesian models featuring price stickiness, but no other distortions, suggest full price stability as optimal to
eliminate this type of distortion. These prescriptions, however, stand in sharp contrast to the fact that central banks around the world do in fact target positive inflation rates. In this regard, the current paper can also be seen as suggesting some mechanism that may reconcile the contrast between theoretical models and actual monetary policy making.

The discussion about the optimal inflation rate, particularly the optimal inflation rate in a low-inflation environment, is a very timely topic both in the literature and in practical monetary policy making. Until the 1990s, policy makers around the world were concerned with high inflation rates. But, with the dramatic falls in inflation rates world-wide, the concern has shifted towards studying the consequences of low inflation rates. The “lost decade” in Japan during the 1990s and the recent global recession only fueled these concerns. Recently, it has been proposed that nominal interest rates have been too low which tied the hands of policy makers when the economy slowed down. As suggested by Blanchard, Dell’Arificia and Mauro (2010), “the crisis has shown that interest rates can actually hit the zero level, and when this happens it is a severe constraint on monetary policy that ties your hands during times of trouble”. Hence, they suggest that setting a higher positive inflation rate (around 4 percent) would have allowed for more room for policy makers to conduct monetary policy using “conventional” means.

Recent work has suggested other precautionary factors that justify a positive inflation rate. Related to the current study, Antinolfi, Azariadis and Bullard (2010) point to the role of positive inflation in deepening asset markets and loosening debt contracts. Kim and Ruge-Murcia (2009) show, assuming a neoclassical labor environment, that the optimal long-run inflation rate is positive (around 0.4 percent annually) if nominal wages are downwardly rigid. Abo-Zaied (2010) reports a significantly higher optimal long-run inflation rate (around 2 percent annually) in a labor search and matching framework in the presence of downward nominal wage rigidity. Fagan and Messina (2009) suggest that the optimal inflation rate for the U.S. ranges between 2 percent and 5 percent when nominal wages are downwardly rigid. This paper contributes to the growing literature on precautionary motives for setting positive long-run inflation rates.

The remainder of the paper proceeds as follows. Section 2 outlines the model economy with the collateral constraint and defines the private-sector equilibrium and the optimal monetary policy problem. Section 3 discusses the labor wedge and the role of inflation in smoothing this wedge. Section 4 describes the calibration and the solution methodology of the model. Section 5

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1 See Schmitt-Grohe and Uribe (2009) and Shimasawa and Sadahiro (2009) for more discussions about the optimal long-run inflation rate. Billi and Kahn (2008) discuss various mechanisms that may justify a positive inflation rate.
presents the optimal long-run inflation rate suggested by this paper. Impulse responses, the implications for the ZLB on the nominal interest rate and the frequency of hitting the collateral constraint limit are also presented in this section. Section 6 presents the results of robustness tests and section 7 concludes.

2 The Model Economy

The model is a variation of the standard New Keynesian model, with the basic structure by which financial frictions are modeled similar to the recent work of Carlstrom, Fuerst and Paustian (2010, CFP hereafter). The economy is populated by households, entrepreneurs that produce intermediate goods (in what follows, I refer to this sector as entrepreneurs and intermediate-good firms interchangeably), and final-good firms. Households consume differentiated final goods and supply labor on spot markets. Entrepreneurs hire labor services to produce homogenous intermediate goods. Entrepreneurs’ labor demand is constrained by the accumulated value of their net worth. This constraint is the source of the financial friction. Final-good firms are monopolistic competitors that purchase intermediate goods from entrepreneurs and costlessly produce final goods. The pricing of a final-good firm is subject to a direct resource cost, which is the source of price rigidity in this model.

2.1 Households

The representative household purchases the differentiated final goods and enjoy utility from a composite consumption index \(c_t\) and supplies labor \(l_t\) in each period \(t\). Households have access to two financial instruments. The first is a standard one-period bond that pays a riskless nominal gross interest rate of \(R_t\). These bonds are in zero net supply, and, as in CFP, they make explicit pricing the nominal interest rate. In period \(t\), households also purchase \(s_t\) shares of final-good firms at a nominal per-share price of \(Q_t\). Total shares pay nominal dividends of \(D_t\), and their market supply is normalized to unity.

Households maximize their expected discounted lifetime utility given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(l_t)],
\]

(1)

where \(\beta < 1\) is the standard subjective discount factor, \(E_0\) is the expectation operator, \(u(c_t)\) is the period utility function from consumption and \(v(l_t)\) is the period disutility function from
supplying labor. These functions satisfy the Inada conditions and the usual properties: 
\[ \frac{\partial u(\cdot)}{\partial c} > 0, \quad \frac{\partial^2 u(\cdot)}{\partial c^2} < 0, \quad \frac{\partial v(\cdot)}{\partial l} > 0 \quad \text{and} \quad \frac{\partial^2 v(\cdot)}{\partial l^2} > 0. \] 
As standard in NK models, consumption \( c_t \) is a Dixit-Stiglitz aggregator of final goods \( c_{jt} \) produced by monopolistically-competitive firms,

\[ c_t = \left( \int_0^1 c_{jt} \frac{\varepsilon_j}{\varepsilon_t - 1} \, dj \right)^{-\varepsilon_t}, \quad (2) \]

where \( \varepsilon_t > 1 \) measures the elasticity of substitution between two varieties of final goods. The elasticity of substitution is allowed to be time-varying in order to allow for markup shocks, or, put differently, cost-push shocks. Other things equal, an increase in \( \varepsilon_t \) leads to a fall in the desired markup (the optimal price over the marginal cost), and hence to less inflationary pressures in equilibrium. I allow for markup shocks both due to their familiarity in New Keynesian models and because they generate a tradeoff for the monetary authority between stabilizing inflation and stabilizing output. In some of the experiments in section 5, I consider constant elasticity of substitution, and the main results are not, qualitatively, sensitive to whether markups are stochastic or not.

Following standard derivations in Dixit-Stiglitz based NK models, the optimal allocation of expenditures on each variety is given by

\[ c_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon_t} c_t, \quad (3) \]

where \( P_t = \left( \int_0^1 P_{jt}^{1-\varepsilon_t} \, dj \right)^{1/1-\varepsilon_t} \) is the Dixit-Stiglitz price index that results from cost minimization.

Maximization is subject to the sequence of budget constraints of the form:

\[ P_t c_t + Q_t s_t + B_t = R_{t-1} B_{t-1} + P_t (1 + \tau) w_t l_t + s_{t-1} (Q_t + D_t) + P_t \Pi_t + P_T T_t, \quad (4) \]

with \( c_t \) denoting consumption of the final good, \( P_t \) is the nominal price of the final good, \( w_t \) is the real wage, \( \tau \) is a labor market subsidy that is introduced to ensure the efficiency of the deterministic steady state (i.e. to achieve the first-best level of output; see Appendix 1-E for details). Finally, \( T_t \) are real lump-sum transfers by the government, and \( \Pi_t \) are profits from the ownership of firms.

The households’ budget constraint may be expressed in real terms as follows:
\[ c_t + q_t s_t + b_t = \frac{R_t b_{t-1}}{\pi_t} + (1 + \tau)w_t l_t + s_{t-1}(q_t + d_t) + \Pi_t + T_t \]  
\[(5)\]

where \( q_t = \frac{Q_t}{P_t} \) denotes the real price of shares, \( b_t = \frac{B_t}{P_t} \) is real bond holdings at the end of period \( t \), and \( d_t = \frac{D_t}{P_t} \) stands for real dividends.

The optimal choices of consumption, bonds, labor supply and shares of final-good firms yield the following optimization conditions (see Appendix 1-A for derivations):

\[ \frac{v_{t,j}}{u_{c,t}} = (1 + \tau)w_t, \]  
\[(6)\]

\[ u_{c,t} = \beta R E_t \left( \frac{u_{c,t+1}}{\pi_{t+1}} \right), \]  
\[(7)\]

\[ u_{c,t} = \beta E_t \left[ u_{c,t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) \right], \]  
\[(8)\]

where \( u_{c,t} \) is the marginal utility of consumption in period \( t \), \( u_{t,j} \) is the marginal disutility of supplying labor in period \( t \), and \( \pi_t = \frac{P_t}{P_{t-1}} \) denotes the gross price inflation rate. Equation (6) is the standard labor-supply condition, and equation (7) is the standard consumption Euler equation. Equation (8) prices shares of final-good firms; it equates the period-\( t \) marginal utility of consumption to the expected utility of expanding future consumption through the gross one-period return on holding shares, given by \( \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) \).

### 2.2 Entrepreneurs/ Intermediate-Good Firms

There is a continuum of long-lived entrepreneurs, each of whom produces intermediate goods. An entrepreneur hires labor services on sport markets in order to produce a homogenous good using the linear production function,

\[ x_t = A_t l_t, \]  
\[(9)\]

with \( A_t \) denoting total factor productivity, which is identical across all entrepreneurs.

Prior to supplying labor to an intermediate-good firm, households require that a part \( \alpha \) of their wages be backed up by collateral. This is the source of the financial friction in the model,
about which more is discussed below. Given that a share of wage payments is collateralized, the intermediate-good firm then hires labor and starts production. Realized operating profits (revenues net of wage costs) and the beginning-of-the-period net worth can then be used to buy shares \((e_t)\) for the next period. Positive operating profits are possible if the collateral constraint binds (see Appendix 1-D for a proof).

The collateral constraint can be motivated by the hold-up problem, as in Kiyotaki and More (1997). Prior to supplying their labor services, households require some “guarantee” from the entrepreneur so that she does not force their wages down ex-post. In other words, the entrepreneur is required to back up the promised wage by some collateral that can be seized if needed. Introducing the financial friction follows CFP and allows me to obtain the main results in a simple way.

Formally, hiring labor is constrained by the-beginning-of-period net worth, as follows

\[
\alpha w_t l_t \leq \kappa e_{t-1} (q_t + d_t) \equiv \kappa n_t, \tag{10}
\]

where, \(e_{t-1}\) stands for the share-holdings by the entrepreneur at the beginning of period \(t\), and \(n_t\) is the real value of net worth. The maximum share of net worth that can be used as collateral is \(\kappa\) (which is equivalent to the loan-to-value ratio in models with borrowing constraints). The parameter \(\alpha\) measures the “significance” of the financial friction: the higher this parameter is, the more “significant” (or “severe”) the financial friction. Clearly, if \(\alpha = 0\) then the model collapses to a standard new Keynesian model with no financial frictions.

The most realistic setup, which is the main focus of this paper, is one in which the collateral constraint may only occasionally bind. For example, the constraint may not bind after a long series of positive shocks (Iacoviello, 2005). Assuming this constraint is always binding, as many New Keynesian models with financial frictions do, including CFP, imposes a restriction on the model’s dynamics. Also, even if the constraint always binds at the deterministic steady state and for small (positive) shocks, it does not necessarily bind for large shocks. Because large shocks are of course sometimes observed in reality, it is important to understand the model’s dynamics when constraints need not always bind.

To my knowledge, allowing the collateral constraint to only occasionally bind is an innovation compared to studies of monetary policy in the presence of financial frictions. Recent studies assume always-binding collateral constraints (e.g. Iacoviello, 2005; Monacelli, 2009 and Carlstrom, Fuerst and Paustian, 2010). Studying optimal monetary policy with occasionally-
binding financial constraints can be viewed as another contribution of the paper. The way I computationally handle the occasionally-binding constraint is discussed in section 4.

I assume that any remaining resources (or “profits”) will be remitted to households in a lump-sum fashion, and that in the process of accumulating shares, entrepreneurs are more impatient than households. For this reason, they discount the future using a discount factor of $\delta \Xi_{t+1}$, where

$$\Xi_{t+1} = \beta \frac{u_{t+1}}{u_t}$$

and $\delta < 1$. The parameter $\delta$ is needed to ensure that an entrepreneur will not accumulate enough assets so that the collateral constraint never binds. Finally, as will be discussed in subsection 2.6, the assumption that entrepreneurs remit their “profits” to households simplifies the objective function of the monetary policy maker; the goal is only maximizing the lifetime utility of households. An entrepreneur thus chooses labor demand and shares to maximize expected present discounted value of profit payouts to households,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \Xi_{t+1} \left[ p_t A_t l_t - w_t l_t + e_{t+1} (q_t + d_t) - e_t d_t \right],$$

subject to the sequence of collateral constraints (10). The variable $p_t$ denotes the relative price of the intermediate good in terms of the final good (and, in equilibrium, equals the marginal cost of final-good firms). The term in the square brackets is what I refer to as “profits,” and it corresponds, in equilibrium, to part of $\Pi_t$ in the budget constraint of households.

Denoting the Lagrange multiplier on (10) by $\mu_t$, the optimal choices of labor and shares by an entrepreneur are characterized by (see Appendix 1-B for details):

$$A_t p_t = w_t (1 + \alpha \mu_t),$$

$$1 = \delta \mathbb{E}_t \left[ \Xi_{t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) (1 + \kappa \mu_{t+1}) \right],$$

where, as in CFP, the variable $\alpha \mu_t$ can be interpreted as a “real interest rate” on a loan required for paying the wage bill of $l$ in advance. Equation (12) states that, at the optimum, the marginal product of labor is equated to the real wage adjusted by a “financial markup” (i.e. the effective real wage from the viewpoint of the firm in the beginning of the period). Hence, if $\alpha > 0$, then labor demand will be distorted by the existence of the collateral constraint if it binds. Ex ante, the

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2 In appendix 1-J, I show that this setup is isomorphic to a model in which part of wages is required to be paid in advance (“working capital”), the entrepreneur obtains intra-period loans to finance this part of wages, and borrowing is constrained by net worth.
cost of hiring a unit of labor is higher the tighter the collateral constraint. Moreover, if $\alpha = 0$, this condition reads $p_r = \frac{W_r}{A_r}$, as is standard in NK models. Finally, equation (13) is a typical asset-pricing condition, but expanded to account for the imposition of the collateral constraint.4

2.3 Final-Good Firms
Firms in this market are monopolistically competitive. A final-good firm $j$ purchases the homogenous intermediate goods from entrepreneurs at a relative price $p_i$ and transforms each unit of the intermediate good into a final good $y_{jt}$ using a one-to-one technology.5 Each firm chooses its own price ($P_{jt}$) to maximize profits subject to the downward-sloping demand for its product (see Appendix 1-G for more details). The pricing of a final-good firm is subject to a quadratic adjustment cost as in Rotemberg (1982), expressed in units of the final good:

$$\frac{\varphi}{2} \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right)^2 y_{jt},$$

where $\varphi$ is a parameter that governs the degree of rigidity. The adjustment cost here is expressed in terms of the final-good output. In a symmetric equilibrium, in which all firms set the same price, Rotemberg pricing leads to the following forward-looking Phillips curve:

$$1 - \varphi(\pi_t - 1)\pi_t + \beta \varphi \mu_t \left[ \frac{u_{ct+1}}{u_{ct}} \right] (\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t} = \epsilon_t (1 - mc_t),$$

where, $mc_t$ is the marginal cost of the final-good firm, which equals $p_j$. As usual, because of the assumptions of one-to-one technology and zero fixed cost, the real marginal cost equals the real

3 Condition (12) makes clear that profits are positive when the collateral constraint binds. In this case, the marginal product of labor exceeds the real wage. Put differently, at the optimum, the intermediate-good firm hires labor to equate the “ex ante” real wage (which includes $\alpha \mu_t$) to the marginal product of labor. However, since ex-post the firm pays only $w_t$, the firm ends up making positive operating profits.

4 Condition (13) is consistent with condition (8) because of the variations of the Lagrange multiplier on the collateral constraint and the additional discount factor ($\delta$). To fix ideas, consider the deterministic steady state versions of the two conditions. In this case, from (13) we get $1 = \delta (1 + \kappa \mu)$, which makes the two conditions consistent.

5 I assume two types of firms in the production sector since the “asset” in this model is shares of final-good firms. To avoid adding an asset (e.g. capital), and hence deviate from the linear-in-labor technology that is typically assumed in NK models, I assume two types of firms and introduce each friction in one sector.
average cost. In the case of fully flexible prices (φ = 0) or fully stable prices (τ_t = 1 for all t),
equation (15) collapses to the familiar condition, \( mc_t = \frac{\varepsilon_t - 1}{\varepsilon_t} \). Hence, in the absence of price
adjustment costs, the real marginal cost equals the inverse of the price markup.

By combining conditions (6) and (12) and using the fact that \( mc_t = p_t \), the Phillips curve
can be written as

\[
(\pi_t - 1)\pi_t = \frac{1 - \varepsilon_t}{\phi} + \beta E \left[ \left( \frac{u_{t+1}}{u_t} \right) (\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t} \right] + \left[ \frac{\varepsilon_t \nu_{1,t}}{\phi(1 + \tau)A_t u_{c,t}} \right] \mu_t \tag{16}
\]

which explicitly shows the relationship between inflation and the financial friction (as measured
by the multiplier \( \mu_t \)). This is a key equation since it directly links inflation and the (binding)
collateral constraint. The left hand side of condition (16) is increasing in \( \mu_t \), which implies that
an increase in \( \mu_t \) leads to an increase in inflation. In this regard, the Lagrange multiplier on the
collateral constraint acts as an endogenous markup shifter that generates inflation. Basically, the
binding collateral constraint in this setup is one way to justify the inclusion of ad-hoc cost-push
shocks. It is also worth nothing that, other things equal, the impact of \( \mu_t \) on inflation is decreasing
in the degree of price rigidity and increasing in the “degree” of the financial friction. With very
high degrees of price rigidity, the channel introduced through the collateral constraint is expected
to be dominated by the cost of deviating from zero inflation.

Finally, due to monopolistic competition, firms in this sector earn positive profits in
equilibrium. These profits are paid in the form of dividends to shareholders. Real dividends are
thus given by:

\[
d_t = y_t - mc_t y_t - \frac{\phi}{2} (\pi_t - 1)^2 y_t \tag{17}
\]

2.4 Market Clearing

In equilibrium, the resource constraint of the economy reads as follows:

\[
y_t = c_t + \frac{\phi}{2} (\pi_t - 1)^2 y_t \tag{18}
\]

Finally, market clearing for shares implies:

\[
e_t + s_t = 1 \tag{19}
\]
2.5 The Private Sector Equilibrium

Definition 1: Given the exogenous processes \( \{R_t, A_t\} \), the private sector equilibrium is a state-contingent sequence of allocations \( \{c_t, l_t, w_t, mc_t, \pi_t, q_t, d_t, e_t, \mu_t\} \) that satisfy the equilibrium conditions (6)-(8), (12)-(13), (15) and (17)-(18), and the complementary slackness condition \( \mu_t(q_{t-1} + d_t) - \alpha w_t l_t = 0 \).

2.6 The Optimal Monetary Policy Problem

I use a Ramsey-type approach to study optimal monetary policy. In particular, the monetary authority in this economy chooses allocations, rather than committing to any Taylor-type rule, to maximize the lifetime utility of households subject to the resource constraint and the private-sector equilibrium conditions.\(^6\) This formulation also assumes commitment to the optimal policy problem (i.e. the monetary authority is constrained by commitments in the initial period).

Definition 2: Given the exogenous process for technology \( A_t \), the monetary authority chooses a sequence of allocations \( \{c_t, l_t, w_t, mc_t, \pi_t, q_t, d_t, e_t, \mu_t\} \) to maximize (1) subject to the conditions (6), (8), (12)-(13), (15) and (17)-(18).

3 Optimal Monetary Policy and the Labor Wedge

This section presents an alternative way, related to basic Ramsey theory, to view the implications of the collateral constraint for optimal monetary policy. In the basic Ramsey theory, the aim of the planner is to smooth distortions (or “wedges”) over time. In this paper, a binding collateral constraint distorts the choice of labor by entrepreneurs and hence leads to suboptimal choice of labor. To see this, consider first the problem of the social planner who maximizes the expected present discounted utility of households subject to the goods-market resource constraint (see Appendix 1-E for details). The condition characterizing the social planner’s problem is given by

\[
\frac{v_{t,t}}{u_{c,t}} = A_t,
\]

which states that the marginal rate of substitution between labor and consumption should be equal to the marginal product of labor. In the decentralized economy, the equivalent condition is given by

\[6\] The fact that “profits” of entrepreneurs are transferred to households simplifies the problem of the monetary policy maker; instead of maximizing some weighted average of the lifetime objective functions of households and entrepreneurs, the objective function of the monetary policy maker is only the lifetime utility of households.
\[
\frac{v_{t,t}}{u_{c,t}} = A_y \left[ \frac{(1+\tau)mc_t}{1+\alpha\mu_t} \right].
\]

The “labor wedge” is given by the term in the brackets (more precisely, it is the difference between 1 and this term). I refer to this wedge as the labor wedge following Shimer (2009). In this paper, the labor wedge is a function of the Lagrange multiplier on the collateral constraint and the monopoly power of monopolistically-competitive firms.

The role of positive inflation can be easily seen by substituting for \( mc_t \) using the Phillips curve (condition 15) as follows:

\[
\frac{v_{t,t}}{u_{c,t}} = A_y \left[ \frac{(1+\tau)\left( \frac{\epsilon_t - 1}{\epsilon_t} + \frac{\sigma}{\epsilon_t} \xi - 1 \right) \rho - \beta \phi E_t \left[ \frac{u_{ct+1}}{u_{ct}}(\pi_{ct+1} - 1)\pi_{ct+1} \frac{y_{ct+1}}{y_t} \right] }{1+\alpha\mu_t} \right].
\]

To fix ideas, assume that the economy is subject only to a TFP shock (i.e. \( \epsilon_t \) is constant) and let the difference between 1 and the term in brackets be defined as “labor-income tax”. Under zero-inflation policy, the nominator of (22) is constant, but the denominator varies with the variation in \( \mu_t \). If the monetary authority implements zero-inflation policy, then a negative shock that leads to an increase in \( \mu_t \) will also result in a higher “tax rate”. This increase in the “tax rate” can be alleviated by appropriate setting of the inflation rate. In this case, by setting a positive inflation rate, the monetary authority can decrease the “tax rate” and smooth its variation.

If \( \epsilon_t \) is allowed to be exogenously time-varying, setting a positive inflation rate has a similar role. Suppose that \( \epsilon_t \) falls (which implies a decrease in the degree of competitiveness in the final-

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7 In general, with labor-income tax, we have \( \frac{v_{t,t}}{u_{c,t}} = A_y (1 - \gamma_t) \), with \( \gamma_t \) being the labor-income tax rate. In our case, the “tax rate” is defined as

\[
\gamma_t = \frac{1}{1+\alpha\mu_t} \left\{ \frac{(1+\tau)\left( \frac{\epsilon_t - 1}{\epsilon_t} + \frac{\sigma}{\epsilon_t} \xi - 1 \right) \rho - \beta \phi E_t \left[ \frac{u_{ct+1}}{u_{ct}}(\pi_{ct+1} - 1)\pi_{ct+1} \frac{y_{ct+1}}{y_t} \right]}{1+\alpha\mu_t} \right\}.
\]

Hence, if \( \pi_t = 1 \) all the time, then any increase in the shadow value of relaxing the collateral constraint will lead to an increase in \( \gamma_t \).
good sector). Under zero-inflation policy, the numerator \( \frac{e_t - 1}{e_t} \) decreases, but the denominator increases. Both effects lead to increases in the “labor-income tax” and thus require greater response by the monetary authority.

More generally, the aim of setting a positive inflation rate is to reduce and smooth the labor wedge, and thus to position the economy as close as possible to the efficient state. In this regard, optimal monetary policy in this paper is in line with basic Ramsey policy of smoothing distortions over the business cycle.

4 Computational Strategy and Calibration

The first subsection presents some discussion about the solution methodology applied in this study. Subsection 4.2 then discusses the parameterization of the model.

4.1 Computational Strategy

Ideally, occasionally-binding constraints should be handled using global computational methods, but this comes at the expense of tractability. Hence, I resort to local methods in order to approximate the solution of the model. However, standard perturbation methods, as they stand, cannot deal with occasionally-binding constraints. Therefore, I modify the problem by using the penalty function approach; this approach allows for any value of \( \alpha w_i l_t \) to be possible in principle, but it imposes penalty once the collateral constraint is violated. Since the constraint is imposed on the labor choice of an entrepreneur, her objective function is modified so that it explicitly includes the penalty on violating the collateral constraint. Once the objective function of an entrepreneur is enlarged with the penalty function, the collateral constraint is removed. Thus, the computational problem that I solve, in place of the problem described in subsection (2.2), is

\[
\max E_0 \sum_{t=0}^\infty \delta^t \mathbb{E}_{0,t} \left\{ p_t A_t l_t - w_t l_t + e_{t-1}(q_t + d_t) - e_t q_t - \frac{1}{\psi^2} \exp[\psi(\alpha w_t l_t - \kappa e_{t-1}(q_t + d_t))] \right\}. \tag{23}
\]

The parameter \( \psi \) governs the curvature of the penalty function and it will be a key parameter in the analyses below. Also, the penalty approaches zero when the collateral constraint is not violated (see Figure 1).

---

8 This function is similar to the one used by Den Haan and Ocaktan (2009).
9 The horizontal axis shows \( \alpha w_i l_t - \kappa e_{t-1}(q_t + d_t) \).
Computationally, the optimality conditions (12) and (13) are replaced by
\[ A_t p_t = w_t (1 + \alpha \Omega_t), \]  
\[ 1 = \delta E_t \left[ \Xi_{t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) (1 + \kappa \Omega_{t+1}) \right], \]  
where, \( \Omega_t = \frac{1}{\psi} \exp \left[ \nu (\alpha\psi l_t - \kappa \psi_{t-1} (q_t + d_t)) \right]. \)

Comparing (24) and (25) with (12) and (13), it is apparent that the approximation method replaces the economic variable \( \mu_t \) by \( \Omega_t \). Also, this variable satisfies the requirement of being nonnegative and it approaches zero when the collateral constraint does not bind.

The decision rules that solve this approximation to the equilibrium are obtained through a second-order approximation to the optimality conditions of the monetary authority. Using a second order approximation, rather than linearization, is necessary in order to capture the asymmetry inherent in the occasionally-binding collateral constraint. A second-order approximation also allows for the mean of a variable to be different from its respective deterministic steady state value. The second-order approximation procedure I apply is the one developed by Schmitt-Grohe and Uribe (2004).

### 4.2 Parameterization

In what follows, I assume a time unit of a quarter and hence the discount factor \( \beta \) is set to 0.99. Following CFP, I set the parameter \( \alpha \) to 0.5 in the benchmark calibration of the model. Following Iacoviello (2005) the maximum loan-to-value ratio \( \kappa \) is set 0.89 in the benchmark calibration. In addition, I assume the following period utility function for households:
\[ u(c_t) - v(l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{l_t^{1+\theta}}{1+\theta}, \]
with the benchmark value of \( \sigma \) being set to 1.5. The parameter \( \theta \) is set to 0.5, implying a labor supply elasticity of 2. This relatively high labor supply elasticity is needed to capture the volatility of total hours in a model with no extensive margin, as is the case in this paper. The parameter \( \chi \) is then calibrated so that the steady state value of \( l \) is 0.3.

Productivity is governed by the following AR(1) process:
\[ \ln(A_t) = (1 - \rho_A) \ln(A) + \rho_A \ln(A_{t-1}) + u_t, \]
with $\rho_A$ being 0.95 in line with standard calibration of the TFP process. The innovation term $u_t$ is normally distributed with zero mean and a standard deviation of $\sigma_u = 0.007$. The mean of $A$ is 1.

Similarly, the elasticity of substitution $\varepsilon_t$ evolves according to the following process:

$$
\ln(\varepsilon_t) = (1 - \rho_\varepsilon)\ln(\varepsilon) + \rho_\varepsilon\ln(\varepsilon_{t-1}) + v_t,
$$

where $\rho_\varepsilon$ is set to 0.9672 and the innovation term $v_t$ is normally distributed with zero mean and a standard deviation of $\sigma_v = 0.0729$, in line with Ireland (2002). The deterministic steady state value of $\varepsilon_t$ is set to 6, implying a deterministic steady state markup of 20 percent.

The benchmark value of $\psi$ is chosen so that the collateral constraint holds with equality in the deterministic steady state of the model. Hence, imposing deterministic steady state on $\Omega$ gives $\Omega = \frac{1}{\psi}$. In addition, combining the steady state versions of (8) and (25) gives $\Omega = \frac{1 - \delta}{\delta \kappa}$. The combination of these two relationships gives $\psi = \frac{\delta \kappa}{1 - \delta}$. The implied value of this parameter is 889.11. There are two reasons for the assumption that the collateral constraint binds in the deterministic steady state. First, for the constraint not to bind in the deterministic steady state, the additional discount factor ($\delta$) should be 1. In this case, however, entrepreneurs accumulate enough assets so that the collateral constraint never binds. Second, starting from a deterministic steady state in which the collateral constraint binds enables good comparison with the case in which the constraint always binds (because in both cases the deterministic steady state is the same). Hence, any differences regarding the optimal long-run inflation rate in the “stochastic” steady state can be attributed to the assumption of occasionally-binding collateral constraint.

The value of $\psi$ is obtained for $\delta = 0.999$. This value of $\delta$ is larger than the values typically assumed in the literature (which, approximately, lie between 0.95 and 0.99). There are two reasons for choosing a higher than usual value for $\delta$. First, in models with always-binding constraints, this parameter is chosen so that the collateral constraint always binds. Other things equal, a lower value of $\delta$ increases the chance for the constraint to bind. This fact is also apparent in the relationship $\Omega = \frac{1 - \delta}{\delta \kappa}$; the value of $\Omega$ in the deterministic steady state is decreasing in $\delta$.

In particular, the constraint does not bind in the deterministic steady state of the model if $\delta = 1$. 

15
Second, the accuracy of the approximation using the penalty function depends on the value of $\varphi$; the higher $\varphi$ is, the closer the penalty function to obtain the L-shape, which clearly improves the approximation (put differently, a higher $\varphi$ reduces the probability of violating the collateral constraint since any violation entails a higher penalty). But the equation $\varphi = \frac{\delta K}{1 - \delta}$ suggests that $\varphi$ depends positively on $\delta$. Hence, setting a higher value of $\delta$ is equivalent to setting a higher value of $\varphi$. Note, however, that in the robustness checks section, I also show the results for $\delta = 0.99$.

Finally, the parameter governing the adjustment cost of prices $\varphi$ is set to 18.47 in my benchmark calibration. This value is based on the recent evidence regarding the duration of price contracts: Bils and Klenow (2004) show that the average duration of prices is between 4.5 and 5.5 months; Ravenna and Walsh (2006) suggest price duration of between 2 and 3 quarters, and Christiano, Eichenbaum and Evans (2005) use price duration of 2.5 quarters. I follow Christiano, Eichenbaum and Evans (2005) and set my benchmark price duration to 2.5 quarters, but I also show the results for various price durations between 2 and 4 quarters.

I map between the price duration and the adjustment cost parameter $\varphi$ using the relationship

$$\varphi = \frac{\lambda(\lambda - 1)(\varepsilon - 1)}{\lambda - \beta(\lambda - 1)}$$

with $\lambda$ denoting the price duration. This approach follows Faia and Monacelli (2007). In short, the price rigidity parameter $\varphi$ is pinned down when the slope of the Philips curve in a linearized model with Calvo (1983)’s parameterization is equalized to the slope of the Philips curve in a linearized model with a Rotemberg (1982)’s parameterization. For more details, refer to Appendix 1-H.

## 5 The Optimal Long-Run Inflation Rate

This section presents the main findings regarding the optimal long-run inflation rate in the presence of financial frictions.

### 5.1 The Optimal Inflation Rate in the Deterministic Steady State

Before turning to present the optimal long-run inflation rate, a note on the deterministic steady state (i.e. the state with constant technology) is in order. Given the parameter $\varphi$, the deterministic steady state of the model is invariant to the degree of price stickiness. The main result is that the
optimal deterministic steady state of inflation is exactly zero (see Appendix 1-K for a proof). This result is as expected: in the absence of shocks, inflation is not beneficial, and due to the resource cost of deviations from zero inflation, the monetary authority completely stabilizes prices in the deterministic steady state. This is true regardless of the degree of price rigidity assumed (since there is no benefit from non-zero inflation but there is a cost of non-zero inflation for any positive value of price rigidity) and regardless of whether there is a labor market subsidy or not. Also, given that the deterministic steady state value of inflation is zero regardless of the degree of price rigidity, the deterministic steady state values of other variables will not vary with the degree of price rigidity.

5.2 The Optimal Long-Run Inflation Rate

This subsection presents the main results regarding the optimal long-run inflation rate. Before presenting the results under occasionally-binding constraint, I comment on the optimal inflation rate with always-binding constraint and no financial frictions. In both cases, the optimal long-run inflation rate is zero regardless of the type of the underlying shock. Furthermore, with the absence of financial frictions, inflation does not respond to TFP shock in the short run.

The results with occasionally-binding collateral constraint are considerably different (Table 1). In this case, optimal monetary policy deviates from full price stability in the long run. When the economy is, simultaneously, subject to TFP and markup shocks, the optimal long-run inflation rate is around 2 percent annually in the benchmark calibration of the model. This is an important result, since in the real world the economy is subject to TFP and markup shocks, among others, at the same time. The optimal inflation rate is also positive and around 1 percent for other empirically-plausible price durations. Hence, regardless of what the actual price duration in the U.S. is, the optimal inflation rate should, generally speaking, lie between 1 percent and 2 percent annually.

<table>
<thead>
<tr>
<th>Price Duration (Quarters)</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP and Markup shocks</td>
<td>2.08</td>
<td>1.82</td>
<td>1.44</td>
<td>1.06</td>
<td>0.81</td>
</tr>
<tr>
<td>TFP shocks only</td>
<td>0.63</td>
<td>0.54</td>
<td>0.43</td>
<td>0.32</td>
<td>0.22</td>
</tr>
<tr>
<td>Markup Shocks only</td>
<td>1.61</td>
<td>1.16</td>
<td>0.91</td>
<td>0.73</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Table 1: The optimal long-run inflation rate (in annualized percentages terms) for various price durations.
Also, for plausible price durations, the optimal inflation target is falling in the degree of price rigidity. This result is due to the higher resource cost associated with higher inflation rate, which in turn negatively affects the mean value of consumption. In both cases, the nominal distortion seems to be less dominant for relatively low degrees of price stickiness, but becomes more dominant as the degree of price rigidity increases. Indeed, as the cost of price adjustment increases, the optimal inflation rate decreases. The optimal inflation rate reaches zero for very high degrees of price rigidity, but this happens outside the empirically plausible range of $\varphi$ considered in this paper.

It is also interesting to consider each shock separately in order to learn about the contribution of each shock in driving the results in Table 1. When the economy is only subject to TFP risk, the optimal annual long-run inflation rate is around 0.5 percent in the baseline calibration of the model, and it is also strictly positive for other plausible price durations. This result suggests that the presence of occasionally-binding constraints not only leads to variations in inflation following productivity shocks (as will be shown in the impulse-response subsection below), but also implies positive inflation on average.

Under only markup shocks, the optimal long-run inflation rate is almost twice as large as under TFP shocks only and decreasing in the degree of price rigidity. Hence, most of the positive inflation rate found above is due to markup shocks. This result is in line with the fact that markup shocks have stronger impact on inflation, through the Phillips curve, and they account for higher portion of the variability in inflation compared to TFP shocks (Ireland, 2002). Furthermore, the existence of the collateral constraint magnifies this effect since, as discussed in subsection (2.3), the variable $\Omega$, acts as an endogenous markup shifter.

5.3 Discussion

When the collateral constraint binds, the Lagrange multiplier is akin to a markup shifter that leads to positive inflation. When the collateral constraint does not bind, the optimal inflation rate is zero. Since the Lagrange multiplier cannot take negative values that offset the effects of positive Lagrange multipliers on inflation and given empirically plausible degrees of price rigidity, a positive inflation rate on average is the outcome.

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10 The results here are, conceptually, in line with the findings of Schmitt-Grohe and Uribe (2007); they show that the optimal inflation rate is highly sensitive to the degree of price rigidity. In their study, there was a tension between the monetary distortion, which calls for a negative inflation rate, and the nominal distortion, which calls for full price stability. In the current study, the tension is between the financial friction, which calls for a positive inflation rate, and the nominal distortion.
5.4 The Labor Wedge
As discussed in section 3, a binding collateral constraint generates a wedge between the marginal rate of substitution between labor and consumption and the marginal product of labor, thus leading to a rise in the “labor-income tax”, as defined above. These effects can be counteracted by setting positive inflation.

Figure 2 shows the volatility of the labor wedge for various optimal inflation rates as the price duration is varied between 2 and 4 quarters. For the sake of exposition, I consider the case with simultaneous markup and TFP shocks, but the results hold under each shock individually. Clearly, the volatility of the wedge is decreasing in the inflation rate (put differently, a lower degree of price rigidity is associated with higher optimal inflation and lower volatility of the labor wedge). The Ramsey planner cannot completely close and/or smooth the wedge due to the lack of a set of policy instruments to completely and simultaneously offset all distortions along the business cycle. This is the case since prices are not fully flexible and hence the Ramsey planner trades-offs between stabilizing the wedge and stabilizing inflation.

More generally, the main results of the paper can be related to basic Ramsey theory of smoothing distortions over time. In this case, smoothing the labor wedge requires smoothing of the “labor-income tax”, which by itself can be achieved through appropriate setting of inflation.

5.5 Impulse Responses
It is useful to observe the behavior of some key variables following TFP and markup shocks in order to gain some insights. Figure 3 displays, under the optimal policy with the positive long-run inflation rate result, the responses of some key economic aggregates following negative and positive markup shocks of the same magnitude. Figure 4 shows the behavior of these variables following TFP shocks. Numbers indicate the percentage deviation of each variable from its deterministic steady state value. The main observation is the asymmetry in the response of these variables to negative and positive shocks of either type. The asymmetry is more apparent for the case of markup shocks, which, as we have seen above, is reflected in a higher inflation rate.

Following a negative one standard deviation shock, the fall in nominal share prices and the increase in good prices lead to a drop in the real price of shares \((q)\) of about 2.5 percent below their steady state value. The asymmetry in the response of net worth is mainly driven by the asymmetry in the real price of shares (notice the similarity of their movements) and, to a lesser extent, the asymmetry in the behavior of dividends. Shares \((e)\) display little asymmetry (and their
overall response is relatively small). Output, consumption, labor and the financial friction variable \(\Omega_t\) all display clear asymmetry under both types of shocks.

Inflation behaves as expected; a negative TFP shocks leads to an increase in the marginal cost and consequently to an increase in inflation. This is apparent from examining condition (16). In this paper, the existence of the collateral constraint is the reason for inflation to respond to TFP shocks (i.e. it breaks down the “divine coincidence”). A negative markup shock (which is modeled here through a fall in the elasticity of substitution between two types of final goods) is akin to a cost-push shock that generate inflation. Clearly, the response of inflation to markup shocks is considerably larger than the response of inflation to TFP shocks.

5.6 The Zero-Lower Bound on the Nominal Interest Rate

In this subsection, I relate my results to the ongoing discussion about the Zero-Lower Bound (ZLB) on the nominal interest rate. Other things equal, a higher inflation rate implies a higher nominal interest rate and hence more room for policy intervention.

Figure 5 presents the frequency of hitting the ZLB for various inflation rates under TFP shocks only, markup shocks only and both TFP and markup shocks. Clearly, the likelihood of hitting the ZLB is decreasing with the optimal inflation rate. Under optimal policy, and given the benchmark calibration of the model, the nominal interest rate may hit the ZLB in about 0.3 percent of the time, 1 percent of the time and about 1.5 percent of the time if the economy is subject to TFP shock only, markup shock only or both shocks, respectively. Under TFP shocks only, the nominal interest rate is unlikely to hit the ZLB if the optimal inflation rate is about 1 percent annually. If the economy is subject to markup shocks only, an inflation rate of slightly below 2 percent will most likely prevent the interest rate from hitting the ZLB. With both shocks, hitting the ZLB is less likely for inflation rates beyond 2 percent.

These results under zero-inflation policy differ markedly. Under zero-inflation policy, the nominal interest rate may hit the ZLB in about 2 percent of the time following a TFP shock only, about 12.5 percent of the time following a markup shock only and about 15.5 percent of the time when the economy is hit by both shocks simultaneously (Table 1A in Appendix 1). Therefore, the frequency of hitting the ZLB is reduced by more than 10 percent if the economy is subject to markup shocks and both shocks and it is reduced by more than 1.5 percent if only TFP shocks hit the economy. Therefore, the main result of the paper can also be seen as aiming for reducing the
frequency of hitting the ZLB. Such a policy is more important during downturns since during those episodes hitting the ZLB and the collateral constraint limit are more likely.

5.7 The Frequency of Hitting the Collateral Constraint Limit

In this subsection, I present another role of inflation in “buffering” the economy from potential adverse shocks. This happens through relaxing the collateral constraint (put differently, reducing the probability of a binding collateral constraint).

To measure the frequency at which the constraint binds (i.e. $\Omega > 0$), I adopt the following strategy. Since $\Omega$ may take very small positive values, I first choose a tolerance value above which $\Omega$ is considered to be positive. I choose to, separately, normalize the value of $\Omega$ in the simulation by the level of output ($y$) and the marginal utility of consumption ($u_c$). The benchmark values are the ratio of $\Omega$ to $y$ and the ratio of $\Omega$ to $u_c$ in the deterministic steady. Respectively, they take the values of 0.375% of output and 0.0185% of the marginal utility of consumption. For this reason, I set a tolerance of 0.1% when the ratio of $\Omega$ to $y$ is considered, and set the tolerance to 0.005% when the ratio of $\Omega$ to $u_c$ is considered.

Figure 6 shows the behavior of the financial friction variable $\Omega$, when normalized by $u_c$ for a given representative simulation under both optimal policy and under zero-inflation policy (the figure when $\Omega$ is normalized to output is similar). Clearly, the frequency at which the collateral constraint binds under optimal policy (that sets positive inflation) is reduced significantly compared to the frequency at which it binds under zero-inflation policy.

Inflation and the financial friction are linked through the Philips curve (condition 16). Other things equal, the marginal cost of final-good firms increases when the collateral constraint binds (since, as shown in Appendix 1-I, $mc_i = \frac{v_{i,t}}{u_{c,t}(1+\tau)A_j} + \alpha \frac{v_{i,t}}{u_{c,t}(1+\tau)A_j} - \Omega_j$). However, higher marginal cost implies lower profits for final-good firms, and hence lower value of dividends for entrepreneurs (the number of shares is determined in the end of the last period and hence the value of these shares depend on the current amount of dividends). The fall in dividends, in turn, implies lower current real price of shares ($q$). These two effects combined lead to a fall in net worth (recall equation 10). But, the fall in net worth tightens the collateral constraint even further and thus induces another round of fall in dividends and hence net worth. Hence, reducing the likelihood of a binding collateral constraint helps in avoiding the increase in the marginal cost.
6 Robustness Checks

6.1 Changing the Impatience Rate of Entrepreneurs

I start by changing the value of the parameter $\delta$ and, consequently, the value of $\psi$. I set $\delta = 0.99$, which implies a subjective discount factor of entrepreneurs ($\beta \delta$) of about 0.98, in line with Iacoviello (2005). The implied value of $\psi$ is 88.11.

The results are reported in Table 2 and they can be summarized as follows. First, the optimal long-run inflation rate with the benchmark price duration (of 2.5 quarters) is positive, ranging between about one third percent when TFP risk is the driving force and 1.2 percent when both TFP and markup shocks hit the economy. Second, in all cases considered, the optimal inflation rate is lower than the optimal inflation rate presented in section subsection 5.2. This fact reflects less degree of asymmetry in the model that results from a lower value for the curvature parameter $\psi$. Third, the optimal inflation is, in general, around 1 percent for most of the price durations considered.

<table>
<thead>
<tr>
<th>Price Duration (Quarters)</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP and Markup shocks</td>
<td>1.38</td>
<td>1.17</td>
<td>0.98</td>
<td>0.86</td>
<td>0.80</td>
</tr>
<tr>
<td>TFP shocks only</td>
<td>0.40</td>
<td>0.35</td>
<td>0.29</td>
<td>0.25</td>
<td>0.20</td>
</tr>
<tr>
<td>Markup Shocks only</td>
<td>0.95</td>
<td>0.81</td>
<td>0.62</td>
<td>0.47</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Table 2: The optimal long-run inflation rate (in annualized percentages terms) for various price durations with $\delta = 0.99$ and $\psi = 88.11$.

6.2 Changing the Financial Friction Parameter

Table 3 presents the results for various values of the financial parameter $\alpha$ under the assumption that the price duration is 2.5 quarters. The optimal inflation rate is increasing in $\alpha$ for all types of shocks, suggesting that the more “severe” the financial friction is, the higher the optimal inflation rate. Needless to say, the optimal long-run inflation rate for $\alpha = 0$ is zero regardless of the source of uncertainty, and hence it is not presented below.
### Table 3: The optimal long-run inflation rate (in annualized percentages terms) for various values of $\alpha$.

<table>
<thead>
<tr>
<th>Financial Friction Parameter</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP and Markup shocks</td>
<td>0.25</td>
<td>0.50</td>
<td>0.75</td>
<td>1.00</td>
</tr>
<tr>
<td>TFP shocks only</td>
<td>0.92</td>
<td>1.82</td>
<td>2.81</td>
<td>3.85</td>
</tr>
<tr>
<td>Markup Shocks only</td>
<td>0.27</td>
<td>0.54</td>
<td>0.87</td>
<td>1.17</td>
</tr>
<tr>
<td>TFP shocks only</td>
<td>0.58</td>
<td>1.16</td>
<td>1.77</td>
<td>2.54</td>
</tr>
</tbody>
</table>

#### 6.3 Increasing the Sizes of the Underlying Shocks

Blanchard, Dell’Ariccia and Mauro (2010) suggest that “the crisis has shown that large shocks to the system can and do happen,” and that “maybe policymakers should therefore aim for a higher target inflation rate in normal times, in order to allow for more room for monetary policy to react to such shocks.” Motivated by this statement, in what follows, I show the optimal long-run inflation rate when the sizes of the underlying shocks are bigger than usual. This is not necessarily the only way to interpret the ideas of Blanchard, Dell’Ariccia and Mauro (2010), but it perhaps the simplest way to capture their suggestions. Also, for the sake of exposition, I consider the case when the shocks are 10 percent bigger than usual (i.e. $\sigma_u = 0.0077$ and $\sigma_v = 0.0802$). The results are summarized in Table 4.

### Table 4: The optimal long-run inflation rate (in annualized percentages terms) for various price durations, with larger risk.

<table>
<thead>
<tr>
<th>Price Duration (Quarters)</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP and Markup shocks</td>
<td>2.06</td>
<td>2.25</td>
<td>1.79</td>
<td>1.28</td>
<td>1.02</td>
</tr>
<tr>
<td>TFP shocks only</td>
<td>0.78</td>
<td>0.67</td>
<td>0.59</td>
<td>0.38</td>
<td>0.26</td>
</tr>
<tr>
<td>Markup Shocks only</td>
<td>1.61</td>
<td>1.48</td>
<td>1.16</td>
<td>0.82</td>
<td>0.73</td>
</tr>
</tbody>
</table>

With larger risk, the optimal inflation rate under simultaneous TFP and markup shocks is roughly 2.5 percent, about half a percent higher than in the benchmark calibration. In overall, the optimal inflation rate is around 2 percent for price durations between 2 and 3 quarters, which are the most empirically-relevant ranges. Hence, even a relatively small increase in the sizes of the shocks lead to considerable increase in the optimal inflation rate. This result suggest that, if the
likelihood of bigger shocks is high, policy makers should target a relatively higher inflation rate to avoid the case of being constrained in conducting conventional monetary policy.

7 Conclusions

The main purpose of this paper is to study the optimal long-run inflation rate in the presence of financial frictions. The model is a variation of the standard New Keynesian framework in which hiring labor by entrepreneurs is constrained by their collateral. This study modifies the assumption of always-binding collateral constraints by assuming that the constraint may only occasionally bind. To do so, the always-binding collateral constraint is replaced by a function that imposes a penalty once the constraint is violated. The main result is that optimal monetary policy sets a strictly positive inflation rate in the long-run (i.e. in the stochastic steady state of the model). The optimal annual long-run inflation rate is about 2 percent when the economy faces both TFP and markup risks of empirically-relevant magnitudes. When the economy is subject only to markup risk, the optimal inflation rate is slightly above 1 percent annually. And, when only TFP shocks hit the economy, the optimal annual inflation rate is about 0.5 percent.

When the collateral constraint binds, the shadow value of relaxing the constraint is equivalent to an endogenous markup shifter. Since the constraint binds on average (and takes non-negative values only), the effects of positive Lagrange multipliers on inflation are not offset in periods of non-binding collateral constraint. That leads to positive inflation on average.

From the normative point of view, optimal monetary policy sets a positive inflation rate on average because of precautionary considerations. A binding collateral constraint distorts labor demand and thus leads to suboptimal level of output. Basically, a binding collateral constraint is akin to a “tax” on labor which can be both reduced and smoothed by setting positive inflation. In addition, through affecting the flow of profits of final-good firms, monetary authority influences the amount of assets held by entrepreneurs and, as a result, their collateral. In turn, that implies less likelihood of hitting the collateral constraint limit once a negative shock hits the economy. The analyses of this study show that, under the optimal policy, hitting the collateral limit is significantly less likely compared to the likelihood of hitting the collateral constraint limit under zero-inflation policy.

In light of the economic events of the last 2 years and the fact that the economy may indeed experience larger-than-usual shocks, it may even be optimal to set a higher inflation rate over
time. In particular, setting an inflation rate of about 2.5 percent has been found to be optimal if the sizes of the underlying shocks are 10% larger than usual. This suggests that if the monetary authority is willing to reduce the risk of being constrained by the zero-lower bound on the nominal interest rate, then a relatively higher inflation rate can help in achieving that goal. In this regard, monetary policy makers may want to aim for preventing the likelihood of having their hands being tied rather than mainly managing crises once they occur.

The current study also contributes to recent literature that attempt to justify the fact that central banks around the world target positive inflation rates. Recent work in this line includes the contributions of Kim and Ruge-Murcia (2009), Abo-Zaid (2010) and Fagan and Messina (2009), who all study the implications of Downward Nominal Wage Rigidity for the long-run inflation target. To my knowledge, the current study is the first to motivate a positive long-run inflation rate in an environment featuring occasionally-binding financial constraints. The study can be extended to evaluate the performances of different Taylor-type rules compared to the optimal policy. Adding money demand to this framework and then studying the resulting optimal inflation target when financial distortions, nominal distortions and monetary distortions are simultaneously present is another important extension. These are left for future work.

References:


Appendix 1: Tables and Graphs

Figure 1: The Penalty Function ($\psi = 889.11$).

Figure 2: The standard deviation of the static wedge for various optimal annual inflation rate (in percents). The driving processes: Markup and TFP Shocks.
Figure 3: Response to negative and positive markup shocks with financial frictions (percentage deviations from SS levels). Inflation is shown in annualized terms.
Figure 4: Response to negative and positive TFP shocks with financial frictions (percentage deviations from SS levels). Inflation is shown in annualized terms.
Figure 5: The frequency of hitting the ZLB for various optimal annual inflation rates (in percents).
Figure 6: Behavior of $\Omega$ normalized to marginal utility of consumption under optimal policy and zero-inflation policy.

<table>
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<th>Price Duration (Quarters)</th>
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<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
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</table>

Table 1A: The frequency of hitting the ZLB under zero-inflation policy for various price durations.
Appendix 2: Mathematical Derivations

A  The Households’ Problem

Households maximize the following objective function

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - v(l_t) \right], \]  

(A1)

subject to the sequence of budget constraints of the form:

\[ c_t + q_t s_t + b_t = \frac{R_{t-1} b_{t-1}}{\pi_t} + (1+\tau) w_t l_t + s_{t-1} (q_{t-1} + d_{t-1}) + \Pi_t + T_t \]  

(A2)

Denoting the Lagrange multiplier on the budget constraint by \( \lambda_t \), the first order conditions with respect to \( c_t, b_t, s_t \) and \( l_t \) are, respectively:

\[ \lambda_t = u_{c,t}, \]  

(A3)

\[ \lambda_t = \beta R_t E_t \left( \frac{\lambda_{t+1}}{\pi_{t+1}} \right), \]  

(A4)

\[ - q_t \lambda_t + \beta E_t [\lambda_{t+1} (q_{t+1} + d_{t+1})] = 0, \]  

(A5)

and,

\[ - v_{l,t} + \lambda_t (1+\tau) w_t = 0. \]  

(A6)

Combining (A3) and (A6) gives equation (6) in the text. Combining (A3) and (A4) gives equation (7), and the combination of equations (A3) and (A5) yields equation (8) in the text.

B  The Entrepreneurs’ Problem

An entrepreneur chooses labor and shares to maximize:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{0,t} \left[ p_t A_t l_t - w_t l_t + e_{t-1} (q_t + d_t) - \alpha w_t l_t \right], \]  

(B1)

subject to \( \kappa e_{t-1} (q_t + d_t) - \alpha w_t l_t \geq 0. \)  

(B2)

Denoting the Lagrange multiplier on constraint (B2) by \( \mu_t \), the choice of labor yields:

\[ A_t p_t - w_t - \alpha w_t \mu_t = 0, \]  

(B3)

or, after collecting terms,

\[ A_t p_t = w_t (1+\alpha \mu_t), \]  

(B4)
which is equation (12) in the text.

Similarly, the first-order condition with respect to \( e_t \) gives:

\[
\delta \Xi_{0,t} (-q_t) + \delta^{t+1} E_t [\Xi_{0,t+1} (q_{t+1} + d_{t+1})] + \delta^{t+1} E_t [\Xi_{0,t+1} (-\kappa (q_{t+1} + d_{t+1})) \mu_{t+1}] = 0, \tag{B5}
\]

By collecting terms and rearranging, condition (B5) can be written as:

\[
1 = \delta E_t \left[ \Xi_{t,t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) (1 + \kappa \mu_{t+1}) \right]. \tag{B6}
\]

This is condition (13) in the text.

\section{The Approximated Entrepreneurs' Problem}

The problem of an entrepreneur in this case is to maximize:

\[
E_0 \sum_{t=0}^{\infty} \delta^t \Xi_{0,t} \left\{ p_t A_t l_t - w_t l_t + e_{t-1} (q_t + d_t) - e_t q_t - \frac{1}{\psi^2} \exp[\psi (\alpha \omega l_t - \kappa \nu (q_t + d_t)) \right\}, \tag{C1}
\]

The first order condition with respect to \( l_t \) yields:

\[
P_t l_t - w_t - \frac{\psi \alpha \omega}{\psi^2} \exp[\psi (\alpha \omega l_t - \kappa \nu (q_t + d_t))] = 0. \tag{C2}
\]

Letting \( \Omega_t = \frac{1}{\psi} \exp[\psi (\alpha \omega l_t - \kappa \nu (q_t + d_t))] \), condition (C2) can now be written as:

\[
P_t A_t - w_t (1 + \alpha \Omega_t) = 0, \tag{C3}
\]

which is equation (21) in the text.

Finally, the first order condition with respect to \( e_t \) yields:

\[
\delta^t \Xi_{0,t} (-q_t) + \delta^{t+1} E_t [\Xi_{0,t+1} (q_{t+1} + d_{t+1})] + \\
\delta^{t+1} E_t \left[ -\frac{1}{\psi} \exp[\psi (\alpha \omega l_t - \kappa \nu (q_{t+1} + d_{t+1}))] \right] (-\kappa (q_{t+1} + d_{t+1})) = 0 \tag{C4}
\]

or, by using the definition of \( \Omega_t \),

\[
\Xi_{0,t} (-q_t) + \delta E_t [\Xi_{0,t+1} (q_{t+1} + d_{t+1})] + \delta E_t [\Xi_{0,t+1} (-\Omega_{t+1}) (-\kappa (q_{t+1} + d_{t+1}))] = 0. \tag{C5}
\]

Rearranging equation (C5) gives equation (22) in the text.
D Operating Profits of Entrepreneurs in the Approximated Model

Let us define the difference between revenues and wage costs by operating profits, as follows:

$$\Pi^o_t = p_tA_t l_t - w_t l_t$$

(Д1)

Recall, from equation (B4) above, that $$p_t A_t = w_t (1 + \alpha \mu_t)$$ . Hence:

$$p_t A_t l_t = w_t l_t (1 + \alpha \mu_t)$$

(Д2)

Using the production function of entrepreneurs ($x_t = A_t l_t$), condition (D2) can be written as:

$$p_t x_t = w_t l_t (1 + \alpha \mu_t)$$

(Д3)

or,

$$w_t l_t = \frac{p_t x_t}{1 + \alpha \mu_t}$$

(Д4)

Substituting (D4) in (D1) and using the production function give:

$$\Pi^o_t = p_t x_t - \frac{p_t x_t}{1 + \alpha \mu_t}$$

(Д5)

which, after collecting terms, can be written as:

$$\Pi^o_t = \frac{\alpha \mu_t}{1 + \alpha \mu_t} p_t x_t$$

(Д6)

Condition (D6) states that operating profits are positive in an equilibrium with a binding collateral constraint. Clearly, if $\alpha$ is zero (i.e. no part of wages is secured by net worth), then operating profits are zero in equilibrium (as one would expect in a perfectly competitive sector). Similarly, if the collateral constraint does not bind, then these profits will be zero as well, since in this case the economy is behaving as if there is no collateral constraint to begin with.

Finally, in the approximated model discussed in section 4, the operating profits will be given by

$$\Pi^o_t = \frac{\alpha \Omega_t}{1 + \alpha \Omega_t} p_t x_t$$

, with $\Omega_t$ as defined in the text. The derivations are similar to the ones just shown, and therefore they are not presented here.
E  Efficient Allocations and the Labor Wedge

The social planner chooses consumption and labor to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(l_t)],$$

subject to the sequence of resource constraints

$$A_t l_t - c_t = 0.$$  \hspace{1cm} (E2)

Let $\eta_t$ be the Lagrange multiplier associated with (E2), then, the first-order conditions with respect to $c_t$ and $l_t$, respectively, read

$$u_{c,t} = \eta_t,$$  \hspace{1cm} (E3)

and

$$v_{l,t} = \eta_t A_t.$$  \hspace{1cm} (E4)

Combining (E3) and (E4) yields

$$\frac{v_{l,t}}{u_{c,t}} = A_t,$$  \hspace{1cm} (E5)

and hence efficiency requires the marginal rate of substitution (the left hand side of condition E5) to be equal to the marginal product of labor (given by the right-hand side of condition E5).

Given this result, one can derive the expression for the intratemporal (static) wedge. To do so, combine labor supply condition (6) and the labor demand condition (12) to get

$$\frac{v_{l,t}}{u_{c,t}} = A_t \left( \frac{(1 + \tau) p_t}{1 + \alpha \mu_t} \right).$$  \hspace{1cm} (E6)

Comparison of (E5) and (E6) reveals that the wedge is defined by the term in the parentheses. Clearly, this wedge is directly affected by the existence of the collateral constraint.

F  The Labor Market Subsidy

The labor market subsidy $\tau$ is introduced to render the deterministic steady state of the model efficient. In particular, this subsidy is chosen so that, in the deterministic steady state, the marginal rate of substitution ($MRS$) between consumption and labor is equal to marginal product of labor ($MPL$). The derivations for labor market subsidy in the approximated model are similar to what follows, but with $\Omega$ replacing $\mu$ wherever it appears.
In what follows, undated variables denoted the deterministic steady state level of the corresponding variables. From equation (6), we have:

\[
\frac{v}{u_c} = (1 + \tau)w, \quad (F1)
\]

The left-hand side of (F1) is the MRS between consumption and labor, hence:

\[
MRS = (1 + \tau)w \quad (F2)
\]

Recalling that (MPL=A), equation (12) in the text implies:

\[
MPL = \frac{w(1 + \alpha \mu)}{p} \quad (F3)
\]

Setting \(MRS=MPL\), and canceling \(w\), yields:

\[
(1 + \tau) = \frac{1 + \alpha \mu}{p} \quad (F4)
\]

which, after rearranging terms, gives:

\[
\tau = \frac{1 + \alpha \mu - p}{p} \quad (F5)
\]

Finally, recall that \(p\) equals the marginal cost of final-good firms (mc), which, in the deterministic steady state, is given by the inverse of the gross markup (i.e. \(mc = \frac{\varepsilon - 1}{\varepsilon}\)). Substituting this result into equation (F5) yields:

\[
\tau = \frac{1 + \alpha \varepsilon \mu}{\varepsilon - 1} \quad (F6)
\]

Therefore, the labor market subsidy depends both on the level of the “real interest rate” and the degree of the monopolistic distortion (represented by \(\varepsilon\)). If no wage is required to be secured (i.e. \(\alpha = 0\)), or if the collateral constraint does not bind (i.e. \(\mu = 0\)), then the labor market subsidy should correct only the monopolistic distortion. On the other hand, when \(\varepsilon\) approaches infinity (which corresponds to perfect competition in the final-good sector), \(\tau\) approaches \(\alpha \mu\). This result is as expected since, with perfect competition, the only inefficiency in the allocation of \(l\) comes from existence of the financial friction. Clearly, if the choice of labor is unconstrained and the final-good sector is perfectly competitive, there is no distortion to correct for, and hence the labor market subsidy is zero.
G Deriving the Philips Curve

The problem of a final-good firm \( j \) is to choose its price \( (P_{jt}) \) to maximize

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \lambda_t \left[ \frac{P_{jt}}{P_t} y_{jt} - mc_{jt} y_{jt} - \frac{\varphi}{2} \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right)^2 \right] \right],
\]

subject to the demand function for its product

\[
y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\gamma_t} y_t.
\]

Rewrite (G1) as

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \lambda_t \left[ \left( \frac{P_{jt}}{P_t} \right)^{-\gamma_t} y_t - mc_{jt} \left( \frac{P_{jt}}{P_t} \right)^{-\gamma_t} y_t - \frac{\varphi}{2} \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right)^2 \right] \right].
\]

The first-order condition with respect to the price \( P_{jt} \) reads

\[
\beta^t \lambda_t \left[ (1 - \epsilon_t) \left( \frac{P_{jt}}{P_t} \right)^{-\gamma_t} y_t + \epsilon_t mc_{jt} \left( \frac{P_{jt}}{P_t} \right)^{-\gamma_t - 1} y_t - \varphi \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right) y_t \right] + \beta^t \lambda_{t+1} \left[ -\varphi \left( \frac{P_{jt+1}}{P_{jt}} - 1 \right) \left( -\frac{P_{jt+1}}{P_{jt+1}} \right) y_{t+1} \right] = 0.
\]

(4)

In equilibrium, all firms set the same price (i.e. \( P_{jt} = P_t \) for all \( j \)). Imposing symmetry on condition (G4) and canceling terms give

\[
\lambda_t \left[ (1 - \epsilon_t) \frac{y_t}{P_t} + \epsilon_t mc_t \frac{y_t}{P_t} - \frac{P_t}{P_{t-1}} y_t \right] + \beta \lambda_t \left[ -\varphi \left( \frac{P_{t+1}}{P_t} - 1 \right) \left( -\frac{P_{t+1}}{P_{t+1}} \right) y_{t+1} \right] = 0.
\]

(G5)

Multiplying by \( \frac{P_t}{y_t} \) yields

\[
\lambda_t \left[ (1 - \epsilon_t) + \epsilon_t mc_t - \varphi \left( \frac{P_t}{P_{t-1}} - 1 \right) \right] + \beta \lambda_{t+1} \left[ -\varphi \left( \frac{P_{t+1}}{P_t} - 1 \right) \left( -\frac{P_{t+1}}{P_{t+1}} \right) y_{t+1} \right] = 0.
\]

(G6)

Defining \( \pi_t = \frac{P_t}{P_{t-1}} \), we get

\[
\lambda_t \left[ (1 - \epsilon_t) + \epsilon_t mc_t - \varphi (\pi_t - 1) \right] + \beta \lambda_{t+1} \left[ \varphi (\pi_{t+1} - 1) \pi_{t+1} \right] = 0,
\]

(7)

or, after rearranging and using the fact that \( \frac{\lambda_{t+1}}{\lambda_t} = \frac{u_{t+1}}{u_t} \), yields

\[
1 - \varphi (\pi_t - 1) \pi_t + \beta \varphi \left( \frac{u_{t+1}}{u_t} \right) (\pi_{t+1} - 1) \pi_{t+1} \frac{y_{t+1}}{y_t} = \epsilon_t (1 - mc_t),
\]

(G8)

which is equation (15) in the text.
H Mapping Between the Price Duration and the Price Rigidity Parameter

I show here the way to map between the price duration and the price rigidity parameter. To do so, let us define the price duration by \( \lambda \) and probability of not resetting the price during a given period by \( \omega \). Hence,

\[
\lambda = \frac{1}{1 - \omega}.
\]

(H1)

The slope of the Philips curve under the Rotemberg’s approach for price rigidity is given by

\[
\frac{(\varepsilon - 1)}{\varphi}.
\]

(H2)

Similarly, the slope of the Philips curve when one follows Calvo’s approach for price rigidity is \( \frac{(1 - \omega)(1 - \beta \omega)}{\omega} \). Substituting (H1) shows that the slope with the Calvo’s approach can be rewritten as

\[
\frac{\lambda - \beta (\lambda - 1)}{\lambda (\lambda - 1)}.
\]

(H3)

Setting equation (H2) equals to equation (H3) and rearranging yields \( \varphi = \frac{\lambda (\lambda - 1)(\varepsilon - 1)}{\lambda - \beta (\lambda - 1)} \), which is the equation reported in the text.

I The link between Inflation and the Lagrange Multiplier on the Collateral constraint

Recall that the labor’s demand function is given by \( A_t p_t = w_t (1 + 2 \mu_t) \) and the real price of an intermediate-good firm equals the real marginal cost of a final-good firm \( p_t = mc_t \). Hence,

\[
mc_t = \frac{w_t (1 + 2 \mu_t)}{A_t}.
\]

(I1)

Also, the labor-supply condition implies \( w_t = \frac{v_{l,t}}{(1 + \tau)u_{c,t}} \). Substituting this result in (I1) gives:

\[
mc_t = \frac{v_{l,t}}{u_{c,t} (1 + \tau)} + \alpha \frac{v_{l,t}}{u_{c,t} (1 + \tau)} \mu_t,
\]

(I2)

and so, the real marginal cost is positively related to \( \mu_t \). This condition suggests the collateral
constraint (represented by \( \mu \) in condition I2) affects inflation through the marginal cost. To see this more explicitly, rewrite the Philips Curve (equation 15 in the text) as

\[
(p_t - 1)p_t = \frac{1 - \epsilon_t}{\phi} + \beta E_t \left[ \frac{u_{t+1}}{u_t} \right] (p_{t+1} - 1)p_{t+1} + \frac{\epsilon_t}{\phi} mc_t,
\]

which shows inflation as an implicit function of the expected future inflation and the current marginal cost. Substituting (I2) in (I3) yields

\[
(p_t - 1)p_t = \frac{1 - \epsilon_t}{\phi} + \beta E_t \left[ \frac{u_{t+1}}{u_t} \right] (p_{t+1} - 1)p_{t+1} + \frac{\epsilon_t}{\phi} \left[ \frac{v_{1,t}}{u_{c,t} (1 + \tau) A_t} \right] + \left[ \frac{\alpha \epsilon_t}{\phi} u_{c,t} (1 + \tau) A_t \right] \mu_t.
\]

Basically, \( \mu_t \) acts as cost-push shock (even when \( \epsilon \) is constant), so that a rise in \( \mu_t \) is associated with an increase in inflation at time \( t \). This is similar to the idea in the log-linearized version of CFP, where \( \mu_t \) manifests itself as an endogenous mark-up shocks.

In the approximated model \( \Omega_t \) replaces \( \mu_t \) wherever it appears.

**J The Equivalence to a Model with Intra-Period Loans**

I show here that there is equivalence between the main setup of the paper and a model where part of the wage bill needs to be paid ahead of production (the standard “working capital” requirement), entrepreneur need to borrow in order pay this part of wages, and the borrowing is constrained by their net worth.

The model is modified in the following way. Households are assumed to lend to entrepreneurs (say through a perfectly-competitive intermediation sector). They deposit \( B_h^t \) in the beginning of period \( t \) and earn an interest rate of \( R_h^t \) in the end of the same period. Their problem will now be

\[
\max E_t \sum \beta^i [u(c_i) - v(l_i)],
\]

subject to the sequence of budget constraints of the form:

\[
P_i c_i + Q_i s_i + B_i + B_h^t = R_{i,1} B_{i-1} + R_h^{t} B_h^{t+1} + P_t (1 + \tau) w_i l_i + s_{t-1} (Q_i + D_i) + P_i \Pi_i + P_i T_i,
\]

where all variables are as in the main test. The households’ budget constraint in real terms reads:

\[
c_i + q_i s_i + b_i + b_h^t = \frac{R_{i,1} b_{i-1}}{\pi_t} + R_h^{t} b_h^{t+1} + (1 + \tau) w_i l_i + s_{t-1} (q_i + d_i) + \Pi_i + T_i
\]

(J3)
The choices of consumption, bonds, labor supply and shares of final-good firms yield the following optimization conditions:

\[
\frac{v_{i,t}}{u_{i,t}} = (1 + \tau)w_i, \quad (J4)
\]

\[
u_{c,t} = \beta R_i E_i \left( \frac{u_{c,t+1}}{\pi_{t+1}} \right), \quad (J5)
\]

\[
u_{l,t} = \beta E_i \left[ u_{c,t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) \right], \quad (J6)
\]

and,

\[R^h_t = 1. \quad (J7)\]

As for entrepreneurs, at the beginning of the period each entrepreneur obtains a loan \(B^e_t\) from households, which is to be paid in the end of the period at a nominal gross interest rate of \(R^e_t\). His borrowing, however, is constrained by the beginning-of-the-period net worth. Formally, an entrepreneur chooses labor, loans and shares to maximize:

\[
E_0 \sum_{t=0}^{\infty} \delta^t \Xi_{0,t} \left[ p_i A_i l_i + b^e_t - R^e_t b^e_t - w_i l_i + e_{t-1} (q_t + d_t) - e_t q_t \right], \quad (J8)
\]

subject to

\[\kappa e_{t-1} (q_t + d_t) - b^e_t \geq 0, \quad (J9)\]

and

\[b^e_t - \alpha w_i l_i \geq 0. \quad (J10)\]

Letting \(\mu_i\) and \(\zeta_i\), denote the Lagrange multiplier on the constraints (J9) and (J10), respectively, the optimality condition with respect to \(b^e_t\) reads:

\[\zeta^e_t = R^e_t + \mu_i - 1. \quad (J11)\]

Similarly, the first order condition with respect to \(l_i\) yields:

\[A_i p_i - w_i (1 + \alpha \zeta_i) = 0. \quad (J12)\]

Finally, the first order condition with respect to \(e_t\) yields

\[\Xi_{0,t} (-q_t) + \delta E_i \left[ \Xi_{0,t+1} (q_{t+1} + d_{t+1}) \right] + \delta E_i \left[ \Xi_{0,t+1} (-\mu_{t+1}) (-\kappa (q_{t+1} + d_{t+1})) \right] = 0. \quad (J13)\]
In equilibrium, the interest rate that households earn on their deposits will be equal to the interest rate that entrepreneurs pay, and hence \( R^e_t = 1 \). Using this fact, equation (J11) becomes:

\[
\zeta_t = \mu_t, \tag{J14}
\]

which, by substituting in (J12) gives

\[
A_t p_t - w_t (1 + \alpha \mu_t) = 0. \tag{J15}
\]

which is exactly as equation (12) in the text. Rearranging condition (J13) gives condition (13) in the text.

**K The Deterministic Steady State**

In this appendix, I present some analytical solutions for the deterministic steady state. The starting point is the assumption that households devote 30 percent of their time for work, and hence \( l \) is set to 0.3 in the SS. In addition, in the absence of shocks, the optimal inflation rate is zero, and hence \( \pi = 1 \). This result can be shown by considering the first-order condition of the optimal Ramsey planner with respect to inflation \( (\pi_t) \) in the deterministic steady state. In this case, this condition reads

\[
\varphi(\lambda_7 + \lambda_8)(\pi - 1)y = 0. \tag{K1}
\]

\( \lambda_7 \) and \( \lambda_8 \) are the Lagrange multipliers on the resource constraint (condition 18) and dividends (equation 19), respectively. Both of these condition holds with equality in the deterministic steady state and hence \( \lambda_7 \) and \( \lambda_8 \) are both positive. Hence, the solution is \( \pi = 1 \).

Imposing deterministic steady state on equation (15) in the text, the deterministic steady state value of \( mc \) equals the inverse of the gross markup (i.e. \( mc = \frac{1}{1 + \frac{\varepsilon - 1}{\varepsilon}} \)). The deterministic steady state value of technology \( (A) \) is set to 1.

Under the assumption that the collateral constraint holds with equality in the deterministic steady state, we have \( \Omega = \frac{1}{\psi} \). By setting \( mc = p \), equation (8) in the text yields \( w = \frac{mc}{1 + \alpha \Omega} \).

Substituting for \( mc \) and \( \Omega \) gives

\[
w = \frac{(\varepsilon - 1)\psi}{\varepsilon(\alpha + \psi)}. \tag{K2}
\]
Imposing SS on equation (17) gives the SS value of dividends \( d = Al(1 - mc) \), which, after substituting for \( A \) and \( mc \), can be written as \( d = \frac{l}{\varepsilon} \).

Equation (8) in the text yields \( q = \frac{\beta}{1 - \beta} d \), and hence

\[
q = \frac{\beta}{1 - \beta} \frac{l}{\varepsilon}.
\]

(K4)

Since the collateral constraint holds with equality in the SS, shares of entrepreneurs can be written as \( e = \frac{\alpha w l}{\kappa(q + d)} \).

After substituting for \( q, d \) and \( w \), we get,

\[
e = \frac{\alpha \psi(1 - \beta)(\varepsilon - 1)}{\kappa(\alpha + \psi)},
\]

(K5)

which is zero when \( \alpha = 0 \). Intuitively, if no wage is required to be backed by collateral, then the entrepreneur has no reason to accumulate assets. Also, the SS value of \( e \) is increasing in \( \psi \), as expected. The higher the curvature parameter in the penalty function the higher the penalty for any violation of the collateral constraint. Hence, in order to avoid occasions where the constraint is violated, the entrepreneur tends to acquire more assets.

Recalling that \( \Omega = \frac{1}{\psi} \), equation (K5) can also be written as

\[
e = \frac{\alpha(1 - \beta)(\varepsilon - 1)}{\kappa(1 + \alpha \Omega)},
\]

(K6)

which implies a negative relationship between \( e \) and \( \Omega \). Intuitively, the more shares entrepreneurs have the more collateral they will have which reduces the value of the \( \Omega \), the equivalent of the Lagrange multiplier.